

Worked Solutions

Q1. i)



Dist in 8th sec = 5.625

$$\Rightarrow S_8 - S_7 = 5.625$$

$$\begin{aligned} S_8 &= ut + \frac{1}{2}at^2 \\ &= (0)(8) + \frac{1}{2}a(8)^2 \\ &= 32a \end{aligned}$$

$$\begin{aligned} S_7 &= ut + \frac{1}{2}at^2 \\ &= (0)(7) + \frac{1}{2}(a)(7)^2 \\ &= \frac{49a}{2} \end{aligned}$$

$$\begin{aligned} S_8 - S_7 &= 32a - \frac{49a}{2} \\ &= \frac{64a}{2} - \frac{49a}{2} \\ &= \frac{15a}{2} \end{aligned}$$

$$\Rightarrow \frac{15a}{2} = 5.625$$

$$15a = 11.25$$

$$a = \frac{11.25}{15} = \boxed{0.75 \text{ m/s}^2}$$

ii) $u = 0$ $t = 20$ $a = 0.75$ $s = ?$

$$s = ut + \frac{1}{2}at^2$$

$$s = (0)(20) + \frac{1}{2}(0.75)(20)^2$$

$$s = \boxed{150\text{m}}$$

Q2

Sec 1:

$$u=0 \quad v=20 \quad a=2 \quad s=?$$

$$v^2 = u^2 + 2as$$

$$(20)^2 = (0)^2 + 2(2)s$$

$$4s = 400$$

$$s = 100 \text{ m}$$

$$v = u + at$$

$$20 = 0 + 2t$$

$$\Rightarrow t_1 = 10 \text{ s}$$

Sec 3:

$$u=20 \quad v=0 \quad a=-4 \quad s=?$$

$$v^2 = u^2 + 2as$$

$$(0)^2 = (20)^2 + 2(-4)s$$

$$8s = 400$$

$$s = 50 \text{ m}$$

$$v = u + at$$

$$0 = 20 - 4t$$

$$t_3 = 5 \text{ s}$$

$$\Rightarrow \text{Time @ constant speed} = 60 \text{ s} - 10 \text{ s} - 5 \text{ s} \\ = 45 \text{ s}$$

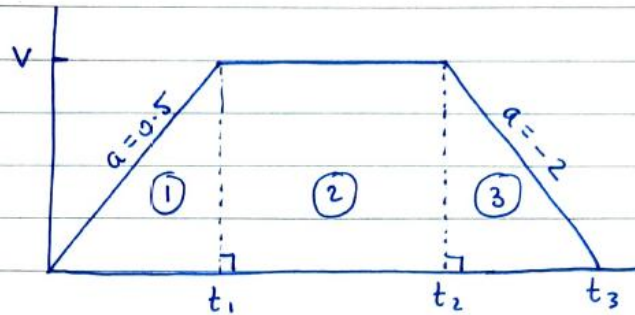
$$\Rightarrow \text{Dist @ constant speed} = \text{Speed} \times \text{Time} \\ = 20 \times 45 \\ = 900 \text{ m}$$

$$\Rightarrow \text{Tot Distance} = 100 + 900 + 50 \\ = 1050 \text{ m}$$

$$\Rightarrow \text{Ave Velocity} = \frac{\text{Tot Dist}}{\text{Tot Time}} \\ = \frac{1050}{60}$$

$$= \boxed{17.5 \text{ m/s}}$$

Q3 i)



ii) Sec 1:

$$u=0 \quad a=0.5 \quad s=256 \quad v=?$$

$$v^2 = u^2 + 2as$$

$$v^2 = (0)^2 + 2(0.5)(256)$$

$$v^2 = 256$$

$$v = \boxed{16 \text{ m/s}}$$

$$v = u + at$$

$$16 = 0 + 0.5(t_1)$$

$$\Rightarrow t_1 = 32 \text{ s}$$

Sec 2

$$\text{Area under graph} = 800 \text{ m}$$

$$16T = 800$$

$$T = 50 \text{ s}$$

Sec 3

$$u=16 \quad v=0 \quad a=-2 \quad t=?$$

$$v = u + at$$

$$0 = 16 - 2t_3$$

$$t_3 = 8 \text{ s}$$

iii) $\text{Tot Time} = 32 + 50 + 8$

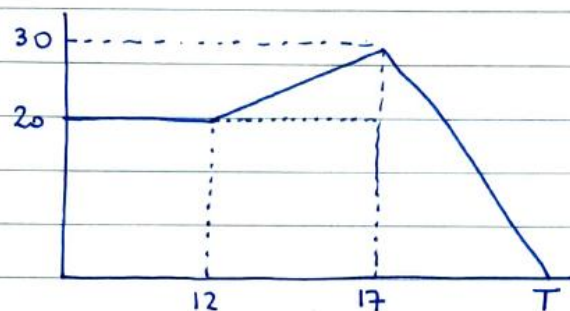
$$= \boxed{90 \text{ s}}$$

iv) $\text{Tot Dist} = \frac{1}{2}(32)(16) + 800 + \frac{1}{2}(8)(16)$

$$= 256 + 800 + 64$$

$$= \boxed{1120 \text{ m}}$$

Q4.



i) $u = 20$ $v = 30$ $a = ?$ $t = 5$

$$v = u + at$$

$$30 = 20 + 5a$$

$$5a = 10$$

$$a = \boxed{2 \text{ m/s}^2}$$

ii) $u = 30$ $v = 0$ $s = 75$ $a = ?$ } $u = 30$ $v = 0$ $a = -6$ $t = ?$

$$v^2 = u^2 + 2as$$

$$(0)^2 = (30)^2 + 2(a)(75)$$

$$150a = -900$$

$$a = -6 \text{ m/s}^2$$

$$\Rightarrow \text{dec} = \boxed{6 \text{ m/s}^2}$$

$$v = u + at$$

$$0 = 30 - 6t$$

$$\Rightarrow t = 5 \text{ s}$$

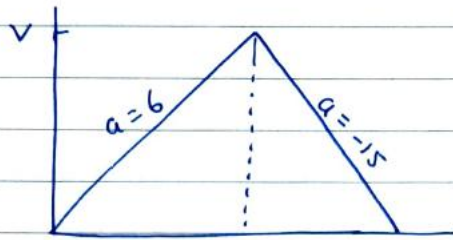
iii) Area under Graph = $(12)(20) + (5)(20) + \frac{1}{2}(5)(10) + \frac{1}{2}(5)(30)$
 $= 240 + 100 + 25 + 75$
 $= \boxed{440 \text{ m}}$

iv) Ave Speed = $\frac{\text{Tot Dist}}{\text{Tot Time}}$

$$= \frac{440}{22}$$

$$= \boxed{20 \text{ m/s}}$$

Q5

Method 1:

Sec 1: $u=0$ $a=6$ $v=v$ $t=t_1$ Sec 2: $u=v$ $v=0$ $a=-15$

$$v = u + at$$

$$v = 6t_1$$

$$\Rightarrow t_1 = \frac{v}{6}$$

$$v = u + at$$

$$0 = v - 15t_2$$

$$t_2 = \frac{v}{15}$$

$$\Rightarrow \text{Tot Time} = \frac{v}{6} + \frac{v}{15} = \frac{7v}{30}$$

$$\Rightarrow \text{Area Under} = \frac{1}{2} \left(\frac{7v}{30} \right) (v) = 1000$$

$$\Rightarrow 7v^2 = 60000$$

$$\Rightarrow v^2 = \frac{60000}{7}$$

$$\Rightarrow v = \sqrt{\frac{60000}{7}} = 92.582$$

$$\Rightarrow \text{Tot Time} = \frac{7(92.582)}{30}$$

$$= \boxed{21.6 \text{ s}}$$

Method 2:

$$a:d = 6:15$$

$$\Rightarrow t_1:t_2 = 15:6 = 5:2$$

$$\Rightarrow t_1 = \frac{5}{7}T \quad t_2 = \frac{2}{7}T$$

Sec 1:

$$u=0 \quad v=v \quad t = \frac{5}{7}T \quad a=6$$

$$v = 0 + 6 \left(\frac{5}{7}T \right)$$

$$v = \frac{30T}{7}$$

$$\text{Area} = \frac{1}{2} \left(\frac{30T}{7} \right) (T) = 1000$$

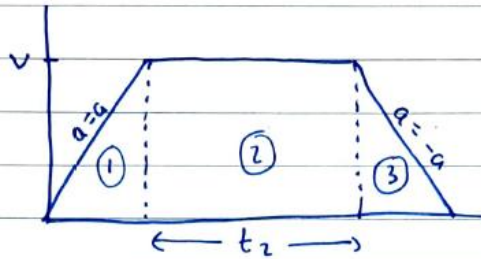
$$\Rightarrow 30T^2 = 14000$$

$$\Rightarrow T^2 = \frac{14000}{30}$$

$$\Rightarrow T = \sqrt{\frac{14000}{30}}$$

$$= \boxed{21.6 \text{ s}}$$

Q6.



i) Sec 1

$$u=0 \quad v=v \quad a=a \quad t=t_1$$

$$v = u + at$$

$$v = at_1$$

$$\Rightarrow t_1 = \frac{v}{a}$$

Sec 3

$$u=v \quad v=0 \quad a=-a \quad t=t_3$$

$$v = u + at$$

$$0 = v - at_3$$

$$\Rightarrow t_3 = \frac{v}{a}$$

$$\Rightarrow \text{Total Time} = t = \frac{v}{a} + \frac{v}{a} + t_2$$

$$\Rightarrow at = 2v + at_2$$

$$2v = at - at_2$$

$$v = \frac{a}{2}(t - t_2)$$

Q.E.D.

ii) Area = s

$$\Rightarrow \frac{1}{2} \left(\frac{v}{a} \right) (v) + t_2 (v) + \frac{1}{2} \left(\frac{v}{a} \right) (v) = s$$

$$\frac{v^2}{2a} + vt_2 + \frac{v^2}{2a} = s$$

$$2v^2 + 2avt_2 = 2as$$

$$v^2 + avt_2 = as$$

$$\left(\frac{a}{2}(t - t_2) \right)^2 + at_2 \left(\frac{a}{2}(t - t_2) \right) = as$$

$$\frac{a^2}{4}(t^2 + t_2^2 - 2tt_2) + \frac{a^2 t_2}{2}(t - t_2) = as$$

$$at^2 + at_2^2 - 2tt_2a + 2at_2(t - t_2) = 4s \quad (\div a \text{ and } \times 4)$$

$$at^2 + at_2^2 - 2\cancel{tt_2a} + 2\cancel{at_2t} - 2at_2^2 = 4s$$

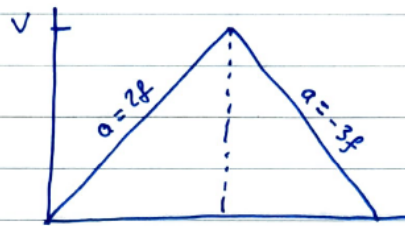
$$at^2 - at_2^2 = 4s$$

$$at_2^2 = at^2 - 4s$$

$$t_2^2 = \frac{at^2 - 4s}{a}$$

$$\Rightarrow t_2 = \sqrt{\frac{at^2 - 4s}{a}}$$

Q7



Sec 1

$$u=0 \quad v=v \quad a=2f \quad t=t_1$$

$$v = u + at$$

$$v = 2ft_1$$

$$\Rightarrow t_1 = \frac{v}{2f}$$

Sec 2

$$u=v \quad v=0 \quad a=-3f \quad t=t_2$$

$$v = u + at$$

$$0 = v - 3ft_2$$

$$\Rightarrow t_2 = \frac{v}{3f}$$

$$\Rightarrow \text{Total Time} = \frac{v}{2f} + \frac{v}{3f}$$

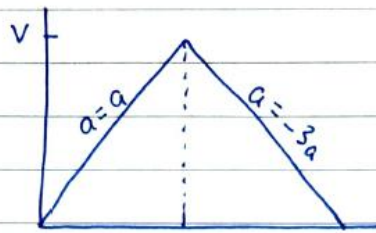
$$t = \frac{5v}{6f} \Rightarrow v = \frac{6ft}{5}$$

$$\Rightarrow \text{Area} = \frac{1}{2} \left(t \right) \left(\frac{6ft}{5} \right) = h$$

$$6ft^2 = 10h$$

$$\Rightarrow h = \frac{6ft^2}{10} = \boxed{\frac{3ft^2}{5}}$$

Q8.



$$a : d = 1 : 3$$

$$\Rightarrow t_1 : t_2 = 3 : 1$$

$$\Rightarrow t_1 = \frac{3}{4}T \quad t_2 = \frac{1}{4}T$$

Sec 1

$$u = 0 \quad v = v \quad a = a \quad t = \frac{3}{4}T$$

$$v = u + at$$

$$v = a\left(\frac{3}{4}T\right)$$

$$v = \frac{3aT}{4}$$

$$\text{Area Under Graph} = \frac{1}{2}(T)\left(\frac{3aT}{4}\right) = s$$

$$\Rightarrow \frac{3aT^2}{8} = s$$

$$\Rightarrow 3aT^2 = 8s : I$$

$$\text{Ave Speed} = \frac{\text{Tot Dist}}{\text{Tot Time}} = \frac{s}{T}$$

$$\Rightarrow \frac{\frac{3aT^2}{8}}{T} = \sqrt{\frac{s}{2}}$$

$$\Rightarrow \left(\frac{3aT}{8}\right)^2 = \left(\sqrt{\frac{s}{2}}\right)^2$$

$$\frac{9a^2T^2}{64} = \frac{s}{2}$$

$$18a^2T^2 = 64s$$

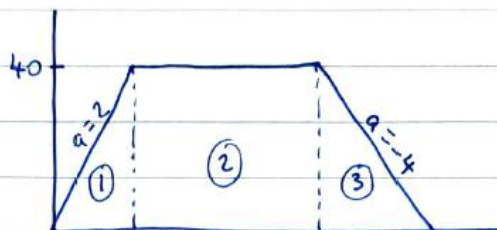
$$6a(3aT^2) = 64s \quad (\text{Using I})$$

$$6a(8s) = 64s$$

$$48as = 64s$$

$$\Rightarrow a = \frac{64}{48} = \boxed{\frac{4}{3} \text{ m/s}^2}$$

Q9. i)



Sec 1

$$u=0 \quad v=40 \quad a=2 \quad t=t_1$$

$$v = u + at$$

$$40 = 2t_1$$

$$t_1 = 20s$$

Sec 3

$$u=40 \quad v=0 \quad a=-4 \quad t=t_2$$

$$v = u + at$$

$$0 = 40 - 4t_2$$

$$t_2 = 10s$$

Area Under Graph = 1000m

$$\Rightarrow \frac{1}{2}(20)(40) + (t_2)(40) + \frac{1}{2}(10)(40) = 1000$$

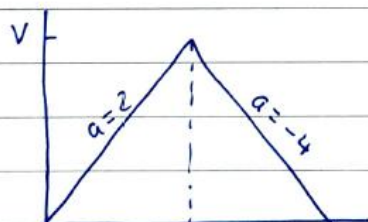
$$400 + 40t_2 + 200 = 1000$$

$$40t_2 = 400$$

$$t_2 = 10s$$

$$\Rightarrow \text{Tot Time} = 20 + 10 + 10 = \boxed{40s}$$

ii)



$$a:d = 1:2$$

$$\Rightarrow t_1:t_2 = 2:1$$

$$\Rightarrow t_1 = \frac{2}{3}T \quad t_2 = \frac{1}{3}T$$

Sec 1

$$u=0 \quad v=v \quad a=2 \quad t=\frac{2}{3}T$$

$$v = u + at$$

$$v = 2\left(\frac{2T}{3}\right)$$

$$v = \frac{4T}{3}$$

Area Under = 384

$$\frac{1}{2}(T)\left(\frac{4T}{3}\right) = 384$$

$$\frac{4T^2}{6} = 384$$

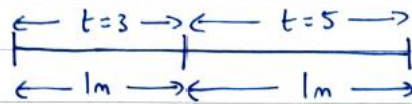
$$4T^2 = 2304$$

$$T^2 = 576$$

$$T = \sqrt{576}$$

$$= \boxed{24s}$$

Q10.



Sec 1:

$$u = u \quad a = a \quad t = 3 \quad s = 1$$

$$s = ut + \frac{1}{2}at^2$$

$$1 = 3u + \frac{1}{2}a(3)^2$$

$$\Rightarrow 6u + 9a = 2 \quad \text{I}$$

Sec 1 & 2:

$$u = u \quad a = a \quad t = 8 \quad s = 2$$

$$s = ut + \frac{1}{2}at^2$$

$$2 = 8u + \frac{1}{2}a(8)^2$$

$$\Rightarrow 8u + 32a = 2 \quad \text{II}$$

Solving I & II:

$$\text{I} \times 4: 24u + 36a = 8$$

$$\text{II} \times -3: -24u - 96a = -6$$

$$-60a = 2$$

$$a = \boxed{-\frac{1}{30} \text{ m/s}^2}$$

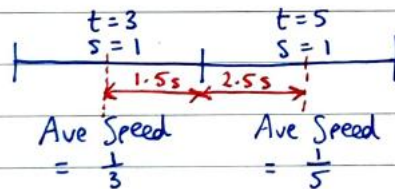
$$\text{I}: 6u + 9\left(-\frac{1}{30}\right) = 2$$

$$6u = 2 + \frac{3}{10}$$

$$6u = \frac{23}{10}$$

$$u = \boxed{\frac{23}{60} \text{ m/s}}$$

OR



$$u = \frac{1}{3} \quad v = \frac{1}{5} \quad t = 1.5 + 2.5 = 4s \quad a = ?$$

$$v = u + at$$

$$\frac{1}{5} = \frac{1}{3} + 4a$$

$$4a = \frac{1}{5} - \frac{1}{3}$$

$$4a = -\frac{2}{15}$$

$$a = \boxed{-\frac{1}{30} \text{ m/s}^2}$$

Remake equation I from method 1 and solve for

$$u = \boxed{\frac{23}{60} \text{ m/s}}$$

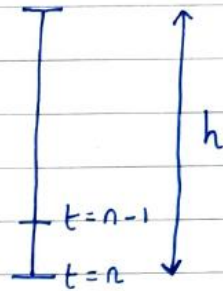
Q11. In n secs:

$$u=0 \quad a=g \quad t=n \quad s=h$$

$$s = ut + \frac{1}{2}at^2$$

$$h = 0 + \frac{1}{2}(g)(n)^2$$

$$h = \frac{1}{2}gn^2 \quad : I$$



In $n-1$ secs:

$$u=0 \quad a=g \quad t=n-1 \quad s = \frac{3}{4}h$$

$$s = ut + \frac{1}{2}at^2$$

$$\frac{3}{4}h = 0 + \frac{1}{2}(g)(n-1)^2$$

$$\frac{3}{4}h = \frac{g}{2}(n^2 - 2n + 1)$$

$$3h = 2g(n^2 - 2n + 1) \quad (\times 4)$$

$$3h = 2gn^2 - 4gn + 2g \quad : II$$

Put I into II:

$$3\left(\frac{1}{2}gn^2\right) = 2gn^2 - 4gn + 2g$$

$$3gn^2 = 4gn^2 - 8gn + 4g$$

$$gn^2 - 8gn + 4g = 0$$

$$n^2 - 8n + 4 = 0$$

$$n = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{48}}{2}$$

$$= 4 \pm 2\sqrt{3}$$

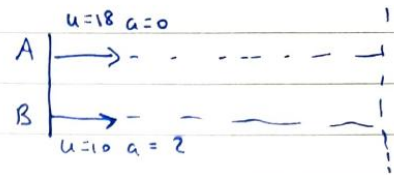
$$\Rightarrow \text{Using I: } h = \frac{1}{2}g(4-2\sqrt{3})^2 \quad \text{or} \quad h = \frac{1}{2}g(4+2\sqrt{3})^2$$
$$= 1.4\text{m} \qquad \qquad \qquad = 273\text{m}$$

Height can't be 1.4m as stone would fall more than this height in 1s

$$\Rightarrow \text{Ans} = \boxed{273\text{m}}$$

Q12.

i) Let T = time to overtake



$$S_A = 18T + \frac{1}{2}(0)T^2 \\ = 18T$$

$$S_B = 10(T) + \frac{1}{2}(2)(T^2) \\ = 10T + T^2$$

Overtaking $\Rightarrow S_A = S_B$

$$18T = 10T + T^2$$

$$T^2 - 8T = 0$$

$$T(T-8) = 0$$

$$T=0 \text{ or } T = \boxed{8s}$$

$$\Rightarrow S_A = 18(8)$$

$$= \boxed{144m}$$

ii) $V_A = 18$ $V_B = 10 + 2T$

Greatest Dist $\Rightarrow V_A = V_B$

$$18 = 10 + 2T$$

$$2T = 8$$

$$T = 4$$

$$S_A = 18(4) = 72$$

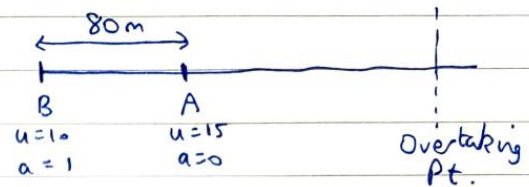
$$S_B = 10(4) + (4)^2 \\ = 56m$$

$$\Rightarrow \text{Furthest ahead} = 72 - 56$$

$$= \boxed{16m}$$

Q13.

i) Let T = time to overtake



$$S_A = 15T + \frac{1}{2}(0)T^2$$

$$= 15T$$

$$S_B = 10T + \frac{1}{2}(1)T^2$$

$$= 10T + 0.5T^2$$

$$S_A + 80 = S_B$$

$$\Rightarrow 15T + 80 = 10T + 0.5T^2$$

$$0.5T^2 - 5T - 80 = 0$$

$$T^2 - 10T - 160 = 0$$

$$T = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-160)}}{2(1)}$$

$$= 18.6 \text{ or } -8.6$$

$$\Rightarrow S_A = 15(18.6)$$

$$= 279\text{m}$$

$\Rightarrow$ Brian passes him 29m beyond finishing line

ii) Time for Brian to finish:

$$80 + 250 = 10t_1 + \frac{1}{2}(1)t_1^2$$

$$\frac{1}{2}t_1^2 + 10t_1 - 330 = 0$$

$$t_1^2 + 20t_1 - 660 = 0$$

$$t_1 = \frac{-20 \pm \sqrt{(20)^2 - 4(1)(-660)}}{2(1)}$$

$$= 17.57\text{s or } -37.6$$

Time for Alan to finish:

$$250 = 15t_2$$

$$t_2 = 16.67\text{s}$$

$$\Rightarrow 17.57 - 16.67 = \boxed{0.9\text{s}}$$

Q14.



$$u = 20 \quad a = -g \quad t = 5.5 \quad s = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$s = 20(5.5) + \frac{1}{2}(-g)(5.5)^2$$

$$= -38.225$$

$$\Rightarrow h = \boxed{38.225 \text{ m}}$$

Q15. i) $u = 0 \quad t = 20 \quad a = 0.25 \quad s = ?$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(0.25)(20)^2$$

$$s = \boxed{50 \text{ m}}$$

$$v = u + at$$

$$v = 0 + 0.25(20)$$

$$v = \boxed{5 \text{ m/s}}$$

ii) $u = 5 \quad a = g \quad s = 50 \quad t = ?$

$$s = ut + \frac{1}{2}at^2$$

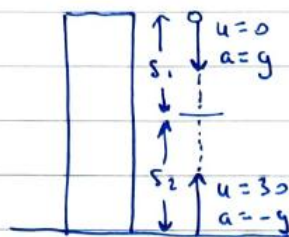
$$50 = 5(t) + \frac{1}{2}(g)(t)^2$$

$$\Rightarrow 4.9t^2 + 5t - 50 = 0$$

$$t = \frac{-5 \pm \sqrt{(5)^2 - 4(4.9)(-50)}}{2(4.9)}$$

$$= \boxed{3.75 \text{ s}}$$

Q16.



$$s_1 = ut + \frac{1}{2}at^2$$

$$= 0(4) + \frac{1}{2}(g)(4)^2$$

$$= 78.4\text{m}$$

$$s_2 = ut + \frac{1}{2}at^2$$

$$= 30(4) + \frac{1}{2}(-g)(4)^2$$

$$= 41.6\text{m}$$

$$\Rightarrow h = 78.4 + 41.6$$

$$= \boxed{120\text{m}}$$

Q17.



Let T = time to collide

$$s_1 = ut + \frac{1}{2}at^2$$

$$= 25(T) + \frac{1}{2}(-g)(T)^2$$

$$= 25T - 4.9T^2$$

$$s_2 = ut + \frac{1}{2}at^2$$

$$= 20(T-3) + \frac{1}{2}(-g)(T-3)^2$$

$$= 20T - 60 - 4.9(T^2 - 6T + 9)$$

$$= 20T - 60 - 4.9T^2 + 29.4T - 44.1$$

$$= -4.9T^2 + 49.4T - 104.1$$

Collide $\Rightarrow s_1 = s_2$

$$25T - 4.9T^2 = -4.9T^2 + 49.4T - 104.1$$

$$-25T + 49.4T = 104.1$$

$$24.4T = 104.1$$

$$T = 4.266\text{s}$$

$$\Rightarrow s_1 = 25(4.266) - 4.9(4.266)^2$$

$$= \boxed{17.5\text{m}}$$

Q18.

$$\begin{array}{l} u=0 \\ a=g \end{array} \quad \begin{array}{l} u=\sqrt{8ga} \\ a=g \end{array}$$

Time for particle 1 to travel 'a':

$$u=0 \quad a=g \quad s=a \quad t=?$$

$$s = ut + \frac{1}{2}at^2$$

$$a = 0(t) + \frac{1}{2}(g)(t)^2$$

$$gt^2 = 2a$$

$$t^2 = \frac{2a}{g}$$

$$\Rightarrow t = \sqrt{\frac{2a}{g}}$$

Particle 2: Let T = time to collide

$$u = \sqrt{8ga} \quad a = g \quad t = T - \sqrt{\frac{2a}{g}} \quad s = ?$$

$$s_2 = ut + \frac{1}{2}at^2$$

$$s_2 = \sqrt{8ga} \left(T - \sqrt{\frac{2a}{g}} \right) + \frac{1}{2}(g) \left(T - \sqrt{\frac{2a}{g}} \right)^2$$

$$= T\sqrt{8ga} - 4a + \frac{g}{2} \left(T^2 - 2\sqrt{\frac{2a}{g}}T + \frac{2a}{g} \right)$$

$$= T\sqrt{8ga} - 4a + \frac{1}{2}gT^2 - g\sqrt{\frac{2a}{g}}T + a$$

$$= 2T\sqrt{2ga} - 4a + \frac{1}{2}gT^2 - g\sqrt{\frac{2a}{g}}T + a$$

Particle 1:

$$u=0 \quad t=T \quad s=? \quad a=g$$

$$s_1 = ut + \frac{1}{2}at^2$$

$$s_1 = \frac{1}{2}gT^2$$

$$s_1 = s_2$$

$$\Rightarrow \frac{1}{2}gT^2 = 2T\sqrt{2ga} - 4a + \frac{1}{2}gT^2 - g\sqrt{\frac{2a}{g}}T + a$$

$$\Rightarrow 0 = 2T\sqrt{2ga} - 3a - T\sqrt{2ga}$$

$$\Rightarrow T\sqrt{2ga} = 3a$$

$$\Rightarrow T = \frac{3a}{\sqrt{2ga}}$$

$$\Rightarrow s_1 = \frac{g}{2} \left(\frac{3a}{\sqrt{2ga}} \right)^2 = \frac{9a^2g}{2(2ag)} = \boxed{\frac{9a}{4}}$$

$$\begin{aligned} g\sqrt{\frac{2a}{g}} &= \sqrt{g}\sqrt{g}\sqrt{\frac{2a}{g}} \\ &= \frac{\sqrt{g}\sqrt{g}\sqrt{2}\sqrt{a}}{\sqrt{g}} \\ &= \sqrt{2ga} \end{aligned}$$

2010

Q1

a) i) $u=14$ $v=0$ $s=98$

$$v^2 = u^2 + 2as$$

$$(0)^2 = (14)^2 + 2a(98)$$

$$196a = -196$$

$$a = -1$$

$$\Rightarrow \text{dec} = \boxed{1 \text{ m/s}^2}$$

ii) Dist in 1 sec = $14(1) = 14\text{m}$

$\Rightarrow 84\text{m}$ from lights now

$$v^2 = u^2 + 2as$$

$$(0)^2 = (14)^2 + 2a(84)$$

$$a = -7/6$$

$$\Rightarrow \text{dec} = \boxed{7/6 \text{ m/s}^2}$$

b)



In 1st second:

$$u=20 \quad s=25 \quad t=1$$

$$s = ut + \frac{1}{2}at^2$$

$$25 = 20(1) + \frac{1}{2}a(1)^2$$

$$\Rightarrow a = 10 \text{ m/s}^2$$

Before last 3 secs:

$$u=20 \quad s = \frac{7}{20}x \quad a=10$$

$$v^2 = u^2 + 2as$$

$$v^2 = (20)^2 + 2\left(\frac{7}{20}x\right)$$

$$v^2 = 400 + 7x$$

In last 3 secs:

$$u = \sqrt{400+7x} \quad a=10 \quad s = \frac{13}{20}x \quad t=3$$

$$s = ut + \frac{1}{2}at^2$$

$$\frac{13}{20}x = 3\sqrt{400+7x} + \frac{1}{2}(10)(3)^2$$

$$\frac{13x}{20} - 45 = 3\sqrt{400+7x}$$

$$(13x - 900)^2 = (60\sqrt{400+7x})^2$$

$$169x^2 - 23400x + 81000 = 1440000 + 25200x$$

$$169x^2 - 48600x - 630000 = 0$$

$$x = \frac{48600 \pm \sqrt{(48600)^2 - 4(169)(-630000)}}{2(169)}$$

$$= \boxed{300\text{m}}$$

2011

Q1

a)



A → B

$$u=0 \quad a=g \quad t=t$$

$$v=u+at$$

$$v=0+gt$$

$$v=gt$$

B → C

$$u=gt \quad a=g \quad t=\frac{2t}{7} \quad s=2.45$$

$$s=ut + \frac{1}{2}at^2$$

$$2.45 = gt\left(\frac{2t}{7}\right) + \frac{1}{2}(g)\left(\frac{2t}{7}\right)^2$$

$$2.45 = \frac{2gt^2}{7} + \frac{2gt^2}{49}$$

$$2.45 = \frac{16gt^2}{49}$$

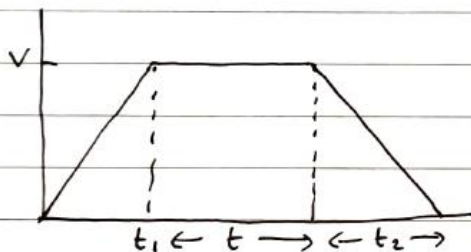
$$16gt^2 = 120.05$$

$$t^2 = \frac{49}{64}$$

$$t = \sqrt{\frac{49}{64}} = \boxed{\frac{7}{8} \text{ s}}$$

b)

i)



ii) Ave Speed = $\frac{\text{Tot Dist}}{\text{Tot Time}}$

$$\begin{aligned} \text{Tot Dist} &= \text{Area Under Curve} = \frac{1}{2}(t_1)(v) + tv + \frac{1}{2}(t_2)(v) \\ &= \frac{vt_1 + 2vt + vt_2}{2} \end{aligned}$$

$$\text{Tot Time} = t_1 + t_2 + t_3$$

$$\Rightarrow \text{Ave Speed} = \frac{\frac{v(t_1 + 2t + t_2)}{2}}{t_1 + t_2 + t_3} = \frac{3v}{4}$$

$$\Rightarrow 2\cancel{v}t_1 + 4\cancel{v}t + 2\cancel{v}t_2 = 3\cancel{v}t_1 + 3\cancel{v}t_2 + 3t_3\cancel{v}$$

$$\Rightarrow t_1 + t_2 = \boxed{t}$$

2015

Q1

a) Dist in 7th sec = $S_7 - S_6$

$$\begin{aligned} S_6 &= ut + \frac{1}{2}at^2 \\ &= (0)(6) + \frac{1}{2}a(6)^2 \\ &= 18a \end{aligned}$$

$$\begin{aligned} S_7 &= ut + \frac{1}{2}at^2 \\ &= (0)(7) + \frac{1}{2}a(7)^2 \\ &= \frac{49}{2}a \end{aligned}$$

$$\Rightarrow \frac{49}{2}a - 18a = 39$$

$$49a - 36a = 78$$

$$13a = 78$$

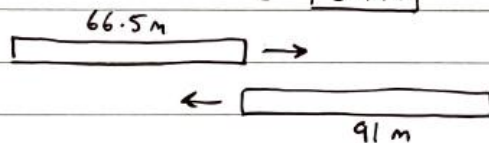
$$a = 6 \text{ m/s}^2$$

$$\begin{aligned} \text{Dist in } 10^{\text{th}} \text{ sec} &= S_{10} - S_9 \\ &= (0)(10) + \frac{1}{2}(6)(10)^2 - \left[(0)(9) + \frac{1}{2}(6)(9)^2 \right] \end{aligned}$$

$$= 300 - 243$$

$$= \boxed{57 \text{ m}}$$

b)



$$i) S_1 = 18t + \frac{1}{2}\left(\frac{4}{7}\right)t^2 = 18t + \frac{2}{7}t^2$$

$$S_2 = 24t + \frac{1}{2}\left(\frac{8}{7}\right)t^2 = 24t + \frac{4}{7}t^2$$

$$S_1 + S_2 = 66.5 + 91 = 157.5$$

$$\Rightarrow 18t + \frac{2}{7}t^2 + 24t + \frac{4}{7}t^2 = 157.5$$

$$126t + 2t^2 + 168t + 4t^2 = 1102.5$$

$$12t^2 + 588t - 2205 = 0$$

$$t = \boxed{3.5 \text{ s}} \quad \text{or } \ominus$$

$$ii) \text{ After } 4.5 \text{ s: } S_1 = 18(4.5) + \frac{2}{7}(4.5)^2 = \frac{1215}{14}$$

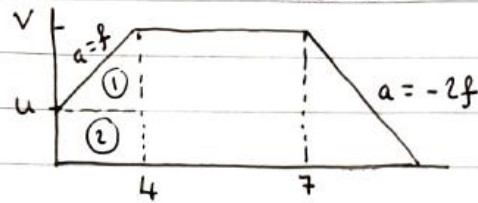
$$S_2 = 24(4.5) + \frac{4}{7}(4.5)^2 = \frac{837}{7}$$

$$\begin{aligned} \Rightarrow \text{Dist between} &= \frac{1215}{14} + \frac{837}{7} - 157.5 \\ &= \boxed{48.86 \text{ m}} \end{aligned}$$

2016

Q1

a) i)



ii) 45m in 3 secs

$$\Rightarrow v = 45/3 = 15 \text{ m/s}$$

$$\text{Area ① + ②} = 40$$

$$4u + \frac{1}{2}(4)(15-u) = 40$$

$$4u + 30 - 2u = 40$$

$$2u = 10$$

$$u = \boxed{5 \text{ m/s}}$$

iii) $u = 5$ $v = 15$ $t = 4$

$$\Rightarrow f = 5/2$$

Sec 3

$$u = 15$$
 $v = 0$ $a = -5$

$$\Rightarrow s = 22.5 \quad (v^2 = u^2 + 2as)$$

$$\Rightarrow \text{Dist} = 40 + 45 + 22.5$$

$$= \boxed{107.5 \text{ m}}$$

b) Particle 1

$$u = u \quad a = -g \quad t = T$$

$$s_1 = uT - \frac{1}{2}gT^2$$

Particle 2

$$u = u \quad a = -g \quad t = T - 2t$$

$$s_2 = u(T - 2t) - \frac{1}{2}g(T - 2t)^2$$

$$= uT - 2ut - \frac{1}{2}gT^2 - 2gt^2 + 2gtT$$

$$s_1 = s_2$$

$$uT - \frac{1}{2}gT^2 = uT - 2ut - \frac{1}{2}gT^2 - 2gt^2 + 2gtT$$

$$2ut + 2gt^2 = 2gtT$$

$$\Rightarrow T = \frac{u + gt}{g}$$

$$\Rightarrow h = u\left(\frac{u + gt}{g}\right) - \frac{g}{2}\left(\frac{u + gt}{g}\right)^2$$

$$= \frac{u^2 + ugt}{g} - \frac{g}{2}\left(\frac{u^2 + 2ugt + g^2t^2}{g^2}\right)$$

$$= \frac{2u^2 + 2ugt}{2g} - \frac{u^2 + 2ugt + g^2t^2}{2g}$$

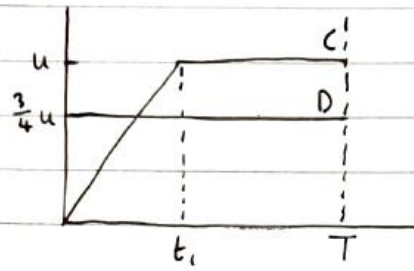
$$= \boxed{\frac{u^2 - g^2t^2}{g}}$$

Q.E.D.

2018

Q1

b) i)



Let T = time to intercept

$$u=0 \quad a=a \quad v=u \quad t=t$$

$$v = u + at$$

$$u = at_1$$

$$\Rightarrow t_1 = u/a$$

$$\begin{aligned} \text{Dist travelled by C} &= \frac{1}{2} \left(\frac{u}{a} \right) (u) + \left(T - \frac{u}{a} \right) (u) \\ &= \frac{u^2}{2a} + uT - \frac{u^2}{2a} \\ &= uT - \frac{u^2}{2a} \end{aligned}$$

$$\begin{aligned} \text{Dist travelled by D} &= \frac{3}{4} uT \\ \Rightarrow uT - \frac{u^2}{2a} &= \frac{3}{4} uT \end{aligned}$$

$$\frac{1}{4} uT = \frac{u^2}{2a}$$

$$T = \frac{2u}{a}$$

$$\begin{aligned} \Rightarrow d &= u \left(\frac{2u}{a} \right) - \frac{u^2}{2a} \\ &= \frac{2u^2}{a} - \frac{u^2}{2a} \end{aligned}$$

$$\Rightarrow d = \boxed{\frac{3u^2}{2a}}$$

$$\Rightarrow 3u^2 = 2ad$$

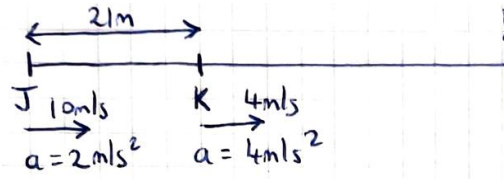
$$\Rightarrow a = \frac{3u^2}{2d}$$

$$\Rightarrow T = \frac{2u}{\frac{3u^2}{2d}} = \frac{2u}{1} \times \frac{2d}{3u^2} = \frac{4d}{3u}$$

$$\Rightarrow \text{Time @ Speed } u = T - t_1$$

$$= \boxed{\frac{4d}{3u} - \frac{u}{a}}$$

Q.E.D.

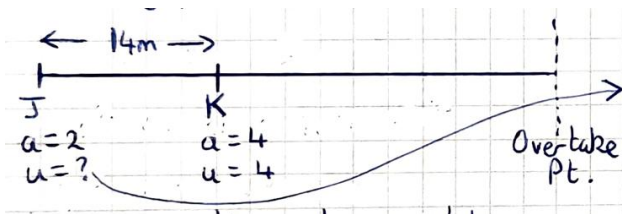
Q1(c)

In 1 second:

$$s_J = 10(1) + \frac{1}{2}(2)(1)^2 = 11m$$

$$s_K = 4(1) + \frac{1}{2}(0)(1)^2 = 4m$$

$\Rightarrow$ John gains 7 m on Kevin and gap reduces to 14 m



Speed of John after 1 sec:

$$v_J = u + at$$

$$= 10 + 2(1) = 12 \text{ m/s}$$

Let T = time to overtake

$$s_J = 12(T) + \frac{1}{2}(2)(T)^2$$

$$s_K = 4(T) + \frac{1}{2}(4)(T)^2$$

$$s_J = 12T + T^2$$

$$s_K = 4T + 2T^2$$

From 2nd diagram above: $s_K + 14 = s_J$

$$\Rightarrow 4T + 2T^2 + 14 = 12T + T^2$$

$$\Rightarrow T^2 - 8T + 14 = 0$$

$$\Rightarrow T = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)}$$

$$\Rightarrow T = 5.41 \text{ s or } T = 2.59 \text{ s}$$

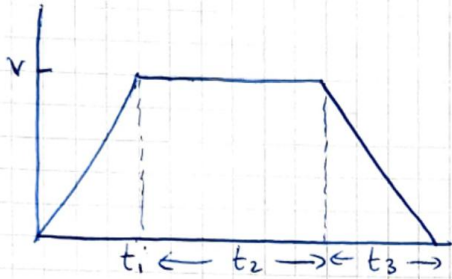
$$\Rightarrow \text{Ans: } T = 6.41 \text{ s or } T = 3.59 \text{ s}$$

Q5(b)

- (b) A train departs from Connolly Station and accelerates uniformly from rest for t_1 seconds until it reaches a speed of v . The train maintains this speed for t_2 seconds, where $t_2 = 2t_1$. The train then decelerates to rest at Pearse Station in a time of t_3 seconds.

The total time taken for the journey is $T = t_1 + t_2 + t_3$.

- (i) Show that $T = \frac{2d - t_2 v}{v}$, where d is the distance between Connolly Station and Pearse Station.



$$\begin{aligned} t_2 &= 2t_1 \Rightarrow t_1 = \frac{t_2}{2} \\ T &= t_1 + t_2 + t_3 \\ \Rightarrow t_3 &= T - t_1 - t_2 \\ \Rightarrow t_3 &= T - \frac{t_2}{2} - t_2 \\ \Rightarrow t_3 &= T - \frac{t_2}{2} - \frac{2t_2}{2} \\ \Rightarrow t_3 &= T - \frac{3t_2}{2} \end{aligned}$$

Total Distance $d = \text{Area under graph}$

$$\begin{aligned} \Rightarrow d &= \frac{1}{2}(t_1)(v) + (t_2)(v) + \frac{1}{2}(t_3)(v) \\ \Rightarrow d &= \frac{1}{2}v\left(\frac{t_2}{2}\right) + vt_2 + \frac{1}{2}v\left(T - \frac{3t_2}{2}\right) \\ \Rightarrow 4d &= vt_2 + 4vt_2 + 2vT - 3vt_2 \\ \Rightarrow 4d &= 2vt_2 + 2vT \\ \Rightarrow 2d &= vt_2 + vT \\ \Rightarrow vT &= 2d - vt_2 \\ \Rightarrow T &= \frac{2d - vt_2}{v} \end{aligned}$$

Q.E.D.

- (ii) If the average speed for the entire journey is $\frac{2v}{3}$, show that $T = 6t_1$.

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{2v}{3}$$

$$\Rightarrow \frac{d}{T} = \frac{2v}{3}$$

$$\Rightarrow 3d = 2vT$$

$$\Rightarrow 3d = 2v\left(\frac{2d - vt_2}{v}\right)$$

$$\Rightarrow 3d = 4d - 2vt_2$$

$$\Rightarrow 2vt_2 = d$$

$$\Rightarrow t_2 = \frac{d}{2v}$$

Subbing into expression from part (i):

$$T = \frac{2d - v\left(\frac{d}{2v}\right)}{v} = \frac{\frac{3d}{2}}{v} = \frac{3d}{2v}$$

$$\text{We know } t_1 = \frac{t_2}{2} = \frac{\frac{d}{2v}}{2} = \frac{d}{4v}$$

$$\Rightarrow 6t_1 = 6\left(\frac{d}{4v}\right) = \frac{3d}{2v}$$

$$\Rightarrow 6t_1 = T \quad \text{Q.E.D.}$$