

Revision Sheet 2 - Worked Solutions

Q1.

$$\frac{1}{x+1} + \frac{1}{x} = \frac{5}{6}$$

$$\frac{1(x) + 1(x+1)}{(x+1)(x)} = \frac{5}{6}$$

$$\frac{x + x + 1}{x^2 + x} = \frac{5}{6}$$

$$\frac{2x + 1}{x^2 + x} = \frac{5}{6}$$

$$\Rightarrow 6(2x+1) = 5(x^2+x)$$

$$12x + 6 = 5x^2 + 5x$$

$$5x^2 - 7x - 6 = 0$$

$$(5x+3)(x-2) = 0$$

$$5x+3=0 \text{ or } x-2=0$$

$$5x = -3 \quad \boxed{x=2}$$

$$\boxed{x = -\frac{3}{5}}$$

Q2. A: $3p + 4q - 2r = 8$

B: $9p + 8q + 2r = -13$

C: $6p - 12q + 14r = -59$

Solving A and B

A: $3p + 4q - 2r = 8$

B: $9p + 8q + 2r = -13$

D: $12p + 12q = -5$

Solving B and C

Bx7: $9p + 8q + 14r = -91$

C: $-6p + 12q - 14r = 59$

E: $57p + 68q = -32$

Solving D and E

Dx17: $204p + 204q = -85$

Ex3: $171p + 204q = -96$

$$33p = 11$$

$$p = \frac{1}{3} = \boxed{\frac{1}{3}}$$

Sub p into D

D: $12\left(\frac{1}{3}\right) + 12q = -5$

$$4 + 12q = -5$$

$$12q = -9$$

$$q = -\frac{9}{12} = \boxed{-\frac{3}{4}}$$

Sub p and q into A

A: $3\left(\frac{1}{3}\right) + 4\left(-\frac{3}{4}\right) - 2r = 8$

$$1 - 3 - 2r = 8$$

$$-2r = 10$$

$$r = \boxed{-5}$$

Q3. L: $x - 4y = -13$

C: $x^2 + 2y^2 + 6xy = 29$

Using L

$$x = 4y - 13 \quad *$$

Sub * into C

C: $(4y - 13)^2 + 2y^2 + 6(4y - 13)y = 29$

$$16y^2 + 169 - 104y + 2y^2 + 24y^2 - 78y - 29 = 0$$

$$42y^2 - 182y + 140 = 0$$

$$6y^2 - 26y + 20 = 0$$

$$3y^2 - 13y + 10 = 0$$

$$(3y - 10)(y - 1) = 0$$

$$3y - 10 = 0 \text{ or } y - 1 = 0$$

$$3y = 10 \quad y = 1$$

$$y = \frac{10}{3}$$

P.T.O.
→

Q8. Method 1:

$$x = \frac{5}{2} \quad \text{or} \quad x = -3$$

$$2x = 5 \quad x + 3 = 0$$

$$2x - 5 = 0$$

$$\Rightarrow (2x - 5)(x + 3) = 0$$

$$2x^2 - 5x + 6x - 15 = 0$$

$$\boxed{2x^2 + x - 15 = 0}$$

Method 2:

$$x^2 - x \left(\begin{smallmatrix} \text{SUM OF} \\ \text{ROOTS} \end{smallmatrix} \right) + \left(\begin{smallmatrix} \text{PRODUCT} \\ \text{OF ROOTS} \end{smallmatrix} \right) = 0$$

$$x^2 - x \left(\frac{5}{2} - 3 \right) + \left(\frac{5}{2} \right)(-3) = 0$$

$$x^2 - x \left(-\frac{1}{2} \right) - \frac{15}{2} = 0$$

$$\boxed{2x^2 + x - 15 = 0}$$

Q9. Roots @ $x = -2, 0, 1, 3$

Triple root Double root

$$\Rightarrow (x+2)^3(x)(x-1)^2(x-3)$$

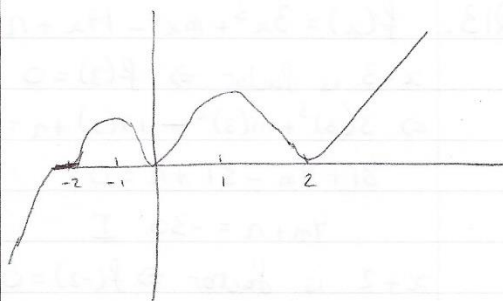
Right hand tail is down
and degree is odd

$\Rightarrow \ominus$ at start

$$\Rightarrow \boxed{-(x+2)^3(x)(x-1)^2(x-3)}$$

Q10. Roots @ -2 (Triple), 0 (Double)
and 2 (double) \Rightarrow degree = 7

The 2 at the front will
double the size of peaks
and troughs but won't
affect the roots (i.e. where the
graph crosses the x-axis)



Q11.

$$x^2 + ax + 4 \quad \begin{array}{l} x + (p-a) \\ \hline x^3 + px^2 + qx + 4b \end{array}$$

$$\begin{array}{l} (-)x^3 + (-)ax^2 + 4x \\ \hline \end{array} \quad \downarrow$$

$$(p-a)x^2 + (q-4)x + 4b$$

$$\begin{array}{l} (-) (p/a)x^2 + a(p-a)x + 4(p-a) \\ \hline \end{array}$$

$$(q-4-ap+a^2)x + (4b-4p+4a)$$

$x^2 + ax + 4$ is a factor

$$\Rightarrow q-4-ap+a^2=0 \quad \text{and} \quad 4b-4p+4a=0$$

$$\Rightarrow b-p+a=0$$

$$\Rightarrow \boxed{p=a+b}$$

Sub p into

$$q-4-a(a+b)+a^2=0$$

$$q-4-a^2-ab+a^2=0$$

$$\boxed{q=4+ab}$$

Q12.

$$x^2 + 4x - 6 = (x+a)^2 + b$$

$$x^2 + 4x - 6 = x^2 + 2ax + a^2 + b$$

$$x^2 + 4x - 6 = x^2 + 2ax + (a^2 + b)$$

Comparing LHS and RHS

x terms must be equal

$$\Rightarrow 4 = 2a$$

$$\Rightarrow \boxed{a=2}$$

$$-6 = a^2 + b \quad \text{also}$$

$$\Rightarrow -6 = (2)^2 + b$$

$$\Rightarrow \boxed{b=-10}$$

Q13. $f(x) = 3x^3 + mx^2 - 17x + n$
 $x-3$ is factor $\Rightarrow f(3) = 0$
 $\Rightarrow 3(3)^3 + m(3)^2 - 17(3) + n = 0$
 $81 + 9m - 51 + n = 0$

$$9m + n = -30 \quad \text{I}$$

$x+2$ is factor $\Rightarrow f(-2) = 0$
 $\Rightarrow 3(-2)^3 + m(-2)^2 - 17(-2) + n = 0$
 $-24 + 4m + 34 + n = 0$

$$4m + n = -10 \quad \text{II}$$

Solving I and II

$$9m + n = -30$$

$$\rightarrow 4m + n = -10$$

$$5m = -20$$

$$m = -4$$

Put m into I

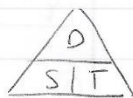
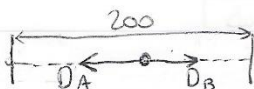
$$9m + n = -30$$

$$9(-4) + n = -30$$

$$-36 + n = -30$$

$$n = 6$$

Q14.



Car A	Car B
Speed = 50 km/h	Speed = 70 km/h
Time = T	Time = T
$D_A = S \times T$ $= 50T$	$D_B = S \times T$ $= 70T$

From diagram above

$$D_A + D_B = 200$$

$$\Rightarrow 50T + 70T = 200$$

$$\Rightarrow 120T = 200$$

$$\Rightarrow T = \frac{200}{120} \text{ hrs}$$

$$= 1.6666 \dots \text{ hrs}$$

$$\Rightarrow T = 1.6666 \times 60 = \boxed{1 \text{ hr } 40 \text{ mins}}$$

Q15. $x^3 - 19x - 30 = 0$

Need to find a root first
 (which will be a factor of 30)

\Rightarrow try $\pm 1, \pm 2, \pm 3, \dots$

$$f(-1): (-1)^3 - 19(-1) - 30 \neq 0$$

$$f(-2): (-2)^3 - 19(-2) - 30 = 0$$

$\Rightarrow x+2$ is a factor

$$\begin{array}{r}
 x^2 - 2x - 15 \\
 x+2 \overline{) x^3 + 0x^2 - 19x - 30} \\
 \underline{-x^3 + 2x^2} \\
 -2x^2 - 19x \\
 \underline{+2x^2 + 4x} \\
 -15x - 30 \\
 \underline{+15x + 30} \\
 0
 \end{array}$$

$$\Rightarrow x^3 - 19x - 30 = 0 \text{ becomes}$$

$$(x+2)(x^2 - 2x - 15) = 0$$

$$(x+2)(x-5)(x+3) = 0$$

$$x+2=0 \text{ or } x-5=0 \text{ or } x+3=0$$

$$\boxed{x = -2}$$

$$\boxed{x = 5}$$

$$\boxed{x = -3}$$