Week 30 Revision Sheet: Functions & Graphs 5th Yr Higher Level

Q1. Draw a sketch of the following functions:

(i) $f(x) = 2x - 3$ (ii) $g(x) = 3x^2 - 5x + 2$

(iii) $h(x) = e^x$ (iv) $f(x) = \sin x$

(v) $g(x) = \ln x$ (vi) $h(x) = |2x - 3|$

Q2. Use your calculator to find the couples for graphing the functions below:

(i) $f(x) = x^3 - 2x^2 + 3x$, domain $-3 \le x \le 4$

(ii) $g(x) = \ln(2x - 3)$, domain $3 \le x \le 6$

(iii) $h(x) = 2\sin 3\theta$, domain $-180 \le \theta \le 180°$

Q3. If $p(x) = 5x + 4$ and $g(x) = 3x - 2$, find: (i) $fg(x)$ (ii) $g^2(x)$

Q4. If $f(x) = x - \frac{3}{2}$ and $g(x) = \frac{2x-5}{3}$, find: (i) $fg(4)$ (ii) $gf(x)$ (iii) $f^2(x)$

Q5. If $f(x) = e^{2x-1}$ and $g(x) = \sqrt{x}, x \ge 0$, find (i) $fg(9)$ (ii) $gf(x)$ (iii) $f^{-1}g(x)$

Q6. Two functions f and g are given by: $f(x) = 6x - 3$ and $g(x) = 5x + q$. If $fg(x) = gf(x)$, find the value of q.

Q7. If $f(x) = 6x - 1$, find the value of the real number p such that

$$f[x + f(x)] = p[f(x)].$$

Q8. $f(x) = 3x - 5$ and $g(x) = 2x + 4$. Solve for x, if $fg(x) = 24$.

Q9. If $f(x) = 3^x$ and $g(x) = x + 2$, find the value of x for which $fg(x) = gf(x)$.

Q10. Write in completed square form:

(i) $f(x) = x^2 + 8x - 3$ (ii) $g(x) = x^2 - 10x + 8$

(iii) $h(x) = x^2 + 12x + 43$

Q11. Write the following in completed square form:

(i) $f(x) = 9x^2 + 36x - 4$

(ii) $g(x) = 4x^2 - 20x + 31$

Q12.

(i) If $f(x) = x^2 - 6x - 5$, where $x \ge 3, x \in R$, write $f(x)$ in the form $(x - h)^2 + k$.

(ii) Write down the coordinates of the minimum point and the axis of symmetry.

(iii) Hence, sketch the graph of $f(x)$.

(iv) Show that the inverse of $f(x)$ is the function $f^{-1}(x) = \sqrt{x + 14} + 3$.

Q13. A function f is defined by $f: x \to x^2 - 6x + 13$, where $x \ge 3$.

(i) Express $f(x)$ in the form $(x + q)^2 + p$.

(ii) Find an expression for $f^{-1}(x)$ and write down its domain.

(iii) The function h is defined by $h(x) = x - 3$, where $x \ge 2$. If p is a function such that $p \circ h = f$, find $p(x)$.

Q14. The function $f(x)$ is defined by

$$f: R \to R: x \to 5x - 3$$

(i) Draw a rough sketch of $f(x)$.

(ii) Is this function injective, surjective or bijective? Explain your answer.

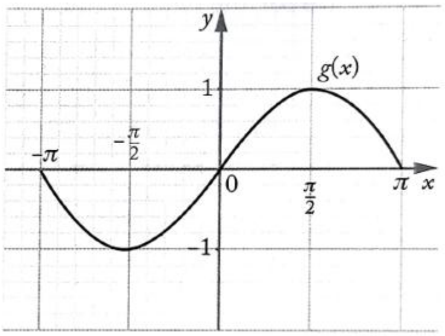
Q15. The function $g(x)$ is defined by

$$g: R \to R: x \to x^2 - 2x$$

(i) Draw a rough sketch of this function.

(ii) Is this function injective, surjective or bijective? Explain your answers.

Q16. The graph of a periodic function is shown below:



Indicate whether $g(x)$ is injective, surjective or bijective. Explain your answers.

Q17.

(i) Complete the table below and use the table to draw the graph of

$$f: R \to R^+: x \to x^2$$

in the interval $-3 \le x \le 3$.

x	-3	-2	-1	0	1	2	3
x^2							

(ii) Explain why this function is surjective but not injective.

(iii) How could the domain of $f(x)$ be changed to make it an injective function?

Q18. Find the inverse of the following:

(i) $f(x) = 5x - 2, x \in R$.

(ii) $g(x) = x^2 - 6x, x \geq 3, x \in R$

(iii) $h(x) = 2(3^x), x \in R$

(iv) $p(x) = \sqrt{2x - 4}$

(v) State the domain and the range of the inverse function of $p(x)$ in part (iv).

(vi) Sketch both $f(x)$ and $f^{-1}(x)$ on the same graph.

Q19. Find the inverse of the following functions:

(i) $f(x) = \frac{x-6}{4}$ (ii) $g(x) = 4x + \frac{2}{3}$

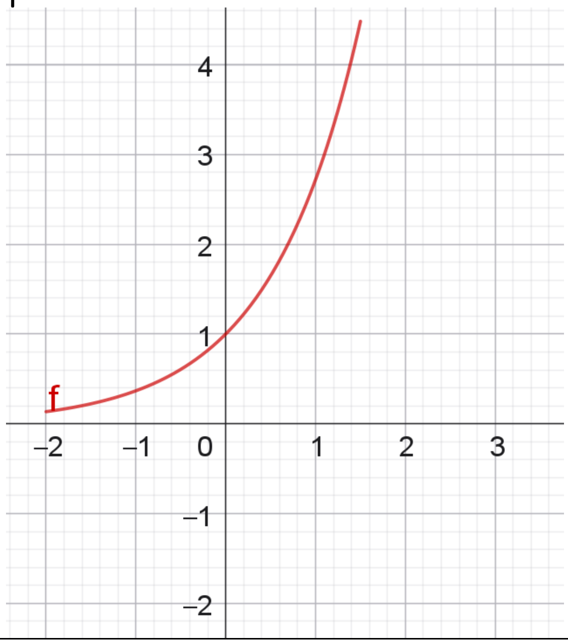
(iii) $h(x) = \sqrt{x+4}, x \geq -4$

Q20. Find the inverse of the following functions and write down the range of the inverse functions:

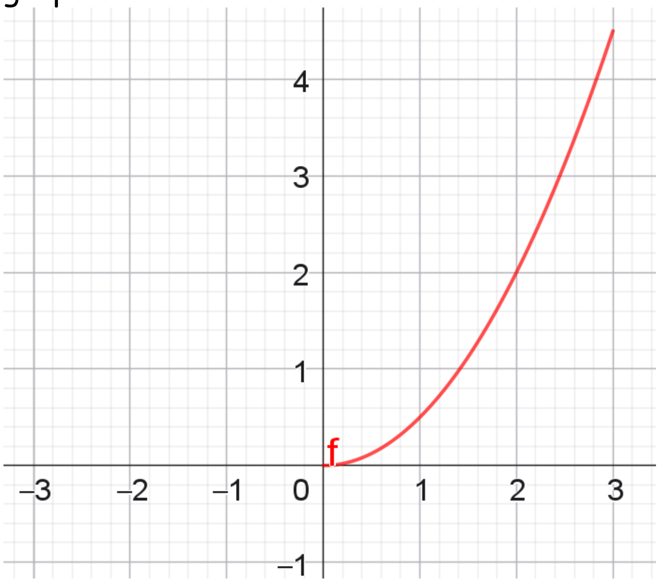
(i) $f(x) = \sqrt{x-3}, x \in R, x \geq 3$

(ii) $g(x) = x^3, x \in R$

Q21. Sketch the inverse of $f(x)$ onto the graph below:



Q22. Sketch the inverse of $f(x)$ onto the graph below:



Answers:

Q2. (i) $(-3-54), (-2,-22) \ldots \ldots \ldots (4,44)$ (ii) $(3,1.0986), (4,1.61), (5,1.195), (6,2.2)$
(iii) $(-180,0), (-150,-2), (-120,0) \ldots \ldots \ldots (150,2), (180,0)$

Q3. (i) 129 (ii) $9x - 8$

Q4. (i) $-\frac{1}{2}$ (ii) $\frac{2x-8}{3}$ (iii) $x - 3$

Q5. (i) e^{17} (ii) $\sqrt{e^{2x-1}}$ (iii) $\frac{1 + \ln\sqrt{x}}{2}$

Q6. $q = -\frac{12}{5}$

Q7. $p = 7$

Q8. $x = \frac{17}{6}$

Q9. $x = -1.262$

Q10. (i) $(x+4)^2 - 19$ (ii) $(x-5)^2 - 17$ (iii) $(x+6)^2 + 7$

Q11. (i) $9(x+2)^2 - 40$ (ii) $4\left(x - \frac{5}{2}\right)^2 + 6$

Q12. (i) $(x-3)^2 - 14$ (ii) $(3,-14), x = 3$ (iv) $f^{-1}(x) = \sqrt{x+14} + 3$

Q13. (i) $(x-3)^2 + 4$ (ii) $f^{-1}(x) = \sqrt{x-3} + 3$, Domain: $x \geq 3$ (iii) $p(x) = x^2 + 4$

Q14. (ii) Bijective

Q15. (ii) Neither injective nor surjective

Q16. Neither injective nor surjective

Q17. (iii) Domain = $x \geq 0$ or $x \leq 0$

Q18. (i) $f^{-1}(x) = \frac{x+2}{5}$ (ii) $g^{-1}(x) = 3 + \sqrt{x}$ (iii) $h^{-1}(x) = \frac{\log(\frac{x}{2})}{\log 3}$ (iv) $p^{-1}(x) = \frac{x^2+4}{2}$
(v) Domain : $x \geq 0$, Range : $y \geq 2$

Q19. (i) $f^{-1}(x) = 4x + 6$ (ii) $g^{-1}(x) = \frac{3x-2}{12}$ (iii) $h^{-1}(x) = x^2 - 4$

Q20. (i) Range of inverse : $x \geq 3$ (ii) Range of inverse : $x \in R$