Question 10 **(50 marks)**

(a) People are chosen at random from the population in Ireland.
The probability that a person in Ireland wears contact lenses is $\frac{2}{25}$.

(i) If 10 people are chosen at random, find the probability that exactly two of them wear contact lenses. Give your answer correct to 4 decimal places.

$P(\text{Contact } L) = \frac{2}{25} \Rightarrow P(\text{No contact } L) = \frac{23}{25}$

$P(2 \text{ wear lenses}) = \binom{10}{2}\left(\frac{2}{25}\right)^2\left(\frac{23}{25}\right)^8$

$= \boxed{0.1478}$

(ii) Find the probability that the 7th person chosen is the **second** person who wears contact lenses. Give your answer correct to 4 decimal places.

$P(1 \text{ person in } 1^{st} \text{ 6 people}) \text{ AND } P(7^{th} \text{ person has lenses})$

$= \binom{6}{1}\left(\frac{2}{25}\right)^1\left(\frac{23}{25}\right)^5 \times \frac{2}{25}$

$= 0.316359 \times \frac{2}{25}$

$= \boxed{0.0253}$

(iii) Find the minimum number of people who would need to be chosen, in order to be at least 90% certain of including at least one person who wears contact lenses.

$P(\text{At least } 1) = 1 - P(\text{None})$

$= 1 - \left(\frac{23}{25}\right)^n \geq 0.9$

$\Rightarrow \left(\frac{23}{25}\right)^n \leq 0.1$

$n \cdot \log\left(\frac{23}{25}\right) \leq \log(0.1)$

$\Rightarrow n \geq \frac{\log(0.1)}{\log(23/25)} = 27.6 \Rightarrow \boxed{n = 28}$

This question continues on the next page.

(b) A company produces eye drops. The eye drops are sold in small bottles.
The amount of liquid in the bottles follows a normal distribution with mean 10 ml and standard deviation 0·18 ml.

(i) Find the percentage of bottles that contain less than 9·85 ml of liquid.
Give your answer correct to the nearest percentage.

$\mu = 10 \qquad \sigma = 0.18$

$z = \dfrac{x - \mu}{\sigma} = \dfrac{9.85 - 10}{0.18} = -0.8333$

$P(z \leq -0.833) = P(z \geq 0.8333)$

$= 1 - P(z \leq 0.8333)$

$= 1 - 0.7967$

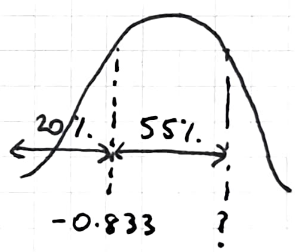
$= 0.2033$

$= \boxed{20\%}$

(ii) It is known that 55% of the bottles contain between 9·85 ml and k ml of liquid, where $k \in \mathbb{R}$ and $k > 9{\cdot}85$.

Find the value of k. Give your answer correct to 2 decimal places.

$P(9.85 < x < k) = 0.55$



$P(x < k) = 0.55 + 0.2$
$= 0.75$

Tables in reverse for 0.75

$\Rightarrow z = 0.67$

$z = \dfrac{x - \mu}{\sigma}$

$0.67 = \dfrac{x - 10}{0.18}$

$\Rightarrow x - 10 = 0.67(0.18)$

$x = 0.1206 + 10 = \boxed{10.12 \text{ mL}}$

(c) The machine used by the company to fill the bottles is serviced and adjusted.
The company want to check if the mean amount of liquid in the bottles is still 10 ml.
They take a random sample of 50 bottles and find the mean amount of liquid is 9·96 ml.

Carry out a hypothesis test at the 5% level of significance to see if this shows a change in the mean amount of liquid in the bottles.
State clearly your null hypothesis, your alternative hypothesis, your conclusion, and a reason for your conclusion.

Null Hypothesis: $H_0 : \mu = 10$ mL

Alternative Hypothesis: $H_1 : \mu \neq 10$ mL

Calculations: $\bar{x} = 9.96$ $\quad \sigma = 0.18$ $\quad n = 50$

Method 1: (95% Confidence Interval)

$$E = \frac{1.96\sigma}{\sqrt{n}} = \frac{1.96(0.18)}{\sqrt{50}} = 0.0498$$

$\Rightarrow$ 95% Conf Int :

$$\bar{x} - E \leq \mu \leq \bar{x} + E$$

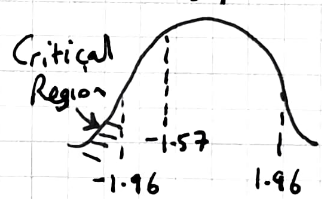
$$9.96 - 0.0498 \leq \mu \leq 9.96 \times 0.0498$$

$$9.91 \leq \mu \leq 10$$

Method 2: (Test Statistic)

$$T = \frac{9.96 - 10}{\frac{0.18}{\sqrt{50}}}$$

$$= -1.57$$



Conclusion: Fail to reject H_0 – evidence suggests mean contents hasn't changed

Reason for your conclusion: Method 1: $\mu = 10$ is in confidence interval
Method 2: Test statistic is not in critical region (see diag)