## Topic 1: Arithmetic

## 1) The Basics

## a) Types of Numbers:

- Natural (N): Positive Whole Numbers: e.g. 1, 2, 3, ......
- Integers (Z): Positive and Negative Whole Numbers: e.g. $-3,-2,-1,0,1,2,3$,.......
- Real (R): All numbers: e.g.s $-3,-1.4,0.2,6,7 / 2, \sqrt{8} \ldots \ldots$.
- Rational (Q): Numbers that can be written in the form e.g.s -5, 3, 1/2, -9/4.....
- Irrational: Numbers that cannot be written in the form $\frac{a}{b}$ e.g.s $\sqrt{3}, \sqrt{2}, \pi$......
- Prime: A natural number bigger than 1 with only itself and 1 as divisors. e.g.s $2,3,5,7,11,13,17 \ldots . . .$.
Composite: A number that is not prime. e.g.s., $6,9,15,20$


## c) Scientific Notation

## Notes:

- A number is in scientific notation if it's in the form $\mathbf{a} \times$ $10^{n}$, where 'a' has to be between 1 and 10 .
Examples: i) $3400=3.4 \times 10^{3}$ ii) $0.004=4 \times 10^{-3}$
> On a Casio calculator the button you will need to type in numbers in scientific notation is:


## $\times 10^{x}$

> To type in $7 \times 10^{4}$, press " 7 " and the button above and then "4"
> To convert numbers into scientific notation on your calculator:

- Type in the number and press = to enter it on the screen.
- Press "Shift" + "Mode" and select "Sci" from the menu. Then press "0".


## e) HCF/LCM using Prime Factors:

## Notes:

> When asked to find the HCF and LCM of 2 numbers using prime factors, use your calculator.
> Type in the number first and then press Shift + Button below.

$>$ For HCF: pick the lowest power of each prime number.
> For LCM: pick the highest power of each prime number.
Example: Find the HCF and LCM of 60 and 80 .
Using calculator: $60=5^{1} \times 3^{1} \times 2^{2} \quad 80=2^{4} \times 5^{1}$
HCF:

- Lowest Power of 2 is $2^{2}$
- Lowest Power of 3 is $3^{1}$......but $3^{1}$ doesn't go into 80 so exclude
Lowest Power of 5 is $5^{1}=>$ HCF of 60 and 80 is $2^{2} \times 5^{1}=10$ LCM:
- Highest Power of 2 is $2^{4}$
- Highest Power of 3 is $3^{1}$
- Highest Power of 5 is $5^{1} \Rightarrow$ LCM of 60 and 80 is $2^{4} \times 3^{1} \times 5^{1}=$ 240


## b) Rounding:

## Rounding to Decimal Places:

- To round to 2 decimal places, look at the 3rd number after the decimal point.
- If it's 5 or more we round UP the 2nd number
- If it's 4 or less we round DOWN the 2nd number
- Similar approach for rounding to other decimal places

Examples: i) $4.768=4.77$ ii) $3.2745=3.27$

## Rounding to Significant Figures:

- To round to 3 significant figures, we look at the $4^{\text {th }}$ significant figure.
- If it's 5 or more we round UP the $3^{\text {rd }}$ number and replace subsequent numbers with 0 s
- If it's 4 or less we round DOWN the $3^{\text {rd }}$ number and replace subsequent numbers with 0 s
Examples: i) $132,421=132,000$ ii) $0.00472543=0.00473$


## d) Foreign Exchange

## Steps:

1. Write the conversion with the currency you want on the right.
2. Get a 1 on the left-hand side, by dividing both sides.
3. Multiply both sides to get the value you want.

Example: If $€ 1=\$ 1.32$, how many euro would you get for $\$ 200$ ?
Step 1: Put euro on the right
$\$ 1.32=€ 1$
Step 2: Get a 1 on the left-hand side

$$
\$ 1=€ \frac{1}{1.32} \quad \text { (dividing both sides by } 1.32 \text { ) }
$$

Step 3: Multiply both sides

$$
\$ 200=\frac{1}{1.32} \times 200=€ 151.52
$$

## f) Speed, Distance and Time:

## Notes:

- For all speed, distance and time calculations remember:
"Dads silly Triangle"


If you want Distance, you cover the $D$ in the triangle, so: $D=S \times T \quad$ (Units are usually m or km )
> If you want Time, you cover the $T$ in the triangle, so: $T=D / S \quad$ (Units are usually secs or hrs)
$\rightarrow$ If you want Speed, you cover the $S$ in the triangle, so: $S=D / T \quad$ (Units might be $\mathrm{m} / \mathrm{s}$ or $\mathrm{km} / \mathrm{h}$ )
> Average speed can be calculated using:

> Careful with units of time. E.g. $1 \mathrm{hr} 45 \mathrm{mins}=1.75 \mathrm{hrs}$ etc.

## g) Ratio

## Notes:

> Ratio shows how to break up a quantity proportionally.
$>$ When given a ratio, add the values in the ratio together to get the total number of parts the quantity is being broken into.
> Write down the fraction each person gets.
Example: Divide $€ 200$ between Alan and Brian in the ratio 3:2.
3:2 means there are $3+2=5$ parts
$\Rightarrow$ Alan gets $\frac{3}{5}$ and Brian gets $\frac{2}{5}=>$ Alan gets $\frac{3}{5}$ of $€ 200=€ 120$ and Brian gets $\frac{2}{5}$ of $€ 200=€ 80$

## a) Percentages:

To find the percentage of a number:
Example: Find $24 \%$ of 250 .
Method 1: Calculate $\frac{24}{100} x \frac{250}{1}=60$
Method 2: Multiply 250 by $0.24=60$

## To find the total when given percentage:

Example: $25 \%$ of the marks in an exam are going for the practical part. If there are 50 marks for the practical, how many marks is the whole exam worth?
Steps:

1) Let \% = value you're given
$25 \%=50$
2) Find what $1 \%$ represents by dividing both sides

$$
1 \%=\frac{50}{25}=2
$$

3) Find $100 \%$ by multiplying by 100 :

$$
100 \%=2 \times 100=200 \mathrm{marks}
$$

## Note:

In this particular example, we could also have just multiplied 50 by 4 , as $25 \%$ represents $1 / 4$ of the total marks

## b) \% Profit / Loss / Discount:



## c) VAT:

## VAT excluded:

Example: Bill comes to $€ 120$. Find final bill with $13.5 \%$ VAT.

$$
\begin{aligned}
& \text { VAT }=13.5 \% \text { of } 120 \\
&=120 \times 0.0135=€ 16.20 \\
& \Rightarrow \text { Final Bill }=€ 120+€ 16.20=€ 136.20
\end{aligned}
$$

## VAT included:

Example: Bill including VAT comes to $€ 340.50$. Find bill without VAT, if VAT is $13.5 \%$.

$$
\begin{aligned}
& \text { Bill }+ \text { VAT }=€ 340.50 \\
& \Rightarrow 113.5 \%=€ 340.50 \\
& \Rightarrow 1 \%=€ 3 \\
& \Rightarrow 100 \%=€ 300
\end{aligned}
$$

3) Income Tax:
a) Income Tax Terminology:

- Gross Income: total pay someone gets before any taxes or deductions are taken
- Net Income: Take home pay or pay that we get after all taxes and deductions
- Rates Of Tax: Higher Rate (usually about $42 \%$ ) and Standard Rate (usually about 20\%)
- Standard Rate Cut-Off Point: Anything you earn up to this is taxed at the standard rate of tax
- Gross Tax: Total tax owing to the government before credits are deducted
- Tax Credits: Money deducted from the gross tax
- Tax Payable: Tax that you actually pay
- Statutory Deductions: Payments that you have to make to the government e.g. income tax (P.A.Y.E.)
- Non-statutory Deductions: Voluntary deductions that somebody pays e.g.s trade union fees or health insurance


## b) Answering Questions:

- The questions are nearly always made up of 3 parts:
- Part 1: Calculation of Gross Tax by adding.... Tax @ Lower Rate + Tax @ Higher Rate
- Part 2: Calculation of Tax Payable using the equation Tax Paid = Gross Tax - Tax Credits
- Part 3: Working out Net Income by taking off all deductions including Tax Paid, USC and PRSI (See below), Union Fees, Health Insurance etc.


## c) USC/PRSI:

USC: Have to be given the rates and bands:

- $2 \%$ of the first $€ 10036=€ 200.72$
- $4 \%$ of the next $€ 5980=€ 239.20$
- $7 \%$ on the balance of income $=>$ need to subtract ( 10036 + 5980) from Gross Income and then get 7\%


## PRSI:

- Usually in class $A, € 127 /$ week is free of PRSI deductions => $€ 127 \times 52=\$ 6604$ (needs to be taken from gross income)
- Then pay $4 \%$ on the remainder of your income.


## 4) Compound Interest/Depreciation:

## a) Terminology:

- Principal: Amount of money invested or borrowed
- Interest: Money added by the bank
- Rate: what percentage the interest is added at
- Amount or Final Value: The value of money at the end of the term it has been borrowed or invested for.
c) Depreciation: (items losing value)
- The formula on the right can be used for Depreciation problems.......just replace the '+' with a '-'......see below.
- The rate must be the same each year to use the formula.

where $F$ is the Final value, $P$ is the starting value, $i$ is the Rate of Depreciation as a decimal (e.g. $2.5 \%=0.025$ ) and $\dagger$ is the time in years.


## b) Answering Compound Interest Questions:

Method 1: Used if rates change from year to year or payments/withdrawals are being made between years

- Lay out Year 1, Year 2, Year 3 etc.
- Work out interest each year and add to Principal at start of the year
Method 2: Formula

where F is the Amount, P is the Principal, i is the Rate of Interest as a decimal (e.g. $3 \%=0.03$ ) and $t$ is the time in years the money is invested/borrowed for.


## Notes:

With utility bills (e.g.s. gas, electricity, water) there is usually a unit rate i.e. a charge per unit used
> To calculate the units used, subtract the previous units reading from the current units reading
> With many bills there is also a standing charge that has to be added on.
>VAT is also added to the bills.
> With Gas Bills, there is also a Carbon Tax that needs to be added on.

Example: Calculate the cost of electricity if the previous meter reading was 21310 and the current reading is 21836 , with a standing charge of $€ 21.60$. The cost per unit is $€ 0.15$ and VAT of $13.5 \%$ is added on.

Units used = Current Reading - Previous Reading
$=21836-21310=526$ units
Cost for electricity $=526 \times € 0.15=€ 78.90$
Standing Charge $=€ 21.60$ => Total Before VAT $=€ 78.90+€ 21.60=€ 100.50$ VAT $=13.5 \%$ of $€ 100.50=€ 13.57$ $\Rightarrow$ Final Bill $=€ 100.50+€ 13.57=€ 114.07$

