

Topic 4: Algebra

1) The Basics:

<p>a) Adding / Subtracting Algebraic Expressions:</p> <p>Notes:</p> <ul style="list-style-type: none">➤ We can only add / subtract 'like terms'.➤ 'Like terms' are terms that have the same letter part or the same variables e.g. $5d$ and $-2d$ are 'like terms' but $5d$ and $5d^2$ are <u>NOT</u> 'like terms' <p>Example 1:</p> $4a + 5 + 2a - 3$ $= 6a + 2$ <p>Example 2:</p> $3x^2y - 4y^2 - x^2y - 3y + 2y^2$ $= 2x^2y - 2y^2 - 3y$	<p>b) Multiplying Expressions:</p> <p>Notes:</p> <ul style="list-style-type: none">➤ when multiplying we follow the order Signs, Numbers, Letters➤ When multiplying the letters together we must remember the first law of indices.....$a^m \times a^n = a^{m+n}$ i.e. Add the Powers <p>Example 1: Multiplying Terms</p> $4a^2 \times 2a^5 \quad (\text{Multiply signs } \dots (+) \cdot (+) = +)$ $= 8a^7 \quad (\text{Multiply Numbers \& Add Powers})$ <p>Example 2: Removing Brackets</p> $2(g + 4)$ $= 2g + 8$ <p>Example 3: Removing Brackets</p> $(2x - 3)(x + 2) \quad (\text{"Split and Repeat"})$ $= 2x(x + 2) - 3(x + 2)$ $= 2x^2 + 4x - 3x - 6$ $= 2x^2 + x - 6$
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2) Algebraic Fractions:

<p>a) Adding/Subtracting Fractions:</p> <p>Note:</p> <ul style="list-style-type: none">➤ When adding/subtracting fractions together we find the common denominator and bring both terms up to the same denominator first. <p>Example 1:</p> $\frac{x+3}{5} - \frac{2x-1}{3}$ $= \frac{3(x+3)}{15} - \frac{5(2x-1)}{15}$ $= \frac{3x+9}{15} - \frac{10x-5}{15}$ $= \frac{3x+9-(10x-5)}{15}$ $= \frac{3x+9-10x+5}{15}$ $= \frac{-7x+14}{15}$	<p>b) Dividing Expressions:</p> <p>Tip: Can we factorise the numerator or the denominator?</p> <p>Example:</p> $\frac{2x+6}{x^2-9} = \frac{2(x+3)}{(x+3)(x-3)} = \frac{2}{x-3}$
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3) Factorising and Manipulation of Formulae:

<p>a) Factorising:</p> <p>1. Taking out the HCF (taking out what's common) e.g.s i) $2x - 10$ $= 2(x - 5)$</p> <p>ii) $3x^2 - 18x$ $= 3x(x - 6)$</p> <p>2. Grouping (always has 4 terms) e.g.s i) $ax + ay + bx + by$ $= a(x + y) + b(x + y)$ $= (x + y)(a + b)$</p> <p>ii) $3p - 3q - pk + kq$ $= 3(p - q) - k(p - q)$ $= (p - q)(3 - k)$</p>	<p>3. Quadratic (always has 3 terms x^2, x, a) e.g.s i) $x^2 + 5x + 6$ $= (x + 3)(x + 2)$</p> <p>ii) $x^2 - 3x - 18$ $= (x - 6)(x + 3)$</p> <p>4. Difference of 2 Squares (always 2 terms with a '-' between) Note: Watch for square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81.... e.g.s $x^2 - 9$ $= (x)^2 - (3)^2$ $= (x - 3)(x + 3)$</p>
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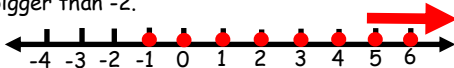
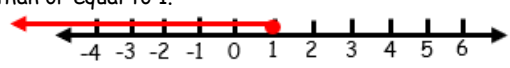
4) Solving Equations:

<p>a) Solving Linear Equations: (x only)</p> <p><u>Steps:</u></p> <ol style="list-style-type: none"> 1. Remove all brackets and any fractions 2. Bring all terms with an 'x' to one side and numbers to the other side 3. Tidy up both sides by putting together 'like terms'. 4. Solve the simple equation remaining. <p><u>Example:</u> $2(x - 3) = 4(x + 1)$</p> $2x - 6 = 4x + 4$ $2x - 4x = 4 + 6$ $-2x = 10$ $x = \frac{10}{-2}$ $\Rightarrow x = -5$	<p>c) Solving Quadratic Eqns by factorising: (Equations with an x^2)</p> <p><u>Steps:</u></p> <ol style="list-style-type: none"> 1. Bring all terms to the left-hand side (LHS) and leave '0' on the RHS 2. Factorise the LHS (See section on Factorising in previous tab) 3. If LHS can't be factorised the 'Quadratic Formula' needs to be used (See Example 3 on the right) 4. Let each factor be = 0 5. Solve the two simple equations to find the two answers. <p><u>Example:</u> $x^2 - 3x - 18 = 0$</p> $(x - 6)(x + 3) = 0$ $x - 6 = 0 \quad \text{or} \quad x + 3 = 0$ $\Rightarrow x = 6 \quad \text{or} \quad x = -3$
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5) Simultaneous Equations:

<p><u>Steps:</u></p> <ol style="list-style-type: none"> 1. Choose a variable to eliminate e.g. 'y' 2. Multiply one or both equations to make no. in front of y the same 3. Multiply the 2nd equation by -1, if necessary, to make signs in front of 'y' different. 4. Add the two equations to eliminate 'y' and solve for 'x'. 5. Put x back into one of the equations to find y. <p><u>Example:</u> Solve the equations below:</p> <p>A: $2x - 3y = 7$ B: $3x + 2y = 4$</p> <p>Ax2: $4x - 6y = 14$ (mult by 2 to get 6 in front of y) Bx3: $9x + 6y = 12$ (mult by 3 to get 6 in front of y)</p> $13x = 26$ $\Rightarrow x = \frac{26}{13}$ $\Rightarrow x = 2$	<p>Putting x into A:</p> <p>A: $2x - 3y = 7$ $\Rightarrow 2(2) - 3y = 7$ $\Rightarrow 4 - 3y = 7$ $\Rightarrow -3y = 7 - 4$ $\Rightarrow -3y = 3$ $\Rightarrow y = \frac{3}{-3}$ (dividing both sides by -3) $\Rightarrow y = -1$</p>
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6) Inequalities:

<p>a) Solving Inequalities:</p> <p><u>Notes:</u></p> <ul style="list-style-type: none"> ➤ Need to know the types of numbers (See Arithmetic 1b) ➤ Same rules as solving linear equations (See Algebra 4a) ➤ One difference: if you have to multiply/divide both sides of an inequality by a NEGATIVE number, we must CHANGE THE DIRECTION of the inequality. <p><u>Example 1:</u> Graph the solution to $3 - 4x < 11, x \in \mathbb{Z}$.</p> $3 - 4x < 11$ $-4x < 11 - 3$ $-4x < 8$ $\frac{-4x}{-4} < \frac{8}{-4} \quad (\text{dividing both sides by } -4)$ $x > -2 \quad (\text{Note sign change because divided by } -4)$ <p>For the number line, we're looking for all the Integers that are bigger than -2.</p> 	<p><u>Example 2:</u> Graph the solution to $3(x - 2) \leq -3, x \in \mathbb{R}$.</p> $3(x - 2) \leq -3$ $3x - 6 \leq -3$ $3x \leq -3 + 6 \quad (\text{adding 6 to both sides})$ $3x \leq 3$ $\frac{3x}{3} \leq \frac{3}{3} \quad (\text{dividing both sides by 3})$ $\Rightarrow x \leq 1$ <p>For the number line, we're looking for all the Real numbers that are smaller than or equal to 1.</p> 
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