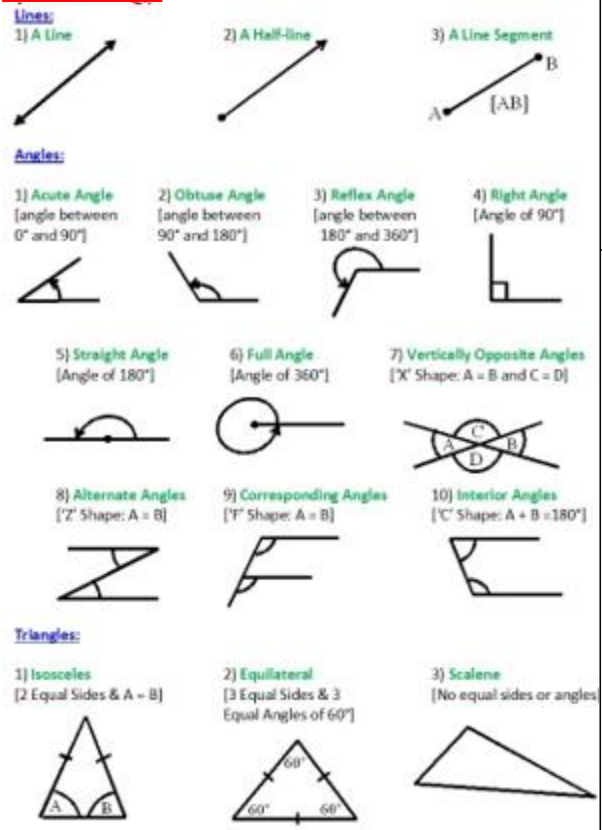
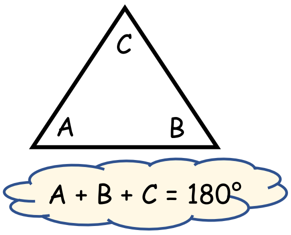
Topic 12: Geometry

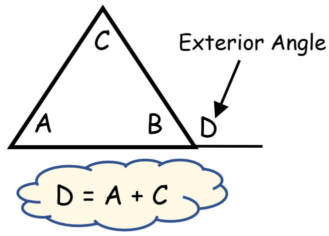
1) The Basics:

a) Terminology:

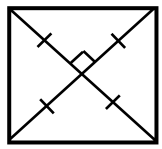


b) Properties of Triangles:

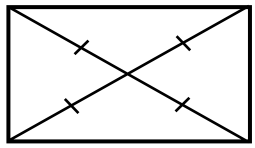




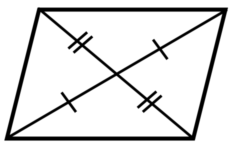
c) Properties of Quadrilaterals:



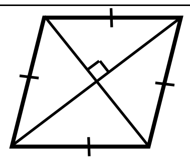
Square
- 4 equal sides
- 4 equal angles of 90°
- Opp sides are parallel
- Diagonals bisect at 90° angles



Rectangle
- Opp sides are equal & parallel
- 4 equal angles of 90°
- Diagonals bisect each other



Parallelogram
- Opp sides are equal & parallel
- Opp angles are equal
- Diagonals bisect each other

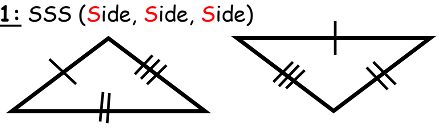


Rhombus
- 4 equal sides & opp sides are parallel
- Opp angles are equal
- Diagonals bisect at 90° angles

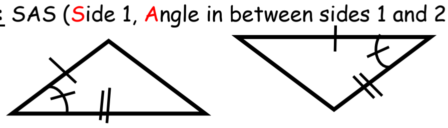
d) Conguent Triangles

- Triangles that sit exactly on top of each other
- Matching sides are called **corresponding sides**

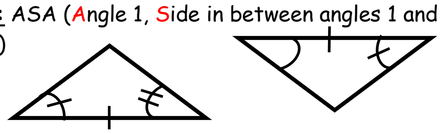
Type 1: SSS (Side, Side, Side)



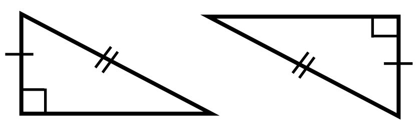
Type 2: SAS (Side 1, Angle in between sides 1 and 2, Side 2)



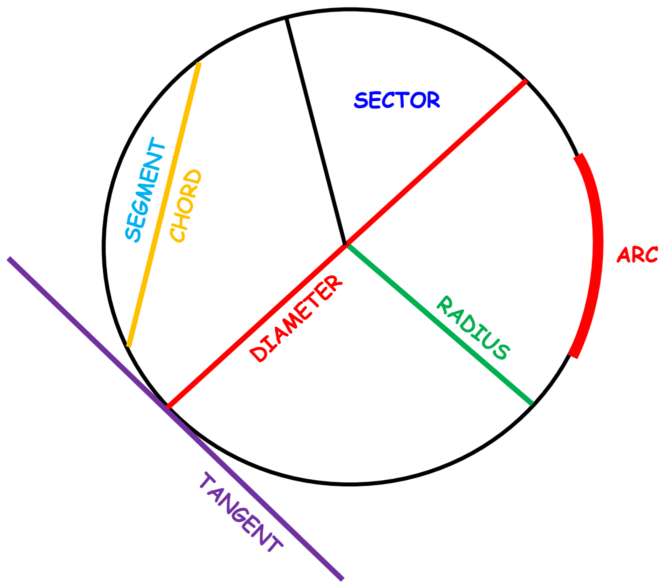
Type 3: ASA (Angle 1, Side in between angles 1 and 2, Angle 2)



Type 4: RHS (Right Angle, Hypotenuse, One other side)



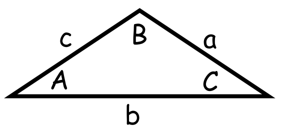
e) Circle Terminology:



2) Theorems: (Note that the formal proofs of Theorems 11, 12 and 13 need to be known)

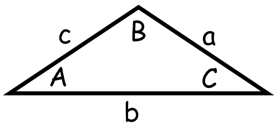
a) Theorems:

1. Vertically opposite angles are equal in measure.
2. In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.
3. If a transversal makes equal alternate angles on two lines then the lines are parallel, (and converse).
4. The angles in any triangle add to 180°.
5. Two lines are parallel if and only if, for any transversal, the corresponding angles are equal.
6. Each exterior angle of a triangle is equal to the sum of the interior opposite angles.
7. In a triangle, the largest angle is opposite the largest side, and the smallest angle is opposite the smallest side.



If a is the largest side, then A is the largest angle, and vice versa.

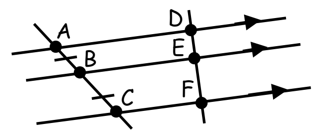
8. In a triangle, two sides of a triangle are always greater than the third.



$a + b > c$
$b + c > a$
$a + c > b$

9. In a parallelogram, opposite sides are equal and opposite angles are equal (and converses).
10. The diagonals of a parallelogram bisect each other.
11. If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

(Proof)

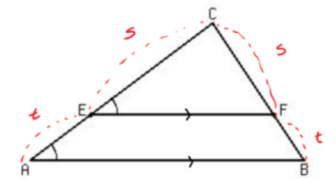


If |AB| = |BC|

=> |DE| = |EF|

12. Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio s:t, then it also cuts [AC] in the same ratio.

(Proof)

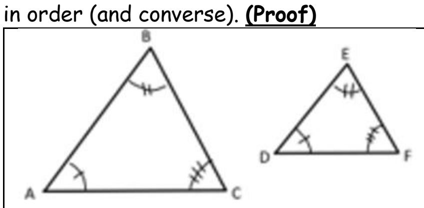


If EF is parallel to AB

=> $\frac{|AE|}{|CE|} = \frac{|BF|}{|CF|}$

or $\frac{|AE|}{|AC|} = \frac{|BF|}{|BC|}$

13. If two triangles are similar, then their sides are proportional, in order (and converse). **(Proof)**

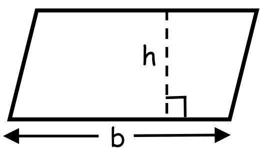


$$\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$$

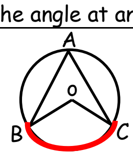
OR

$$\frac{|DE|}{|AB|} = \frac{|EF|}{|BC|} = \frac{|DF|}{|AC|}$$

14. [Theorem of Pythagoras] In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.
15. If the square of one side of a triangle is the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.
16. For a triangle, base x height doesn't depend on the choice of base.
17. A diagonal of a parallelogram bisects the area.
18. The area of a parallelogram is base x height.

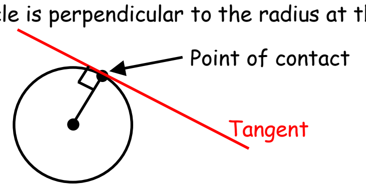


19. The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on that arc.

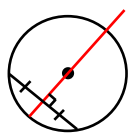


$|< BOC| = 2(|< BAC|)$

20. Each tangent to a circle is perpendicular to the radius at the point of contact.



21. The perpendicular from the centre of a circle to a chord bisects the chord.

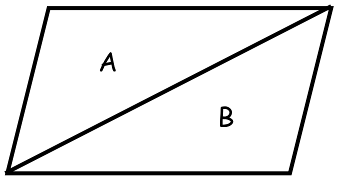


b) Other Theorem Terminology:

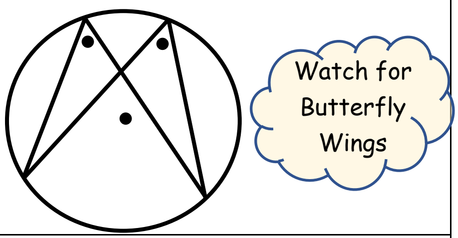
1. **Collinear** points are points that lie on the same line.
2. An **axiom** is a statement that we accept without any proof: e.g. There is exactly one line through any two given points.
3. A **theorem** is a rule that you can prove by following a certain number of logical steps or using a previous theorem or axiom. E.g. Pythagoras' Theorem
4. A **proof** is a series of logical steps that we use to show a theorem is true.
5. A **corollary** is a statement that follows readily from a previous theorem.
6. The **converse** of a statement is formed by reversing the order in which the statement is made. e.g. Statement: If P, then Q   Converse: If Q, then P.
7. '**Implies**' is a term we can use in a proof when we write down a fact or conclusion that follows from previous statements. The symbol is for implies is: =>

3) Corollaries: (The 6 results below follow on from the 21 theorems above)

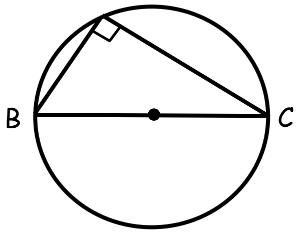
1. A diagonal divides a parallelogram into 2 congruent triangles i.e. triangles A and B below are congruent.



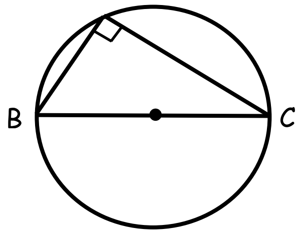
2. All angles at points of a circle, standing on the same arc, are equal, (and converse).



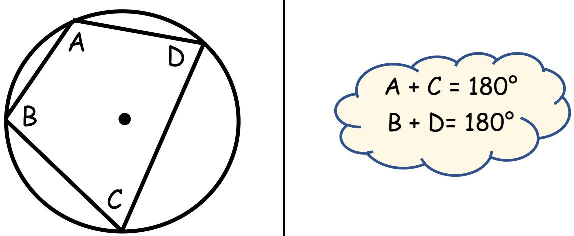
3. Each angle in a semi-circle is a right angle.



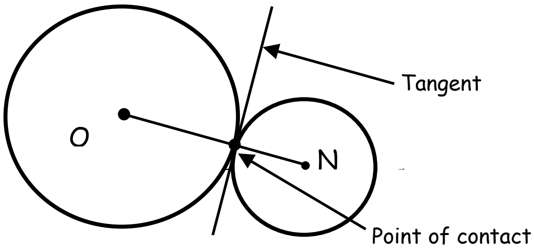
4. If the angle standing on a chord [BC] at some point, of the circle is a right-angle, then [BC] is a diameter.



5. In a cyclic quadrilateral, then opposite angles sum to 180, (and converse).



6. If two circles intersect at one point, their centres and the point of contact are collinear.



4) Constructions:

General Tips:

1. Keep your work neat and tidy.
2. Choose an appropriate pencil to draw the construction, not too dark and not too light.
3. Draw rough sketches of construction first, especially for triangles and rectangles.
4. Show all your construction lines & label your construction.

- There are 21 constructions on the course for Leaving Cert Ordinary Level. (**See Booklet from class for step by step instructions**)

Constructions List:

1. Bisector of a given angle, using only compass and straight edge.
2. Perpendicular bisector of a segment, using only compass and straight edge.
3. Line perpendicular to a given line l, passing through a given point not on l.
4. Line perpendicular to a given line l, passing through a given point on l.
5. Line parallel to a given line, through a given point.
6. Division of a line segment into 2 or 3 equal segments, without measuring it.
7. Division of a line segment into any number of equal segments, without measuring it.
8. Line segment of a given length on a given ray.
9. Angle of a given number of degrees with a given ray as one arm.

10 – 12. Triangle, given i) SSS ii) SAS or iii) ASA data

13. Right-angled triangle, given the length of the hypotenuse and one other side.
14. Right-angled triangle, given one side and one of the acute angles (several cases).
15. Rectangle, given side lengths.
16. Circumcentre and circumcircle of a given triangle, using ruler and compass.
17. Incentre and incircle of a given triangle, using ruler and compass.
18. Angle of 60°, without using a protractor or set square.
19. Tangent to a given circle at a given point on it.
20. Parallelogram, given the length of the sides and the measure of the angles.
21. The centroid of a triangle.
22. The orthocentre of a triangle.

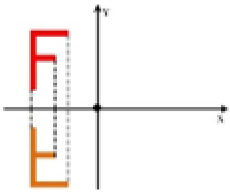
5) Transformations/Symmetries/Enlargements:

a) Transformations:

Note: In each of the pictures below, the red shape is the **object** and the second coloured shape is the **image**.

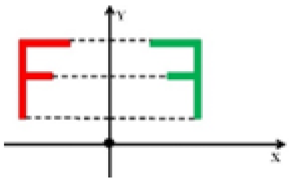
Axial Symmetry in the X-axis: (S_x)

- Shapes are mirrored/reflected in the X-axis. See example below.



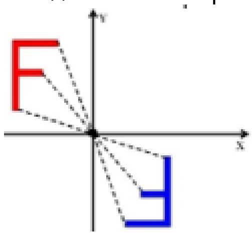
Axial Symmetry in the Y-axis: (S_y)

- Shapes are mirrored / reflected in the Y-axis. See example below.



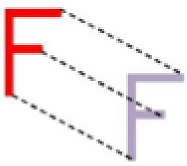
Central Symmetry in the Origin: (S_o)

- Shapes end up flipped and rotated as shown below.
- Central symmetry in a point other than the origin would have the same effect on the shape i.e. flipped and rotated



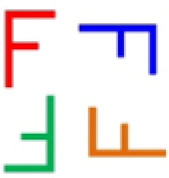
Translation:

- Note that shapes don't change when translated as the shape just '**slides**' to another position



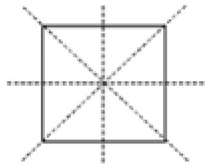
Rotations:

- The shape in blue below is a rotation of the red shape 90° clockwise. The green is a rotation of 180°. Note that it looks similar to the central symmetry in a point image from above. The orange is a rotation of 270° clockwise.

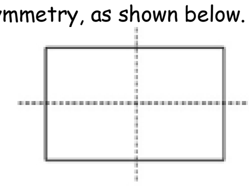


b) Axes of Symmetries of Shapes:

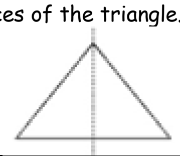
Square:
A square has 4 axes of symmetry, as shown below.



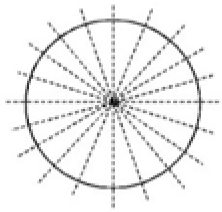
Rectangle:
A rectangle has 2 axes of symmetry, as shown below.



Triangle:
An isosceles triangle has 1 axis of symmetry, as shown below. If the triangle was an equilateral triangle it would have two more axes of symmetry from the other two vertices of the triangle.



Circle:
A circle has an infinite number of axes of symmetry, as shown below.



c) Enlargements:

Notes:

- An **enlargement** is a scaled up/down version of an object.
- The **scale factor 'k'** tells by how much the image has been scaled.
  - If $k > 1$ => scaled up
  - If $0 < k < 1$ => scaled down
- To find the scale factor from a given enlargement, divide a side of the image by its corresponding side in the object.
- The area of the image can be found by using:

$$Area\ Image = k^2 \times Area\ of\ Object$$

Not in Tables

- The **centre of enlargement** is the point where the object is being enlarged from.

Steps for constructing an enlargement:

1. Using a ruler, draw dashed construction lines from the centre of enlargement o, out through some of the main points of the object.
2. Measure the length of |OA|.
3. Multiply |OA| by the scale factor k and then measure out that distance from o along the dashed line to find location of A'.
4. Repeat for the other key points B, C, D, E etc.
5. Join up A', B', C', D' etc. to form image.
6. Check |A'B'| ÷ |AB| should be = the scale factor k.

Example: Enlargement for k = 3 of small L shape is shown below.