Topic 1: Arithmetic

1) The Basics:

a) Types of Numbers:

- Natural (N): Positive Whole Numbers: e.g. 1, 2, 3,
- Integers (Z): Positive and Negative Whole Numbers: e.g. -3, -2, -1, 0, 1, 2, 3,......
- **Real (R):** All numbers: e.g.s -3, -1.4, 0.2, 6, 7/2, $\sqrt{8}$
- **Rational (Q):** Numbers that <u>can</u> be written in the form $\frac{a}{b}$ e.g.s -5, 3, 1/2, -9/4.....
- **Irrational:** Numbers that <u>cannot</u> be written in the form $\frac{a}{b}$ e.q.s $\sqrt{3}$, $\sqrt{2}$, π
- Prime: A natural number bigger than 1 with only itself and 1 as divisors. e.g.s 2, 3, 5, 7, 11, 13, 17......

Composite: A number that is not prime. e.g.s., 6, 9, 15, 20

b) Rounding:

Rounding to Decimal Places:

- To round to 2 decimal places, look at the 3rd number after the decimal point.
 - If it's 5 or more we round UP the 2nd number
 - If it's 4 or less we round DOWN the 2nd number
- Similar approach for rounding to other decimal places

Examples: i) 4.768 = 4.77 ii) 3.2745 = 3.27

Rounding to Significant Figures:

- To round to 3 significant figures, we look at the 4th significant figure.
 - If it's 5 or more we round UP the 3rd number and replace subsequent numbers with Os
 - If it's 4 or less we round DOWN the 3rd number and replace subsequent numbers with Os

Examples: i) 132,421 = 132,000 ii) 0.00472543 = 0.00473

c) Scientific Notation

Notes:

- A number is in scientific notation if it's in the form $a \times a$ 10°, where 'a' has to be between 1 and 10.
 - **Examples:** i) $3400 = 3.4 \times 10^3$ ii) $0.004 = 4 \times 10^{-3}$
- On a Casio calculator the button you will need to type in numbers in scientific notation is:



- To type in 7×10^4 , press "7" and the button above and then
- To convert numbers into scientific notation on your calculator:
- Type in the number and press = to enter it on the screen. - Press "Shift" + "Mode" and select "Sci" from the menu. Then

d) Foreign Exchange

Steps:

- 1. Write the conversion with the currency you want on the right.
- 2. Get a 1 on the left-hand side, by dividing both sides.
- 3. Multiply both sides to get the value you want.

Example: If €1 = \$1.32, how many euro would you get for \$200? Step 1: Put euro on the right

Step 2: Get a 1 on the left-hand side

\$1 =
$$\epsilon \frac{1}{1.32}$$
 (dividing both sides by 1.32)

Step 3: Multiply both sides

\$200 =
$$\frac{1}{1.32}$$
 x 200 = €151.52

e) HCF/LCM using Prime Factors:

Notes:

- When asked to find the HCF and LCM of 2 numbers using prime factors, use your calculator.
- Type in the number first and then press Shift + Button below.



- For **HCF**: pick the **lowest** power of each prime number.
- For LCM: pick the highest power of each prime number.

Example: Find the HCF and LCM of 60 and 80.

Using calculator:
$$60 = 5^1 \times 3^1 \times 2^2$$
 $80 = 2^4 \times 5^1$

HCF:

- Lowest Power of 2 is 22
- Lowest Power of 3 is 31.....but 31 doesn't go into 80 so
- Lowest Power of 5 is $5^1 \Rightarrow$ HCF of 60 and 80 is $2^2 \times 5^1 = 10$

LCM:

- Highest Power of 2 is 24
- Highest Power of 3 is 31
- Highest Power of 5 is $5^1 \Rightarrow LCM$ of 60 and 80 is $2^4 \times 3^1 \times 5^1 =$ 240

f) Speed, Distance and Time:

For all speed, distance and time calculations remember:

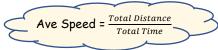
"Dads Silly Triangle"



If you want **Distance**, you cover the **D** in the triangle, so:

 $D = S \times T$ (Units are usually m or km)

- If you want Time, you cover the T in the triangle, so:
 - T = D / S (Units are usually secs or hrs) If you want **Speed**, you cover the **S** in the triangle, so:
 - S = D / T (Units might be m/s or km/h)
- Average speed can be calculated using:



Careful with units of time. E.g. 1hr 45mins = 1.75hrs etc.

g) Ratio

Notes:

- Ratio shows how to break up a quantity proportionally.
- When given a ratio, add the values in the ratio together to get the total number of parts the quantity is being broken into.
- Write down the fraction each person gets.

Example: Divide €200 between Alan and Brian in the ratio 3:2.

- 3:2 means there are 3 + 2 = 5 parts
- => Alan gets \(\frac{3}{2}\) and Brian gets \(\frac{2}{2}\) => Alan gets \(\frac{3}{2}\) of \(\inftige 200\) = \(\inftige 120\) and Brian gets \(\frac{2}{2}\) of \(\inftige 200\) = \(\inftige 80\)

2) Percentages/Profit/Loss/VAT:

a) Percentages:

To find the percentage of a number:

Example: Find 24% of 250. Method 1: Calculate $\frac{24}{100}x \frac{250}{1} = 60$ Method 2: Multiply 250 by 0.24 = 60

To find the total when given percentage:

Example: 25% of the marks in an exam are going for the practical part. If there are 50 marks for the practical, how many marks is the whole exam worth?

Stens:

1) Let % = value you're given

25% = 50

2) Find what 1% represents by dividing both sides

$$1\% = \frac{50}{25} = 2$$

3) Find 100% by multiplying by 100:

 $100\% = 2 \times 100 = 200$ marks

Note:

In this particular example, we could also have just multiplied 50 by 4, as 25% represents $^1\!/_4$ of the total marks

b) % Profit / Loss / Discount:

% Profit or Mark-Up =
$$\frac{Profit}{Cost \ Price} \times 100 \%$$

% Profit Margin = $\frac{Profit}{Selling \ Price} \times 100 \%$
% Loss = $\frac{Loss}{Cost \ Price} \times 100 \%$
% Discount = $\frac{Discount}{Cost \ Price} \times 100 \%$

c) VAT:

VAT excluded:

Example: Bill comes to €120. Find final bill with 13.5% VAT.

VAT = 13.5% of 120 = 120 × 0.0135 = €16.20

=> Final Bill = €120 + €16.20 = €136.20

VAT included:

Example: Bill including VAT comes to €340.50. Find bill without VAT, if VAT is 13.5%.

Bill + VAT = €340.50

=> 113.5% = €340.50

=> 1% = €3

=> 100% = €300

3) Income Tax:

a) Income Tax Terminology:

- Gross Income: total pay someone gets before any taxes or deductions are taken
- Net Income: Take home pay or pay that we get after all taxes and deductions
- Rates Of Tax: Higher Rate (usually about 42%) and Standard Rate (usually about 20%)
- Standard Rate Cut-Off Point: Anything you earn up to this is taxed at the standard rate of tax
- Gross Tax: Total tax owing to the government before credits are deducted
- Tax Credits: Money deducted from the gross tax
- Tax Payable: Tax that you actually pay
- Statutory Deductions: Payments that you have to make to the government e.g. income tax (P.A.Y.E.)
- Non-statutory Deductions: Voluntary deductions that somebody pays e.g.s trade union fees or health insurance

b) Answering Questions:

- The questions are nearly always made up of 3 parts:
- Part 1: Calculation of Gross Tax by adding...
 Tax @ Lower Rate + Tax @ Higher Rate
- Part 2: Calculation of Tax Payable using the equation
 Tax Paid = Gross Tax Tax Credits
- Part 3: Working out Net Income by taking off all deductions including Tax Paid, USC and PRSI (See below), Union Fees, Health Insurance etc.

c) USC/PRSI:

USC: Have to be given the rates and bands:

- 2% of the first €10036 = €200.72
- 4% of the next €5980 = €239.20
- 7% on the balance of income => need to subtract (10036 + 5980) from Gross Income and then get 7%

PRSI:

- Usually in class A, €127/week is free of PRSI deductions => €127 x 52 = \$6604 (needs to be taken from gross income)
- Then pay 4% on the remainder of your income.

4) Compound Interest/Depreciation:

a) Terminology:

- Principal: Amount of money invested or borrowed
- Interest: Money added by the bank
- Rate: what percentage the interest is added at
- Amount or Final Value: The value of money at the end of the term it has been borrowed or invested for.

c) Depreciation: (items losing value)

- The formula on the right can be used for Depreciation problems......just replace the '+' with a '-'.....see below.
- The rate must be the same each year to use the formula.

$$F = P(1-i)^t$$

See Tables

where ${\bf F}$ is the Final value, ${\bf P}$ is the starting value, ${\bf i}$ is the Rate of Depreciation as a **decimal** (e.g. 2.5% = 0.025) and ${\bf t}$ is the time in years.

b) Answering Compound Interest Questions:

Method 1: Used if rates change from year to year or payments/withdrawals are being made between years

- Lay out Year 1, Year 2, Year 3 etc.
- Work out interest each year and add to Principal at start of the year

See Tables

pg 30

Method 2: Formula

 $F = P(1+i)^t$

where **F** is the Amount, **P** is the Principal, **i** is the Rate of Interest as a **decimal** (e.g. 3% = 0.03) and **t** is the time in years the money is invested/borrowed for.

5) Household Bills:

Notes:

- With utility bills (e.g.s. gas, electricity, water) there is usually a unit rate i.e. a charge per unit used
- > To calculate the units used, subtract the previous units reading from the current units reading
- > With many bills there is also a standing charge that has to be added on.
- > VAT is also added to the bills.
- With Gas Bills, there is also a Carbon Tax that needs to be added on.

Example: Calculate the cost of electricity if the previous meter reading was 21310 and the current reading is 21836, with a standing charge of €21.60. The cost per unit is €0.15 and VAT of 13.5% is added on.

Units used = Current Reading - Previous Reading = 21836 - 21310 = 526 units Cost for electricity = 526 × €0.15 = €78.90 Standing Charge = €21.60 => Total Before VAT = €78.90 + €21.60 = €100.50 VAT = 13.5% of €100.50 = €13.57 => Final Bill = €100.50 + €13.57 = €114.07