Topic: Trigonometry in Book 2   (Topics 33 to 42)

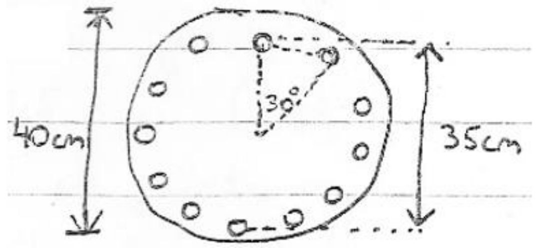
Q1. The diameter of a wheel of a bicycle is 40cm. Calculate the number of rotations of this wheel as the bicycle travels a distance of 100 metres. Give your answer as a whole number. **Ans:** 80

Q2. Prove that $1 - \frac{\sin^2\theta}{1-\cos\theta} = -\cos\theta$.

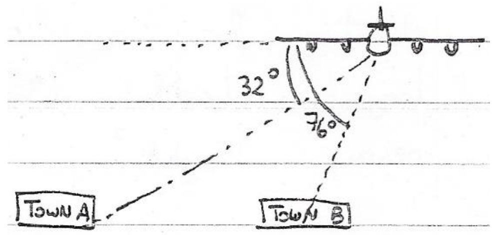
Q3. Show that $\sin 2A + \tan 2A = \frac{\sin 2A \tan 2A}{\tan A}$.

Q4. Write $\tan^2 30 + \sin^2 60$ in surd form. **Ans:** $\frac{13}{12}$

Q5. The figure shows a circular piece of sheet metal of diameter 40cm. The sheet contains 12 equally spaced bolt holes. Determine the straight-line distance between the centres of two consecutive bolt holes. **Ans:** 9.06cm



Q6. A passenger in an airplane flying at an altitude of 37000 ft sees two towns directly to the west of the airplane. The angles of depression to the towns are 32° and 76°. How far apart are the towns?
Ans: 49987ft



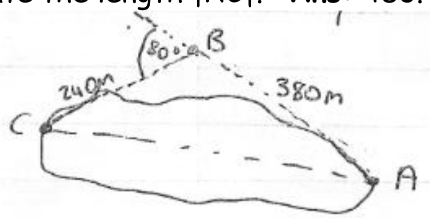
Q7. Solve the equation $\sin 2x = -\frac{\sqrt{3}}{2}$, where x is in radians and $x \in R$.   **Ans:** $x = \frac{2\pi}{3} + n\pi, \frac{5\pi}{6} + n\pi$

Q8. Find all the solutions of the equation $\cos 2A = 0.3420$, correct to the nearest degree.
Ans: A = $35° + n(180°)$, or
A = $145° + n(180°), n \in Z$

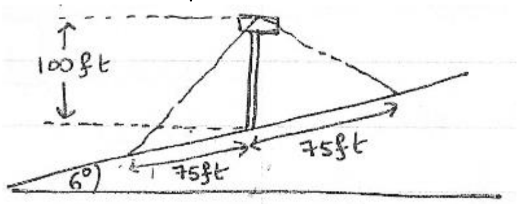
Q9. The course for a boat race starts at point A and proceeds in the direction S52°W to point B, then in the direction S40°E to point C and finally back to A. Point C lies 8km directly south of point A. Approximate the total distance of the race course.   **Ans:** 19.45km

Q10. If $\sin A = \frac{2t}{t^2+1}$, where $t > 1$, show that $\cos A = \frac{t^2-1}{t^2+1}$.

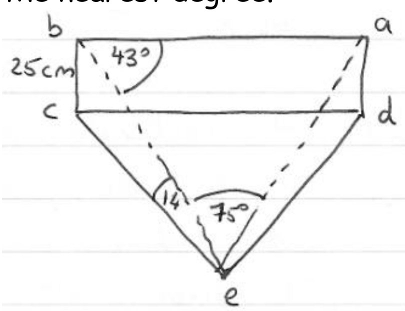
Q11. To approximate the length of a marsh, a surveyor walks 380m from point A to point B. Then the surveyor turns 80° and walks 240m to point C. Approximate the length |AC|.   **Ans:** 483.4m



Q12. A 100ft vertical tower is to be erected on the side of a hill that makes a 6° angle with the horizontal. Find the length of each of the two guide wires that will be anchored 75ft uphill and downhill from the base of the tower.
Ans: 131.12ft, 118.56ft



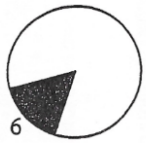
Q13. ced is a triangle on horizontal ground. abcd is a vertical rectangular wall. |bc| = 25cm, |< abe| = 43°, |< aeb| = 75° and |< bec| = 14°. Find |< aed| correct to the nearest degree.



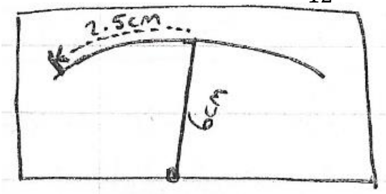
Ans: 18°

Q14. How many degrees does the minute hand of a clock gain on the hour hand in a minute? Hence, find the time, to the nearest second, between 1 o'clock and 2 o'clock when the two hands overlap.
Ans: 5.5°, 1:05:27

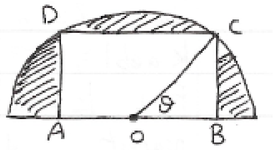
<u>Q15.</u> In the shaded sector in the diagram, the arc is 6cm long, and the angle of the sector is 0.75 radians. Find the area of the sector. **Ans:** 24cm^2



<u>Q16.</u> A voltmeter's pointer is 6cm in length. Find, in radians, the angle through which it rotates when it moves 2.5cm on the scale. **Ans:** $\frac{5}{12}$



<u>Q17.</u> A rectangle ABCD is inscribed in a semi-circle of centre o and radius r.



(i) Given that |< DCA| = θ, show that the perimeter of ABCD is $2r(\sin\theta + 2\cos\theta)$.
(ii) Show that the area of the shaded region is $\frac{r^2}{2}(\pi - 2\sin 2\theta)$.

<u>Q18.</u> The height, x, of a sound wave at time t, in radians, is given by $x = 6\sin 8t$.
(i) Find an expression for all the times at which x = 0.
(ii) Find the first five times at which x = -3.

Ans: (i) $t = \frac{n\pi}{4}$ or $t = \frac{\pi}{8} + \frac{n\pi}{4}, n \in Z$
(ii) $\frac{7\pi}{48}, \frac{11\pi}{48}, \frac{19\pi}{48}, \frac{23\pi}{48}, \frac{31\pi}{48}$

<u>Q19.</u> The average height of water in a harbour varies with the tide and is given by a sinusoidal curve i.e. a sine curve or a cosine curve. If x m represents the height t hours after a maximum height of 12m, then the next lowest height of 7m occurs 6 hours later. Express x as a function of t, where the angle is in degrees.
Ans: $x = \frac{19}{2} + \frac{5}{2}\cos 30t$

<u>Q20.</u> A triangle has sides of length a, a + 1, a + 2, where a > 0. If A is the angle opposite the side of length a, show that $\cos A = \frac{a+5}{2a+4}$.