

Topic 8: Calculus (Differentiation)

1) The Basics:

<p>a) Differentiating Expressions:</p> <p>Notes:</p> <ul style="list-style-type: none"> ➤ Symbols: $\frac{dy}{dx}$ or $f'(x)$ ➤ The derivative of a constant = 0 ➤ In general, to differentiate, we: <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto; background-color: #ffffcc;"> <p>Multiply by the power and reduce the power by 1.</p> </div> <p>Examples: Differentiate i) $y = 3x^2 - 5x + 2$ and ii) $y = \sqrt{x}$, @ the point (1, 3).</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px;"> <p>i) If $y = 3x^2 - 5x + 2$</p> <p>$\Rightarrow \frac{dy}{dx} = 2(3)x^{2-1} - 5x^{1-1} + 0$</p> <p>$\Rightarrow \frac{dy}{dx} = 6x^1 - 5x^0 + 0$</p> <p>$\Rightarrow \frac{dy}{dx} = 6x^1 - 5(1)$</p> <p>$\Rightarrow \frac{dy}{dx} = 6x - 5$</p> </td> <td style="width: 50%; padding: 5px;"> <p>ii) If $y = \sqrt{x}$, rewrite y first:</p> <p>$y = x^{\frac{1}{2}}$</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$ (rule 1)</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$</p> <p>@ the point (1, 3) means $x = 1$</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(1)^{-\frac{1}{2}} = \frac{1}{2}$</p> </td> </tr> </table>	<p>i) If $y = 3x^2 - 5x + 2$</p> <p>$\Rightarrow \frac{dy}{dx} = 2(3)x^{2-1} - 5x^{1-1} + 0$</p> <p>$\Rightarrow \frac{dy}{dx} = 6x^1 - 5x^0 + 0$</p> <p>$\Rightarrow \frac{dy}{dx} = 6x^1 - 5(1)$</p> <p>$\Rightarrow \frac{dy}{dx} = 6x - 5$</p>	<p>ii) If $y = \sqrt{x}$, rewrite y first:</p> <p>$y = x^{\frac{1}{2}}$</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$ (rule 1)</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$</p> <p>@ the point (1, 3) means $x = 1$</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(1)^{-\frac{1}{2}} = \frac{1}{2}$</p>	<p>b) Second Derivative: (Differentiating twice)</p> <p>Examples:</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px;"> <p>i) $y = 3x^3 - 4x^2 + 5x$</p> <p>$\Rightarrow \frac{dy}{dx} = 9x^2 - 8x + 5$</p> <p>$\Rightarrow \frac{d^2y}{dx^2} = 18x - 8$</p> </td> <td style="width: 50%; padding: 5px;"> <p>ii) $f(x) = 2x^4 + 3x^2 - 3x + 2$</p> <p>$\Rightarrow f'(x) = 8x^3 + 6x - 3$</p> <p>$\Rightarrow f''(x) = 24x^2 + 6$</p> </td> </tr> </table>	<p>i) $y = 3x^3 - 4x^2 + 5x$</p> <p>$\Rightarrow \frac{dy}{dx} = 9x^2 - 8x + 5$</p> <p>$\Rightarrow \frac{d^2y}{dx^2} = 18x - 8$</p>	<p>ii) $f(x) = 2x^4 + 3x^2 - 3x + 2$</p> <p>$\Rightarrow f'(x) = 8x^3 + 6x - 3$</p> <p>$\Rightarrow f''(x) = 24x^2 + 6$</p>
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<div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 0 auto; background-color: #ffffcc;"> <p>Slope of the Tangent to the Curve = $\frac{dy}{dx}$</p> </div>					
<p>d) First principles:</p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto; background-color: #ffffcc;"> $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ </div> <p>Example: Differentiate $f(x) = 3x^2$ from first principles.</p> <ol style="list-style-type: none"> 1. $f(x) = 3x^2$ 2. $f(x+h) = 3(x+h)^2 = 3(x^2 + 2hx + h^2) = 3x^2 + 6xh + h^2$ 3. $f(x+h) - f(x) = 3x^2 + 6xh + h^2 - 3x^2 = 6xh + h^2$ 4. $\frac{f(x+h) - f(x)}{h} = \frac{6xh + h^2}{h} = 6x + h$ 5. $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = 6x + 0 = 6x$ 	<p>c) Slopes of Tangents:</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px;"> <p>Example 1: If $y = 3x^2 + 5x - 8$, find the slope of the tangent to the curve at the point (-1, 2).</p> <p>$y = 3x^2 + 5x - 8$</p> <p>$\Rightarrow \frac{dy}{dx} = 6x + 5$</p> <p>@ (-1, 2)</p> <p>$\Rightarrow \frac{dy}{dx} = 6(-1) + 5 = -1$</p> </td> <td style="width: 50%; padding: 5px;"> <p>Example 2: If $y = 2x^3 - 4x + 1$, find the equation of the tangent to the curve at the point (-2, -6). Find slope first: $\frac{dy}{dx} = 6x^2 - 4$</p> <p>Slope @ (-2, -6): $\frac{dy}{dx} = 6(-2)^2 - 4 = 20$</p> <p>Find the equation using equation formula and the point (-2, -6):</p> <p>$y - y_1 = m(x - x_1)$</p> <p>$y - (-6) = 20(x - (-2))$</p> <p>$y + 6 = 20(x + 2)$</p> <p>$y = 20x + 34$</p> </td> </tr> </table>	<p>Example 1: If $y = 3x^2 + 5x - 8$, find the slope of the tangent to the curve at the point (-1, 2).</p> <p>$y = 3x^2 + 5x - 8$</p> <p>$\Rightarrow \frac{dy}{dx} = 6x + 5$</p> <p>@ (-1, 2)</p> <p>$\Rightarrow \frac{dy}{dx} = 6(-1) + 5 = -1$</p>	<p>Example 2: If $y = 2x^3 - 4x + 1$, find the equation of the tangent to the curve at the point (-2, -6). Find slope first: $\frac{dy}{dx} = 6x^2 - 4$</p> <p>Slope @ (-2, -6): $\frac{dy}{dx} = 6(-2)^2 - 4 = 20$</p> <p>Find the equation using equation formula and the point (-2, -6):</p> <p>$y - y_1 = m(x - x_1)$</p> <p>$y - (-6) = 20(x - (-2))$</p> <p>$y + 6 = 20(x + 2)$</p> <p>$y = 20x + 34$</p>		
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2) Product Rule/Quotient Rule:

<p>a) Product Rule:</p> <p>Notes:</p> <ul style="list-style-type: none"> ➤ Used when two functions are multiplied together <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto; background-color: #ffffcc;"> <p>If $y = uv$ then $\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$</p> </div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>See Tables pg25</p> </div> <p>Example: If $y = (x^2 + 3)(2x - 1)$, find $\frac{dy}{dx}$.</p> <p>Let $u = x^2 + 3$ and $v = 2x - 1$</p> <p>$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$</p> <p>$\Rightarrow \frac{dy}{dx} = (2x - 1)(2x) + (x^2 + 3)(2)$</p> <p>$\Rightarrow \frac{dy}{dx} = 4x^2 - 2x + 2x^2 + 6$</p> <p>$\Rightarrow \frac{dy}{dx} = 6x^2 - 2x + 6$</p>	<p>b) Quotient Rule:</p> <p>Notes:</p> <ul style="list-style-type: none"> ➤ Used when two functions are divided by each other <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto; background-color: #ffffcc;"> <p>If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$</p> </div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>See Tables pg25</p> </div> <p>Example: If $y = \frac{3x - 4}{x^2 + 1}$, find $\frac{dy}{dx}$.</p> <p>Let $u = 3x - 4$ and $v = x^2 + 1$.</p> <p>$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{(x^2 + 1)(3) - (3x - 4)(2x)}{(x^2 + 1)^2}$</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{3x^2 + 3 - 6x^2 + 8x}{(x^2 + 1)^2}$</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{-3x^2 + 8x + 3}{(x^2 + 1)^2}$</p>
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3) Chain Rule:

a) Chain Rule:

Notes:

- Used for functions embedded in each other.
- Start on the outside and then work on the inside.

Examples: Differentiate the following i) $f(x) = (3x^2 - 2x)^3$

ii) $g(x) = \sqrt{2x^2 - 3}$

i) Differentiating the power 1st:

$$= 3(3x^2 - 2x)^{3-1}$$

$$= 3(3x^2 - 2x)^2$$

- Now differentiate the function inside i.e. $3x^2 - 2x$:

$$= 2(3x^{2-1}) - 2(1)x^{1-1}$$

$$= 6x - 2x^0$$

$$= 6x - 2$$

- Now multiply both derivatives together:

$$= (6x - 2)(3)(3x^2 - 2x)^2$$

Finally, we tidy up the terms at the front to get:

$$= (18x - 6)(3x^2 - 2x)^2$$

ii) Rewrite the function:

$$g(x) = (2x^2 - 3)^{\frac{1}{2}}$$

➤ Differentiate the power on the outside:

$$= \frac{1}{2}(2x^2 - 3)^{\frac{1}{2}-1}$$

$$= \frac{1}{2}(2x^2 - 3)^{-\frac{1}{2}}$$

➤ Differentiate the function inside i.e. $2x^2 - 3$:

$$= 2(2)x^{2-1} - 0$$

$$= 4x$$

Now, multiply the two derivatives together:

$$= 4x \left(\frac{1}{2}(2x^2 - 3)^{-\frac{1}{2}} \right)$$

➤ Tidying up gives:

$$= 2x(2x^2 - 3)^{-\frac{1}{2}}$$

➤ Or we could rewrite as:

$$= \frac{2x}{\sqrt{(2x^2 - 3)}}$$

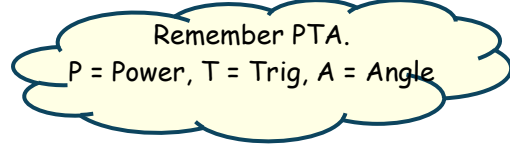
b) Trigonometric Functions:

Notes:

➤ To differentiate we use the Tables pg 25:

$f(x)$	$f'(x)$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$

➤ Use the Chain Rule then following the order below:



Examples: Differentiate i) $\cos 2x$ and ii) $\tan^3(x^2 + 3)$.

i) $y = \cos 2x$

$$\Rightarrow \frac{dy}{dx} =$$

$$(-\sin 2x)(2)$$

$$\Rightarrow \frac{dy}{dx} = -2\sin 2x$$

ii) $y = \tan^3(x^2 + 3)$

$$\Rightarrow \frac{dy}{dx} = (3 \tan^2(x^2 +$$

$$3))(\sec^2(x^2 + 3))(2x)$$

$$\Rightarrow \frac{dy}{dx} = 6x \tan^2(x^2 + 3) \sec^2(x^2 + 3)$$

c) Log/Exponential Functions:

Notes:

- Use the Chain Rule.
- To differentiate we use the Tables pg 25:

$f(x)$	$f'(x)$
$\ln x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}
a^x	$a^x \ln a$

Examples: Differentiate the following i) $f(x) = e^{-3x}$ ii)

$f(x) = \log_e 4x$.

i) $f(x) = e^{-3x}$

We use the Chain Rule to differentiate the exponential function first, and then the power:

$$\Rightarrow f'(x) = e^{-3x}(-3)$$

$$\Rightarrow f'(x) = -3e^{-3x}$$

ii) $f(x) = \log_e 4x$

We use the Chain Rule to differentiate the log function first, and then the 4x:

$$\Rightarrow f'(x) = \frac{1}{4x} \times (4)$$

$$\Rightarrow f'(x) = \frac{4}{4x}$$

$$\Rightarrow f'(x) = \frac{1}{x}$$

d) Inverse Trig Functions:

Notes:

- Use the Chain Rule.
- To differentiate we use the Tables pg 25:

$f(x)$	$f'(x)$
$\cos^{-1} \frac{x}{a}$	$-\frac{1}{\sqrt{a^2 - x^2}}$
$\sin^{-1} \frac{x}{a}$	$\frac{1}{\sqrt{a^2 - x^2}}$
$\tan^{-1} \frac{x}{a}$	$\frac{1}{a^2 + x^2}$

Examples: Differentiate i) $\cos^{-1} 4x$ and ii) $3x \cdot \sin^{-1} \frac{x}{3}$

i) Rewrite the function:

$$y = \cos^{-1} \frac{4x}{1}$$

Use the Chain Rule to differentiate the inverse Trig function first, and then the angle:

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{\sqrt{(1)^2 - (4x)^2}} \times$$

$$(4)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-4}{\sqrt{1 - 16x^2}}$$

ii) Use Product Rule:

$$\text{Let } u = 3x \text{ and } v = \sin^{-1} \frac{x}{3}$$

$$\Rightarrow \frac{du}{dx} = 3 \text{ and } \frac{dv}{dx} =$$

$$\frac{1}{\sqrt{(3)^2 - (x)^2}}$$

$$\Rightarrow \frac{du}{dx} = 3 \text{ and } \frac{dv}{dx} = \frac{1}{\sqrt{9 - x^2}}$$

$$\Rightarrow \frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = (\sin^{-1} \frac{x}{3})(3) +$$

$$(3x) \left(\frac{1}{\sqrt{9 - x^2}} \right)$$

$$\Rightarrow \frac{dy}{dx} = 3\sin^{-1} \frac{x}{3} + \frac{3x}{\sqrt{9 - x^2}}$$

4) Curve Sketching:

a) Increasing/Decreasing Functions:

$$\text{Increasing} \Rightarrow \frac{dy}{dx} > 0$$

$$\text{Decreasing} \Rightarrow \frac{dy}{dx} < 0$$

Example: Find the range of values of x for which the curve

$$f(x) = x^2 - 3x + 4 \text{ is increasing.}$$

$$f'(x) = 2x - 3$$

$$\text{Increasing} \Rightarrow f'(x) > 0$$

$$2x - 3 > 0$$

$$2x > 3$$

$$\Rightarrow x > \frac{3}{2}$$

b) Max/Min Points (Turning Points):

$$\text{Max/Min Points} \Rightarrow \frac{dy}{dx} = 0$$

If $\frac{d^2y}{dx^2} < 0$, then it's a local maximum point.

If $\frac{d^2y}{dx^2} > 0$, then it's a local minimum point.

Example: Find the max/min points of the curve $f(x) = x^3 - 2x^2 + 4$.

$$f'(x) = 3x^2 - 4x$$

$$\text{Max/Min} \Rightarrow f'(x) = 0$$

$$3x^2 - 4x = 0$$

$$x(3x - 4) = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{4}{3}$$

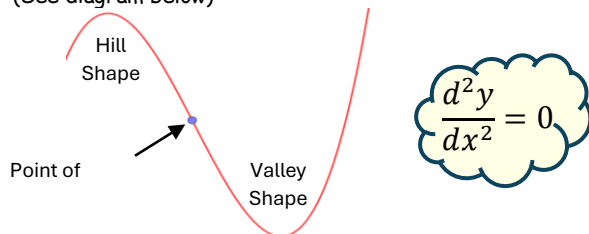
Sub into function at start to find y values of 4 and $\frac{76}{27}$.

\Rightarrow Turning Points are $(0, 4)$ and $(\frac{4}{3}, \frac{76}{27})$

c) Points of Inflection:

Notes:

- The point where a graph begins to change direction i.e. bend in a different direction, is known as a **point of inflection**. (See diagram below)



Example: Find the point of inflection of the curve $y = x^3 - 3x^2 + 4$.

$$\frac{d^2y}{dx^2} = 6x - 6$$

$$\Rightarrow 6x - 6 = 0$$

$$\Rightarrow x = 1$$

- So, the y value of the point of inflection will be:

$$y = (1)^3 - 3(1)^2 + 4$$

$$\Rightarrow y = 2 \Rightarrow \text{the point of inflection is } (1, 2).$$

5) Implicit Differentiation:

Notes:

- Differentiating more than one variable with respect to another
- When differentiating y with respect to x , differentiate as normal and multiply by $\frac{dy}{dx}$.

Example: Find $\frac{dy}{dx}$ for the following curve: $x^2 + y^2 = 36$.

- The derivative of y^2 term is $2y$ but we have to multiply by $\frac{dy}{dx}$ as we are differentiating with respect to x :

$$\Rightarrow \frac{d(y^2)}{dx} = 2y \cdot \frac{dy}{dx}$$

- We now differentiate each term, with respect to x .

$$\Rightarrow \text{if } x^2 + y^2 = 36$$

$$\Rightarrow 2x + 2y \cdot \frac{dy}{dx} = 0$$

- We now rearrange to get $\frac{dy}{dx}$ on its own:

$$\Rightarrow 2y \cdot \frac{dy}{dx} = -2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{2y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{y} \quad (\text{divide above and below by } 2)$$

6) Rates of Change:

a) Max/Min Problems:

Steps:

1. Get an expression for quantity to be maximised/minimised.
2. Differentiate.
3. Let derivative = 0 and solve to find max/min value.
4. Sub max/min value back into expression from step 1, if needed.

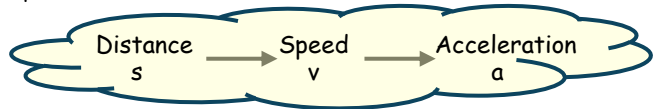
Example: A farmer want to enclose a field with 100m of fencing. Find the maximum area of the field.

$$\begin{aligned} \text{Let Width} = x \Rightarrow \text{Length} &= 50 - x \\ \Rightarrow \text{Area} = L \times W = x(50 - x) &= 50x - x^2 \\ \Rightarrow A = 50x - x^2 \\ \Rightarrow \frac{dA}{dx} = 50 - 2x \quad (\text{differentiating expression for area}) \\ 50 - 2x &= 0 \\ \Rightarrow x &= 25 \\ \Rightarrow \text{Max Area will be } 50(25) - (25)^2 &= 625\text{m}^2 \end{aligned}$$

b) Distance/Speed/Acceleration:

Tip:

Differentiate the expression for distance to get expressions for speed and acceleration.



Example: A body moves a distance given by the function $s = t^3 - 3t^2 + 7$, find the body's acceleration after 3 seconds.

$$\begin{aligned} \text{Distance} &= t^3 - 3t^2 + 7 \\ \Rightarrow \text{Speed} &= 3t^2 - 6t \quad (\text{differentiating distance expression}) \\ \Rightarrow \text{Acceleration} &= 6t - 6 \quad (\text{differentiating speed expression}) \\ \text{So, after 2 secs: Acceleration} &= 6t - 6 = 6(3) - 6 = 12\text{m/s}^2. \end{aligned}$$

c) Rates of Change:

Steps:

1. Write down what you're given.
2. Write down what you're trying to find.
3. Use a formula to link steps 1 and 2.
4. Use Implicit Differentiation to differentiate expression from step 3
5. Fill in what you were given and solve

Example 1: The rate of change of a radius of a circle is 2cm/s. Find the rate of change of the area of the circle when the radius is 3cm.

- First, we will write down the information we were given i.e. the rate of change of a radius

$$\frac{dr}{dt} = 2$$
- Now we write down an expression for what we are looking for i.e. the rate of change of area:

$$\frac{dA}{dt}$$
- And then we need a way to connect the two previous expressions together i.e. the formula for the area of a circle:

$$\text{Area of a circle} = A = \pi r^2$$
- We're looking for $\frac{dA}{dt}$ so we need to differentiate our expression for the area of a circle:

$$\begin{aligned} A &= \pi r^2 \\ \Rightarrow \frac{dA}{dt} &= \pi 2r \left(\frac{dr}{dt} \right) \quad (\text{using implicit differentiation}) \\ \Rightarrow \frac{dA}{dt} &= \pi 2r(2) \\ \Rightarrow \frac{dA}{dt} &= 4\pi r \end{aligned}$$
- Finally, we know the radius is 3cm

$$\begin{aligned} \Rightarrow \frac{dA}{dt} &= 4\pi(3) \\ \Rightarrow \frac{dA}{dt} &= 12\pi \text{ cm}^2/\text{s} \end{aligned}$$

Example 2: The rate of change of volume of a sphere is $6 \text{ cm}^3/\text{s}$. Find the rate of change of the radius at $r = 3$.

- Again, we will write down what we've been given first:

$$\frac{dV}{dt} = 6$$

- We're looking for the rate of change of the radius:

$$\frac{dr}{dt}$$

- Now we need a formula to connect the two i.e. the volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

- And we differentiate to generate our expressions above:

$$V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow 6 = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{6}{4\pi r^2}$$

- Finally, we can fill in the value of r:

$$\Rightarrow \frac{dr}{dt} = \frac{6}{4\pi(3)^2}$$

$$\Rightarrow \frac{dr}{dt} = \frac{6}{4\pi(9)}$$

$$\Rightarrow \frac{dr}{dt} = \frac{6}{36\pi}$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{6\pi} \text{ cm/s}$$