Complex Nos.

Q1. $\dfrac{5-2i}{4+3i} \times \dfrac{4-3i}{4-3i}$

$= \dfrac{(5-2i)(4-3i)}{(4+3i)(4-3i)}$

$= \dfrac{20-15i-8i+6i^2}{16+\cancel{12i}-\cancel{12i}-9i^2}$

$= \dfrac{20-23i+6(-1)}{16-9(-1)}$

$= \dfrac{14-23i}{25} = \boxed{\dfrac{14}{25} - \dfrac{23}{25}i}$

Q2 $2(p+qi)+i(p-qi) = 5+i$

$2p+2qi+pi-qi^2 = 5+i$

$2p+2qi+pi+q = 5+i$

$\underline{Re = Re}$ $\underline{Im = Im}$

$\Rightarrow 2p+q = 5$ $2q+p = 1$

Solving two equations above gives:

$\cancel{2p}+q = 5$

$\underline{(-)\cancel{2p}\overset{(-)}{+}4q \overset{(-)}{=} 2}$

$-3q = 3$

$\boxed{q=-1} \Rightarrow \boxed{p=3}$

Q3. $z^2-6z+10=0$

$z = \dfrac{6 \pm \sqrt{(-6)^2-4(1)(10)}}{2(1)}$

$= \dfrac{6 \pm \sqrt{36-40}}{2}$

$= \dfrac{6 \pm \sqrt{-4}}{2}$

$= \dfrac{6 \pm \sqrt{4}\sqrt{-1}}{2} = \dfrac{6 \pm 2i}{2}$

$= \boxed{3 \pm i}$

Q4. $\sqrt{5}|w|+iw = 3+i$

Let $w = a+bi$

$\Rightarrow |w| = \sqrt{a^2+b^2}$

$\Rightarrow \sqrt{5}\left(\sqrt{a^2+b^2}\right)+i(a+bi) = 3+i$

$\sqrt{5a^2+5b^2}+ai+bi^2 = 3+i$

$\sqrt{5a^2+5b^2}-b+ai = 3+i$

$\underline{Re = Re}$ $\underline{Im = Im}$

$\sqrt{5a^2+5b^2}-b = 3$ $\boxed{a=1}$

If $a=1$

$\Rightarrow \sqrt{5(1)^2+5b^2}-b = 3$

$\sqrt{5b^2+5}-b = 3$

$\sqrt{5b^2+5} = 3+b$

Sq both sides:

$5b^2+5 = b^2+6b+9$

$4b^2-6b-4 = 0$

$2b^2-3b-2 = 0$

$(2b+1)(b-2) = 0$

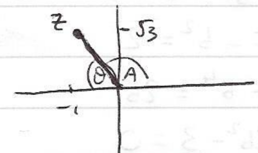
$2b+1 = 0$ or $b-2=0$

$2b = -1$ $\boxed{b=2}$

$\boxed{b = -\tfrac{1}{2}}$

$\Rightarrow \boxed{w = 1+2i \text{ or } 1-\tfrac{1}{2}i}$

Q5.



$|z| = \sqrt{(-1)^2+(\sqrt{3})^2}$

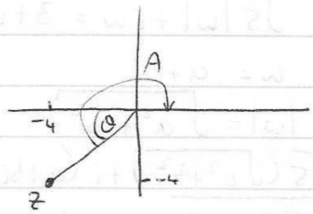
$= 2$

$\tan\theta = \sqrt{3}/1 \Rightarrow \theta = \tan^{-1}\sqrt{3} = 60°$

$\Rightarrow A = 180-60 = 120°$

$\Rightarrow -1+i\sqrt{3} = \boxed{2(\cos 120° + i\sin 120°)}$

Q6.



$|z| = \sqrt{(-4)^2 + (-4)^2}$

$= 4\sqrt{2}$

$\tan\theta = 4/4 = 1$

$\Rightarrow \theta = \tan^{-1} 1 = 45°$

$\Rightarrow A = 180 + 45 = 225°$

$\Rightarrow -4-4i = \boxed{4\sqrt{2}(\cos 225 + i \sin 225)}$

Q7. $z^2 = 2 - 2\sqrt{3}i$

Method 1:

Let $z = a + bi$

$\Rightarrow (a+bi)^2 = 2 - 2\sqrt{3}i$

$a^2 + b^2 i^2 + 2abi = 2 - 2\sqrt{3}i$

$a^2 - b^2 + 2abi = 2 - 2\sqrt{3}i$

Re = Re　　　　Im = Im

$a^2 - b^2 = 2$　　　　$2ab = -2\sqrt{3}$

$ab = -\sqrt{3}$

$\Rightarrow \left(-\sqrt{3}/b\right)^2 - b^2 = 2$　　　　$a = -\sqrt{3}/b$

$\frac{3}{b^2} - b^2 = 2$

$3 - b^4 = 2b^2$

$b^4 + 2b^2 - 3 = 0$

$(b^2 - 1)(b^2 + 3)$

$b^2 = 1$　　$\cancel{b^2 = -3}$

$b = \sqrt{1}$

$b = \pm 1 \Rightarrow a = \pm\sqrt{3}$

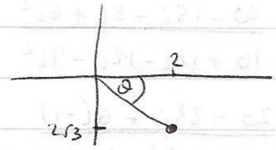
$\Rightarrow \boxed{z = \sqrt{3} - i \text{ or } -\sqrt{3} + i}$

Method 2: De Moivre

$z^2 = 2 - 2\sqrt{3}i$

$z = (2 - 2\sqrt{3}i)^{1/2}$

$2 - 2\sqrt{3}i$ into polar form



$|z| = \sqrt{(2)^2 + (-2\sqrt{3})^2} = 4$

$\tan\theta = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$\Rightarrow \theta = 60°$

Write z in general polar form:

$2 - 2\sqrt{3}i = 4(\cos(300 + 360n) + i\sin(300 + 360n))$

$\Rightarrow (2 - 2\sqrt{3}i)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left[\cos(150 + 180n) + i\sin(150 + 180n)\right]$

$= 2\left[\cos(150 + 180n) + i\sin(150 + 180n)\right]$

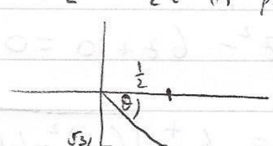
$n = 0$

$z = 2(\cos 150 + i \sin 150) = \boxed{-\sqrt{3} + 1}$

$n = 1$

$z = 2(\cos 330 + i \sin 330) = \boxed{\sqrt{3} - i}$

Q8. $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ in polar form:



$|z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = 1$

$\tan\theta = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

$\Rightarrow \theta = 60°$

$\Rightarrow \frac{1}{2} - \frac{\sqrt{3}}{2}i = 1(\cos 300 + i \sin 300)$

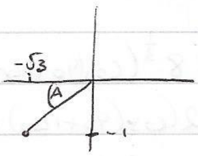
$\Rightarrow \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{15} = 1^{15}(\cos 15(300) + i \sin 15(300))$

$= 1(\cos 4500 + i \sin 4500)$

$= \boxed{-1 + 0i}$

Q9. $(-\sqrt{3} - i)^{10}$

$-\sqrt{3} - i$ in polar form



$|z| = \sqrt{(-\sqrt{3})^2 + (-1)^2} = 2$

$\tan A = \frac{1}{\sqrt{3}} \Rightarrow A = 30°$

$\Rightarrow -\sqrt{3} - i = 2(\cos 210 + i\sin 210)$

$\Rightarrow (-\sqrt{3} - i)^{10} = 2^{10}(\cos 2100 + i\sin 2100)$

$= 2^{10}\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= 2^{10}\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$

$= \frac{2^{10}}{2}(1 - \sqrt{3} i)$

$= \boxed{2^9(1 - \sqrt{3} i)}$

Q10. $\sin 3\theta \Rightarrow$ expand $(\cos\theta + i\sin\theta)^3$

using binomial distribution:

$= \binom{3}{0}\cos^3\theta + \binom{3}{1}\cos^2\theta(i\sin\theta)$

$+ \binom{3}{2}\cos\theta(i\sin\theta)^2 + \binom{3}{3}(i\sin\theta)^3$

Since $i^2 = -1$ and $i^3 = -i$

$\Rightarrow \cos^3\theta + 3\cos^2\theta\sin\theta\, i - 3\cos\theta\sin^2\theta$

$- i\sin^3\theta$

Grouping real + imaginary parts:

$(\cos^3\theta - 3\cos\theta\sin^2\theta) + (3\cos^2\theta\sin\theta - \sin^3\theta)i$

From De Moivre's Theorem

$\cos 3\theta$ = real parts $\sin 3\theta$ = im parts

$\Rightarrow \sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$

As $\cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow \sin 3\theta = 3(1 - \sin^2\theta)\sin\theta - \sin^3\theta$

$= 3\sin\theta - 3\sin^3\theta - \sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

Q.E.D.

Q11. $z = 5 - ki$

$z^2 - 10z = -29$

$\Rightarrow z^2 - 10z + 29 = 0$

$\Rightarrow z = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(+29)}}{2(1)}$

$= \frac{10 \pm \sqrt{-16}}{2}$

$= \frac{10 \pm 4i}{2}$

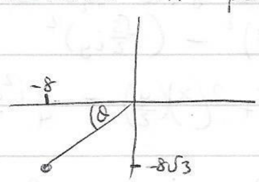
$= 5 \pm 2i$

$\Rightarrow \boxed{k = \pm 2}$

Q12. $z^4 = -8 - 8\sqrt{3} i$

$\Rightarrow z = (-8 - 8\sqrt{3} i)^{1/4}$

$-8 - 8\sqrt{3} i$ in polar form



$|z| = \sqrt{(-8)^2 + (-8\sqrt{3})^2} = 16$

$\tan\theta = 8\sqrt{3}/8 = \sqrt{3}$

$\Rightarrow \theta = \tan^{-1}\sqrt{3} = 60°$

$\Rightarrow$ Write z in general polar form

$z = 16(\cos(240° + 360n) + i\sin(240 + 360n))^{\frac{1}{4}}$

$= 16^{1/4}(\cos(60 + 90n) + i\sin(60 + 90n))$

$= 2(\cos(60 + 90n) + i\sin(60 + 90n))$

Now fill in values of $n = 0, 1, \ldots$

$n=0$

$z = 2(\cos 60 + i\sin 60) = \boxed{1+\sqrt{3}i}$

$n=1$

$z = 2(\cos 150 + i\sin 150) = \boxed{-\sqrt{3}+i}$

$n=2$

$z = 2(\cos 240 + i\sin 240) = \boxed{-1-\sqrt{3}i}$

$n=3$

$z = 2(\cos 330 + i\sin 330) = \boxed{\sqrt{3}-1}$

Q13. $w_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$w_2 = (w_1)^2 = \left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$

$= \frac{1}{4} - \frac{\sqrt{3}}{4}i - \frac{\sqrt{3}}{4}i + \frac{3}{4}i^2$

$= \frac{1}{4} - \frac{\sqrt{3}}{2}i - \frac{3}{4}$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$\underline{(x - w_1 y)(x - w_2 y)}$

$\left(x - y\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\right)\left(x - y\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)\right)$

$\left(\underbrace{x + \frac{y}{2}} - \underbrace{\frac{\sqrt{3}}{2}iy}\right)\left(\underbrace{x + \frac{y}{2}} + \underbrace{\frac{y\sqrt{3}}{2}i}\right)$

Difference of 2 squares:

$\left(x+\frac{y}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}iy\right)^2$

$x^2 + \frac{y^2}{4} + 2(x)\left(\frac{y}{2}\right) - \frac{3}{4}i^2y^2$

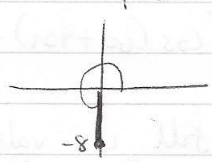
$x^2 + \frac{y^2}{4} + xy + \frac{3}{4}y^2$

$x^2 + xy + y^2$ Q.E.D.

Q14. i) $z^3 + 8i = 0$

$z^3 = 0 - 8i \Rightarrow z = (0-8i)^{1/3}$

$0-8i$ in polar form:



$0-8i = 8(\cos 270 + i\sin 270)$

$\Rightarrow$ in general polar form

$0-8i = 8(\cos(270+360n) + i\sin(270+360n))$

$\Rightarrow (0-8i)^{\frac{1}{3}} = 8^{\frac{1}{3}}(\cos(90+120n) + i\sin(90+120n))$

$= 2(\cos(90+120n) + i\sin(90+120n))$

$n=0$

$\Rightarrow 2(\cos 90 + i\sin 90) = 0 + 2i$

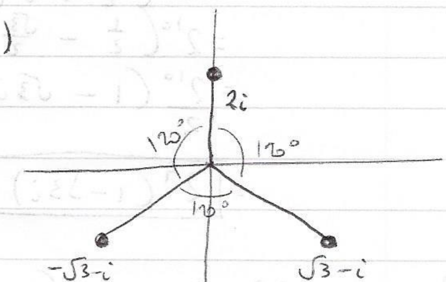
$n=1$

$\Rightarrow 2(\cos 210 + i\sin 210) = -\sqrt{3} - i$

$n=2$

$\Rightarrow 2(\cos 330 + i\sin 330) = \sqrt{3} - i$

ii)



Roots will have 120° between them

Q15. Let $w = a+bi \Rightarrow \bar{w} = a-bi$

$w.\bar{w} - 2iw = 7-4i$

$(a+bi)(a-bi) - 2i(a+bi) = 7-4i$

$a^2 - b^2i^2 - 2ai - 2bi^2 = 7-4i$

$(a^2+b^2+2b) - 2ai = 7-4i$

$\underline{Re = Re}$ $\underline{Im = Im}$

$a^2+b^2+2b = 7$ $-2a = -4$

$(2)^2+b^2+2b = 7$ $\Rightarrow \boxed{a=2}$

$b^2+2b-3=0$

$(b+3)(b-1)=0$

$\boxed{b=1}$ or $\boxed{b=-3}$

$\Rightarrow \boxed{w = 2+i \text{ or } 2-3i}$

Q16. i) $z = \frac{5}{2+i} - 1$

$= \frac{5 - 1(2+i)}{2+i}$

$= \frac{5-2-i}{2+i}$

$= \frac{3-i}{2+i} \times \frac{2-i}{2-i}$

$= \frac{6-2i+i^2-3i}{4+1}$

$= \frac{5-5i}{5}$

$= \boxed{1-i}$

ii) $z^6 = (1-i)^6$

$= \left[\sqrt{2}(\cos 315 + i\sin 315)\right]^6$

$= \sqrt{2}^6(\cos 6(315) + i\sin 6(315))$

$= 8(0+i)$

$= \boxed{8i}$

Q17. $z = 2+i$

$z + \frac{d}{z} = 2+i + \frac{d}{2+i} \times \frac{2-i}{2-i}$

$= 2+i + \frac{2d-di}{4+1}$

$= 2+i + \frac{2d-di}{5}$

$= \left(2 + \frac{2d}{5}\right) + i\left(1 - \frac{d}{5}\right)$

Real $\Rightarrow Im = 0$

$\Rightarrow 1 - \frac{d}{5} = 0$

$\Rightarrow 1 = \frac{d}{5}$

$\Rightarrow \boxed{d = 5}$

Q18. $P(z) = z^3 + (2-2i)z^2 + (3-3i)z - 1 - 4i$

$P(z) = (z-i)(z^2+az+b)$

$= z^3 + az^2 + bz - iz^2 - aiz - bi$

$= z^3 + z^2(a-i) + z(b-ai) - bi$

Comparing to $P(z)$

$a - i = 2-2i$	$b - ai = 3-3i$
$a = 2-2i+i$	$b = 3-3i+ai$
$\boxed{a = 2-i}$	$b = 3-3i+i(2-i)$
	$= 3-3i+2i-i^2$
	$= \boxed{4-i}$

Q19.

1. z and $\bar{z}$ must be images in the x-axis $\Rightarrow$ they must be 'd' and 'e'.
2. $z+\bar{z}$ would complete the parallelogram made by z and $\bar{z}$ $\Rightarrow$ this must be 'f'.
3. $z+w$ would complete the parallelogram made by z and w so z must be 'd' and hence w must be 'b' and $z+w$ = 'c'
4. As z is 'd', $\bar{z}$ must be 'e' from 1 above
5. Finally, $z.w$ must be 'a'

Q20 (1) z and $\bar{z}$ must be images in the x-axis so they must be 'a' and 'b'

(2) $2w$ would be twice as long as w and in the same direction so w must be 'e' and $2w$ must be 'f'!

(3) $z+w$ must complete the parallelogram made up of z and w, so z must be 'a' and $z+w$ must be 'd' then.

(4) $\bar{z}$ must be 'b' if z is 'a'. from (1) above

(5) Finally 'c' must be $z-\bar{z}$.

Q21 As coefficients are all real

$\Rightarrow$ Conjugate Root Theorem applies

$\Rightarrow$ If $3-2i$ is a root, $3+2i$ is too

And if $1+i$ is a root, $1-i$ is too

Form 2 quadratic factors using

$z^2 - z(\text{sum of roots}) + (\text{product})$:

$z^2 - z[3-\cancel{2i}+3+\cancel{2i}] + (3-2i)(3+2i)$

$z^2 - z[6] + 13$

and

$z^2 - z[1+\cancel{i}+1-\cancel{i}] + (1+i)(1-i)$

$z^2 - z(2) + 2$

Multiply these 2 factors together:

$(z^2-6z+13)(z^2-2z+2)$

$= z^2(z^2-2z+2) - 6z(z^2-2z+2) + 13(z^2-2$

$= z^4 - 2z^3 + 2z^2 - 6z^3 + 12z^2 - 12z + 13z^2 - 26z + 26$

$= z^4 - 8z^3 + 27z^2 - 38z + 26 = 0$

$\Rightarrow$ $\boxed{a=-8 \quad b=27 \quad c=-38 \quad z=26}$

Q22.

$$
\begin{array}{r l}
 & z + (1+i) \\
z^2-16 \,\big) & \cancel{z^3} + (1+i)z^2 - 16\cancel{z} - 16(1+i) \\
 & (-)\cancel{z^3} \overset{(-)}{+} 0z^2 \overset{(+)}{-} \cancel{16z} \\
\hline
 & (1+i)z^2 - 16(1+i) \\
 & (-)(1+i)z^2 \overset{(+)}{-} 16(1+i) \\
\hline
 & 0
\end{array}
$$

As remainder $= 0 \Rightarrow z^2-16$ is factor

$\Rightarrow z^3 + (1+i)z^2 - 16z - 16(1+i) = 0$

$\Rightarrow (z^2-16)(z+1+i) = 0$

$(z-4)(z+4)(z+1+i) = 0$

$z-4=0$ or $z+4=0$ or $z+1+i=0$

$\boxed{z=4}$ $\boxed{z=-4}$ $\boxed{z=-1-i}$

Q23.

i) $(a+bi)^2 = 15+8i$

$a^2-b^2+2abi = 15+8i$

Re = Re	Im = Im
$a^2-b^2=15$	$2ab=8$
$\left(\frac{4}{b}\right)^2 - b^2 = 15$	$ab=4$
$\frac{16}{b^2} - b^2 = 15$	$a = 4/b$

$16 - b^4 = 15b^2$

$b^4 + 15b^2 - 16 = 0$

$(b^2-1)(b^2+16) = 0$

$b^2 = 1$ or $\cancel{b^2 = -16}$

$b = \pm 1 \Rightarrow a = \pm 4$

$\Rightarrow$ $\boxed{\text{Ans: } 4+i, \ -4-i}$

ii) $iz^2 + (2-3i)z + (-5+5i) = 0$

$$z = \frac{-(2-3i) \pm \sqrt{(2-3i)^2 - 4(i)(-5+5i)}}{2i}$$

$$= \frac{3i-2 \pm \sqrt{4+9i^2-12i+20i-20i^2}}{2i}$$

$$= \frac{3i-2 \pm \sqrt{15+8i}}{2i}$$

From part (i) $\sqrt{15+8i} = \pm 4 \pm i$

If $\sqrt{15+8i} = 4+i$

$$\Rightarrow z = \frac{3i-2+4+i}{2i}$$

$$= \frac{4i+2}{2i} \cdot \frac{i}{i}$$

$$= \frac{2i-4}{-2}$$

$$= \frac{4-2i}{2}$$

$$= \boxed{2-i}$$

If $\sqrt{15+8i} = -4-i$

$$\Rightarrow z = \frac{3i-2-4-i}{2i}$$

$$= \frac{2i-6}{2i} \cdot \frac{i}{i}$$

$$= \frac{-2-6i}{-2}$$

$$= \boxed{1+3i}$$

Q24.

$z_1 = a+bi \qquad z_2 = c+di$

$\overline{z_1} = a-bi \qquad \overline{z_2} = c-di$

$$z_1 + z_2 = a+bi+c+di$$
$$= (a+c) + (b+d)i$$
$$\overline{z_1+z_2} = (a+c) - (b+d)i$$

$$\overline{z_1} + \overline{z_2} = a-bi+c-di$$
$$= (a+c) - (b+d)i$$

Q.E.D.

Q25. $2iz^2 + (6+2i)z + (3-6i) = 0$

$$z = \frac{-6-2i \pm \sqrt{(6+2i)^2 - 4(2i)(3-6i)}}{2(2i)}$$

$$= \frac{-6-2i \pm \sqrt{36+\cancel{24i}+4i^2-\cancel{24i}+48i^2}}{4i}$$

$$= \frac{-6-2i \pm \sqrt{36-52}}{4i}$$

$$= \frac{-6-2i \pm \sqrt{-16}}{4i}$$

$$= \frac{-6-2i \pm 4i}{4i}$$

$$= \frac{-6-2i+4i}{4i} \quad \text{or} \quad \frac{-6-2i-4i}{4i}$$

$$= \frac{2i-6}{4i} \cdot \frac{i}{i} \quad \text{or} \quad \frac{-6-6i}{4i} \cdot \frac{i}{i}$$

$$= \frac{-2-6i}{-4} \quad \text{or} \quad \frac{6-6i}{-4}$$

$$= \boxed{\frac{1+3i}{2}} \quad \text{or} \quad \boxed{\frac{-3+3i}{2}}$$