End of Chapter Revision Sheet: Projectiles 5th Year

Level 1:

Q1. A particle is projected with an initial velocity of $10\vec{\imath} + 15\vec{\jmath}\ m/s$.

(i) Find the two times when the particle is at a height of $8\ m$.

(ii) Find the speed and direction of its velocity after 3 seconds.

Q2. A particle is projected with an initial velocity of $16\ m/s$ at an angle of $60°$ to the horizontal.

(i) Write its initial velocity in terms of $\vec{\imath}$ and $\vec{\jmath}$.

(ii) Find the maximum height reached by the particle.

(iii) Find the particle's range.

(iv) Calculate the speed of the particle when it lands.

Q3. A stone is projected with a speed of $18\sqrt{2}\ m/s$ at an angle of $45°$ to the horizontal from the top of a cliff of height $120\ m$.

(i) Find the maximum height above the sea the stone reaches.

(ii) Find the distance from the base of the cliff to where it enters the sea.

Q4. A plane is flying horizontally with a speed of $200\ m/s$. As it flies at a height of $120\ m$ over a cannon, a shell is fired at a speed of $250\ m/s$.

(i) Find the angle of projection required in order for the shell to hit the plane.

(ii) Find the time it takes for the shell to hit the plane.

Level 2:

Q5. (i) Show that the maximum height reached by a projectile with an initial speed of u and an angle of projection of A is given by $\frac{u^2 \sin^2 A}{2g}$.

(ii) Show that the range reached by a projectile with an initial speed of u and an angle of projection of A is given by $\frac{2u^2 \cos A \sin A}{g}$.

(iii) Show that the maximum range reached by a projectile with an initial speed of u and an angle of projection of A is given by $\frac{u^2}{g}$.

Q6. Find the angle of projection for a particle which has its horizontal range equal to double its maximum height.

Q7. Find the angle of projection for a particle which has its horizontal range equal to four times its maximum height.

Q8. A particle is projected with a speed of $98\ m/s$ on the horizontal plane. Find the two possible angles of projection, if the range of the particle is $280\ m$.

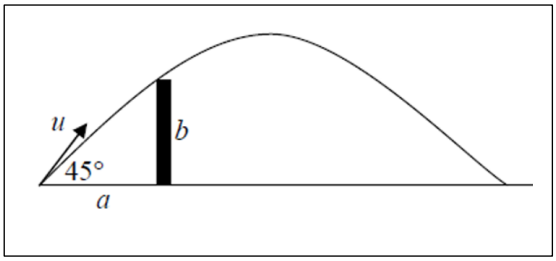
Q9. A gaelic football is kicked with an initial speed of $28\ m/s$ so that it clears the crossbar on a goalpost. If the crossbar is $2.8\ m$ high and the posts are $45\ m$ away from the kicker, find two possible angles of projection of the ball.

Level 3:

Q10. A particle is projected inside a tunnel which is $10\ m$ high with an initial speed of u. Show that the maximum range inside the tunnel is given by $\frac{28\sqrt{u^2 - 196}}{g}$.

Q11. A particle is projected with a speed of $49\ m/s$ at an angle A to the horizontal. The particle strikes the plane again after 8 seconds. At a time t_1 during the motion, its direction of motion was perpendicular to its original direction of motion. Find t_1.

Q12. A ball is projected from a point on the ground at a distance a from the foot of a vertical wall o height b, the velocity of projection being u at an angle $45°$ to the horizontal.



(i) If the ball just clears the wall, prove that the greatest height reached is $\frac{a^2}{4(a-b)}$.

(ii) Find, in terms of a and b, an expression for the horizontal range of the ball.

Answers:

Q1. (i) $0.69\ s, 2.37\ s$ (ii) $17.53\ m/s, E55.22°S$				
Q2. (i) $8\vec{i}+8\sqrt{3}\vec{j}$ (ii) $\frac{96}{g}m$ *or* $9.8\ m$ (iii) $\frac{128\sqrt{3}}{g}$ *or* $22.62\ m$ (iv) $16\ m/s$			Q3. (i) $136.53\ m$ (ii) $128.16\ m$	
Q4. (i) $36.87°$ (ii) $0.82\ s$	Q6. $63.4°$	Q7. $45°$	Q8. $8.3°, 81.7°$	Q9. $21.07°, 72.49°$
Q11. $6.25\ s$	Q12. (ii) $\frac{a^2}{a-b}$			

Past Exam Questions:

2009 Q3(a)

3. **(a)** A straight vertical cliff is 200 m high.
A particle is projected from the top of the cliff.
The speed of projection is $14\sqrt{10}$ m/s at an angle α to the horizontal.
The particle strikes the level ground at a distance of 200 m from the foot of the cliff.

(i) Find, in terms of α, the time taken for the particle to hit the ground.

(ii) Show that the two possible directions of projection are at right angles to each other.

2011 Q3(a)

3. **(a)** A particle is projected from a point P on horizontal ground.
The speed of projection is 35 m s^{-1} at an angle $\tan^{-1} 2$ to the horizontal.
The particle strikes a target whose position vector relative to P is $x\vec{i}+50\vec{j}$.

Find **(i)** the value of x

(ii) a second angle of projection so that the particle strikes the target.

2012 Q3(a)

3. **(a)** A particle is projected with a speed of 98 m s^{-1} at an angle α to the horizontal.
The range of the particle is 940·8 m. Find

(i) the two values of α

(ii) the difference between the two times of flight.

2017 Q3(a)

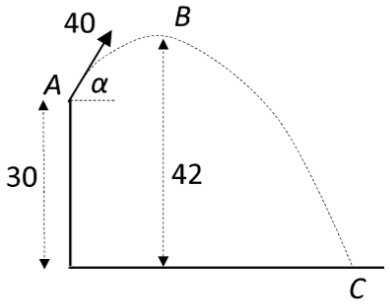
3. **(a)** A particle is projected with speed $\sqrt{\frac{9gh}{2}}$ from a point P on the top of a cliff of height h.
It strikes the ground a horizontal distance $3h$ from P.

(i) Find the two possible angles of projection.

(ii) For each angle of projection find, in terms of h, the time it takes the particle to reach P.

2019 Q3(a)

3. **(a)** A particle is projected with speed 40 m s^{-1} from a point A on the top of a vertical cliff of height 30 m.
The maximum height reached by the particle is 42 m above the horizontal ground, at point B.
It strikes the ground at C.



Find

(i) the value of α, the angle of projection

(ii) the horizontal range of the particle

(iii) the speed of the particle as it hits the ground at C.

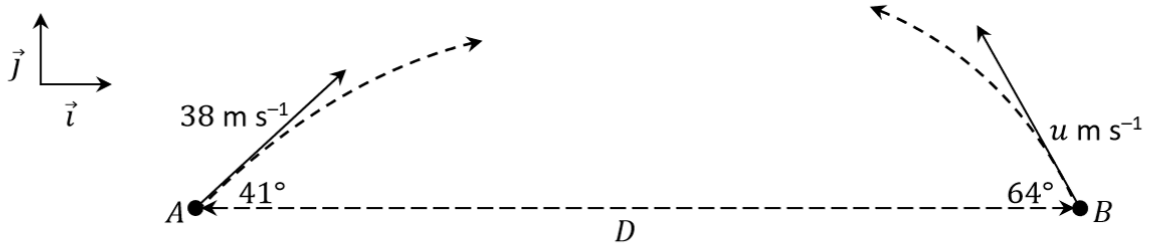
2023 Q8

Question 8

Two balls, P and Q, are projected into the air from points A and B, which are a distance D apart along the horizontal $\vec{\imath}$ axis. The motion of the balls may be modelled as projectile motion in a vertical plane, ignoring the effects of air resistance.

P is projected from point A at time $t = 0$ s with initial velocity 38 m s^{-1} at 41° to AB.

Q is projected from point B at time $t = 1$ s with initial velocity u m s^{-1} at 64° to BA.



P and Q collide in mid-air when $t = 3$ s.

(i) Show that $u = 28$ m s^{-1} to the nearest whole number.

(ii) Calculate D.

(iii) In terms of $\vec{\imath}$ and $\vec{\jmath}$, calculate $\overrightarrow{v_P}$, the velocity of P, and $\overrightarrow{v_Q}$, the velocity of Q, when the balls collide, i.e. when $t = 3$ s.

(iv) Calculate the dot product of $\overrightarrow{v_P}$ and $\overrightarrow{v_Q}$ when $t = 3$ s.

(v) Hence or otherwise calculate the acute angle between $\overrightarrow{v_P}$ and $\overrightarrow{v_Q}$ when $t = 3$ s.

Past Exam Questions:

2009: (i) $\frac{200}{14\sqrt{10}\cos\alpha}$ (ii) $\tan\alpha = 1 \pm \sqrt{2}$	2011: (i) 50 (ii) 71.6°	2012: (i) 36.87°, 53.13° (ii) 4 s
2017: (i) 0°, 71.6° (ii) $\sqrt{\frac{2h}{g}}, \sqrt{\frac{20h}{g}}$ *or* $4.48\sqrt{\frac{h}{g}}$	2019: (i) 22.5° (ii) 166 m (iii) 46.8 m/s	
2023: (ii) 110.53 m (iii) $\vec{p} = 28.68\vec{\imath} - 4.47\vec{\jmath}, \vec{Q} = -12.32\vec{\imath} + 5.6\vec{\jmath}$ (iv) -378.3696 (v) 15.57°		