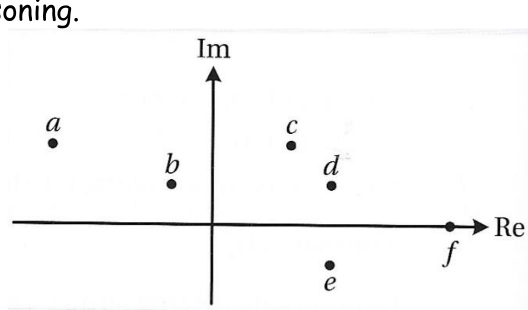
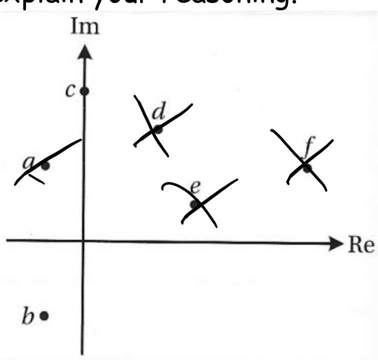
Topic: Complex Numbers (Topics 91 to 96)

Q1. Express $\frac{5-2i}{4+3i}$ in the form $p + qi$. **Ans:** $\frac{14}{25} - \frac{23}{25}i$	**Q2.** Find the real numbers p and q such that $2(p + qi) + i(p - qi) = 5 + i$. **Ans:** p = 3, q = -1
Q3. Solve for z: $z^2 - 6z + 10 = 0$. **Ans:** $3 \pm i$	**Q4.** Solve for w: $\sqrt{5}\lvert w\rvert + iw = 3 + i$. Write answer in the form $u + iv$. **Ans:** $1 + 2i$ or $1 - \frac{1}{2}i$
Q5. Express $-1 + i\sqrt{3}$ in polar form. **Ans:** $2(\cos 120 + i \sin 120)$	**Q6.** Express $-4 - 4i$ in polar form. **Ans:** $4\sqrt{2}(\cos 225 + i \sin 225)$
Q7. Solve for z: $z^2 = 2 - 2\sqrt{3}i$. **Ans:** $\sqrt{3} - i$ or $-\sqrt{3} + i$	**Q8.** Express $(\frac{1}{2} - \frac{\sqrt{3}}{2}i)^{15}$ in the form $a + bi$. **Ans:** $-1 + 0i$
Q9. Use DeMoivre's Theorem to express $(-\sqrt{3} - i)^{10}$ in the form $2^n(1 - i\sqrt{k})$. **Ans:** $2^9(1 - \sqrt{3}i)$	**Q10.** Express $\sin 3\theta$ in terms of $\sin\theta$ using DeMoivre's Theorem. **Ans:** $3\sin\theta - 4\sin^3\theta$
Q11. If $z = 5 - ki$, where $k \in R$ and $i^2 = -1$. If $z^2 - 10z = -29$, find the possible values of k. **Ans:** k = -2 or 2	**Q12.** Solve the equation $z^4 = -8 - 8\sqrt{3}i$ to find its 4 roots. **Ans:** $1 + \sqrt{3}i, -1 - \sqrt{3}i, \sqrt{3} - i, -\sqrt{3} + i$
Q13. Let $w_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $w_2 = (w_1)^2$. Verify that $x^2 + xy + y^2 = (x - w_1 y)(x - w_2 y)$, where $x, y \in R$.	**Q14.** (i) Solve $z^3 + 8i = 0$. (ii) Sketch the three roots and discuss the geometrical properties of the roots. **Ans:** (i) $2i, \sqrt{3} - i, -\sqrt{3} - i$
Q15. w is a complex number such that $w\bar{w} - 2iw = 7 - 4i$, where $\bar{w}$ is the complex conjugate of w. Find the two possible values of w. Express each in the form $p + qi$, where $p, q \in R$. **Ans:** $w = 2 - 3i, 2 + i$	**Q16.** (i) Let $z = \frac{5}{2+i} - 1$, where $i^2 = -1$. Express z in the form $a + bi$ and plot it on an Argand diagram. (ii) Use De Moivre's theorem to evaluate z^6. **Ans:** (i) $1 - i$ (ii) $8i$
Q17. Given that $z = 2 + i$, where $i^2 = -1$, find the real number d such that $z + \frac{d}{z}$ is real. **Ans:** 5	**Q18.** $P(z) = z^3 + (2 - 2i)z^2 + (3 - 3i)z - 1 - 4i$. Find the values of $a, b \in C$ if $P(z) = (z - i)(z^2 + az + b)$. **Ans:** $a = 2 - i, b = 4 - i$
Q19. The Argand Diagram below shows the points a, b, c, d, e and f representing complex numbers. If these complex numbers are $z, \bar{z}, z + \bar{z}, w, z + w$ and $z.w$, determine which point represents each complex number. Copy the diagram and use it to explain your reasoning.  	**Q20.** The Argand Diagram shows the points a, b, c, d, e and f representing complex numbers. If these complex numbers are $z, \bar{z}, w, 2w, z - \bar{z}$ and $z + w$, determine which point represents each complex number. Copy the diagram and use it to explain your reasoning. 

Q21. If $3-2i$ and $1+i$ are two of the roots of the equation $z^4+az^3+bz^2+cz+d=0$ find the values of $a,b,c,d \in R$. **Ans:** a = -8, b = 27, c = -38, d = 26	**Q22.** Show that z^2-16 is a factor of $z^3+(1+i)z^2-16z-16(1+i)$ and hence find the three roots of $z^3+(1+i)z^2-16z-16(1+i)=0$. **Ans:** z = 4, -4 or $-1-i$
Q23. (i) Find the two complex numbers $a+bi$ for which $(a+bi)^2=15+8i$. (ii) Solve the equation $iz^2+(2-3i)z+(-5+5i)=0$. **Ans:** $2-i, 1+3i$	**Q24.** $z_1=a+bi$ and $z_2=c+di$, where $i^2=-1$. Show that $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$, where $\bar{z}$ is the complex conjugate of z.
Q25. Solve the quadratic equation $2iz^2+(6+2i)z+(3-6i)=0$, where $i^2=-1$. **Ans:** $\frac{1+3i}{2}, \frac{-3+3i}{2}$	**Q26.** If ki is a root of the equation $3z^3-z^2+12z-4=0$, find the values of $k \in C$ and the other root of the equation. **Ans:** k = 2, -2 and 3rd root = $\frac{1}{3}$