End of Chapter Revision Sheet: Difference Equations 6th Year

1st Order Difference Equations:

Q1. Solve the following difference equations:

(i) $u_n = 3u_{n-1} + 5, u_0 = 2$ (ii) $u_n = \frac{1}{4}u_{n-1} + 2, u_0 = 12$ (iii) $u_n = -2u_{n-1} + 7, u_1 = 8$

(iv) $u_n = 7u_{n-1} - 30, u_0 = 4$ (v) $u_n = -\frac{1}{2}u_{n-1} + 3, u_1 = 10$ (vi) $u_n = \frac{2}{3}u_{n-1} - 1, u_0 = 18$

Q2. (i) Solve the difference equation $u_n = \frac{2}{3}u_{n-1} + 9, u_0 = 162$

(ii) Find the 9^{th} term in the sequence.

(iii) Find the steady state value of u_n as $n \to \infty$

Q3. (i) Solve the difference equation $u_n = \frac{1}{4}u_{n-1} + 90, u_1 = 800$

(ii) Find the 6^{th} term in the sequence.

(iii) Find which term in the sequence is the first to be less than 125.

(iv) Find $\lim_{n\to\infty} u_n$.

2nd Order Difference Equations:

Q4. Solve the following difference equations:

(i) $u_{n+2} + 7u_{n+1} = -12u_n$, $u_0 = 2$ and $u_1 = -9$ (ii) $2T_n - 5T_{n-1} = 12T_{n-2}, u_1 = 1$ and $u_2 = 20$

(iii) $2T_{n+1} + T_{n+2} = 15T_n$, $T_1 = 1$ and $T_2 = 14$ (iv) $u_{n+2} - 6u_{n+1} = -9u_n, u_0 = 3$ and $u_1 = 6$

Q5. (i) Solve the difference equation $u_n - 3u_{n-1} = n - 5$ with $u_0 = 1$.

(ii) Hence find u_9.

Q6. (i) Solve the difference equation $u_n = 2n^2 - 1 - 2u_{n-1}$ with $u_1 = 5$.

(ii) Hence find u_7.

Q7. (i) Solve the difference equation $T_n = 7T_{n-1} - 12T_{n-2} + 8 - \frac{1}{2}(2^n)$ with $T_1 = 2$ and $T_2 = 3$.

(ii) Hence find T_{10}.

Word Problems:

Q8. In a certain country the population is 5 million people. The birth rate is 1.6% per annum and the death rate is 1.2% per annum. There is an annual inward immigration of $40{,}000$ people per annum and an annual outward emigration of $25{,}000$ people per annum.

(i) Show that the population can be represented by the difference equation $u_n = 1.004u_{n-1} + 15000$.

(ii) Solve the difference equation.

(iii) What will the population of the country be in 30 years' time?

Q9. Jimmy takes out a loan of €$10{,}000$ at a monthly interest rate of 1.2%. He repays €300 per month.

(i) Show that the amount owed for any given month can be represented by the difference equation: $u_n = 1.012u_{n-1} - 300$ with $u_0 = 10000$.

(ii) Solve the difference equation.

(iii) How many months does it take for Jimmy to clear the loan?

(iv) How much does Jimmy have to pay in the final month?

Q10. Aimee takes out a mortgage for €$300{,}000$ from the credit union at a monthly interest rate of 0.3%. She will pay it back in equal monthly instalments over 35 years.

(i) Write down a difference equation to represent the amount owing for any given month by Aimee based on the previous month. Use A to represent the amount of her monthly repayment.

(ii) Solve this difference equation to find A to the nearest euro.

Q11. In a zoo there were initially 8 pairs (8 males and 8 females) of a particular breed of raccoon. Assume that none of the raccoons reproduce in the first year, but each pair produces 4 raccoons (2 male and 2 female) in the following years and none of the raccoons died.

(i) Show that the number of pairs in the raccoon's population can be described by the difference equation: $R_n = R_{n-1} + 2R_{n-2}$ with $R_0 = R_1 = 8$.

(ii) Solve the difference equation and hence find the number of raccoons after 5 years.

Q12. In 2023 the Irish government announced changes to Ireland's deer management strategy, to include reviewing Open Season windows in a bid to curb the animal's population. The plan involved targeting deer in five "hotspot" counties by establishing local management units. A deer population for a particular year P_n can be modelled using the difference equation below, based on the populations for the previous 2 years.

$$P_n = \frac{1}{4}(3P_{n-1} + P_{n-2})$$

The deer population was recorded in 2025 as 70,000 deer and a year later, it was recorded as 95,000.

(i) Solve the difference equation.

(ii) Find how many deer would be recorded in 2030.

(iii) Over the years, the deer population begins to stabilise by itself, due to a number of factors. What is the annual steady state value for the deer population?

Q13. A salesman sells gaming controllers. Two years ago when he started the business, he sold 100 controllers (C_1) and the following year he sold 175 controllers (C_2). The salesman has derived a difference equation to model the growth of his sales, to try and predict how many he can expect to sell over the coming years. His model is: $C_n = C_{n-1} + 0.3C_{n-2} + 5n + 40$.

(i) Use the difference equation to predict the salesman's sales of controllers for the next 2 years.

(ii) Solve the difference equation and hence predict the sales of controllers in his 10th year.

Answers:

Q1. (i) $u_n = \frac{9}{2}(3)^n - \frac{5}{2}$ (ii) $u_n = \frac{28}{3}(\frac{1}{4})^n + \frac{8}{3}$ (iii) $u_n = \frac{-17}{6}(-2)^n + \frac{7}{3}$ (iv) $u_n = -1(7)^n + 5$ (v) $u_n = -16(-\frac{1}{2})^n + 2$ (vi) $u_n = 21(\frac{2}{3})^n - 3$
Q2. (i) $u_n = 135(\frac{2}{3})^n + 27$ (ii) $\frac{7841}{243}$ (iii) 27
Q3. (i) $u_n = 2720(\frac{1}{4})^n + 120$ (ii) $\frac{15445}{128}$ or 120.664 (iii) $n = 5$ (iv) 120
Q4. (i) $u_n = 3(-4)^n - 1(-3)^n$ (ii) $u_n = \frac{64}{33}(-\frac{3}{2})^n + \frac{43}{44}(4)^n$ (iii) $u_n = \frac{11}{40}(-5)^n + \frac{19}{24}(3)^n$ (iv) $u_n = (3-n)(3)^n$
Q5. (i) $u_n = -\frac{1}{6}(3)^n - \frac{1}{2}n + \frac{7}{6}$ (ii) $-\frac{19703}{6}$ or -3283.83
Q6. (i) $u_n = \frac{7}{9}(-2)^n + \frac{2}{3}n^2 + \frac{8}{9}n - \frac{5}{27}$ (ii) $-\frac{1643}{27}$ or -60.85
Q7. (i) $T_n = -\frac{7}{12}(4)^n + \frac{5}{3}(3)^n - 2^n + \frac{4}{3}$ (ii) $-514{,}277$
Q8. (i) $u_n = 1.004u_{n-1} + 15{,}000$ (ii) $u_n = 8{,}750{,}000(1.004)^n - 3{,}750{,}000$ (iii) 6,113,236
Q9. (i) $u_n = 1.012u_{n-1} - 300$ (ii) $u_n = 25{,}000 - 15{,}000(1.012)^n$ (iii) 43 *months* (iv) €247.37
Q10. (i) $D_n = 1.003D_{n-1} - A$ (ii) €1,257.32
Q11. (ii) $R_n = \frac{16}{3}(2)^n + \frac{8}{3}(-1)^n$, $R_5 = 168$
Q12. (i) $P_n = 90{,}000 - 20{,}000(-\frac{1}{4})^n$ (ii) 90,020 (iii) 90,000
Q13. (i) $C_3 = 260, C_4 = 373$ (ii) $C_n = 278.25(\frac{5+\sqrt{55}}{10})^n + 27.33(\frac{5-\sqrt{55}}{10})^n - \frac{50n}{3} - \frac{2000}{9}$, $C_{10} = 2034$

Past Exam Questions:

SEC HL Sample Q8

A group of scientists are investigating the population, P, of rabbits on a certain island. They estimate that there are 8000 rabbits on the island and that the population is growing at a constant rate of 3% per year.

The scientists plan to remove a number of rabbits from the island every year, to help populate another habitat. They develop mathematical models to predict how P will change if B rabbits are removed from the island every year.

The first model which the scientists develop uses a difference equation to express the population of rabbits in year $n + 1$ in terms of the population in year n.

The difference equation is:

$$P_{n+1} = 1.03P_n - B$$

where $n \geq 0, n \in \mathbb{Z}$ and $P_0 = 8000$.

(i) Solve this difference equation to find an expression for P_n in terms of n and B.

2024 Q9

A geologist is carrying out a survey of an opal mine.

During the first month of mining, a mass of 200 kg of opal was removed. During the second month of mining, a mass of 245 kg of opal was removed.

The geologist predicts that M, the mass of opal removed in any month, can be expressed by the second-order homogeneous difference equation:

$$M_{n+2} = M_{n+1} + \frac{3M_n}{4}$$

where $n \geq 0, n \in \mathbb{Z}$, $M_0 = 200$ and $M_1 = 245$.

(i) Write down the values of M_2 and M_3.

(ii) Solve the difference equation to find an expression for M_n in terms of n.

(iii) Calculate the total mass of opal that is predicted to be removed during the first six months of mining.

After the government introduces stricter mining laws, the geologist changes their predictions by estimating that the mass of opal mined in month n will be reduced by 2^n kg.

The geologist predicts that P, the mass of opal removed in any month, can now be expressed by the second-order inhomogeneous difference equation:

$$P_{n+2} = P_{n+1} + \frac{3P_n}{4} - 2^{n+2}$$

where $n \geq 0, n \in \mathbb{Z}$, $P_0 = 199$ and $P_1 = 243$.

(iv) Solve this new difference equation to find an expression for P_n in terms of n.

(v) Calculate the total mass of opal that is now predicted to be removed during the first six months of mining.

Past Exam Questions:

Sample Paper: (i) $P_n = (1.03)^n \left(\frac{24000 - 100B}{3}\right) + \frac{100}{3}B$
2024: (i) $M_2 = 395\ kg, M_3 = 578.75\ kg$ (ii) $M_n = 172.5(1.5)^n + 27.5(-0.5)^n$ (iii) $3602.8125\ kg$ (iv) $P_n = 175.25(1.5)^n + 26.95(-0.5)^n - 3.2(2)^n$ (v) $3458\ kg$