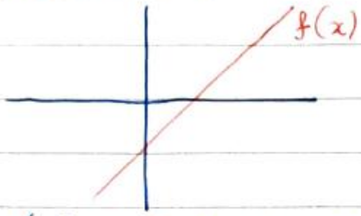
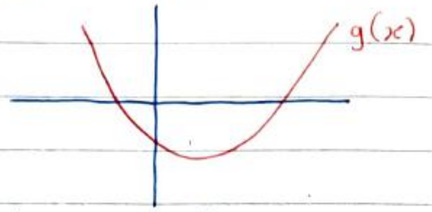
Worked Solutions

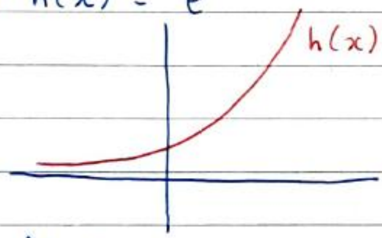
Q1. i) $f(x) = 2x - 3$



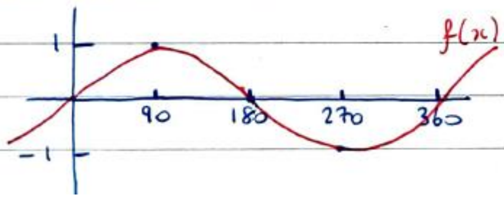
ii) $g(x) = 3x^2 - 5x + 2$



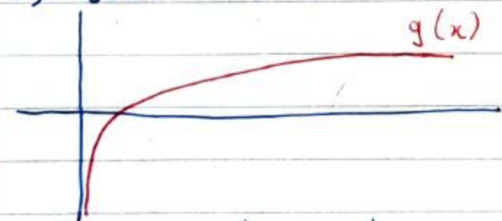
iii) $h(x) = e^x$



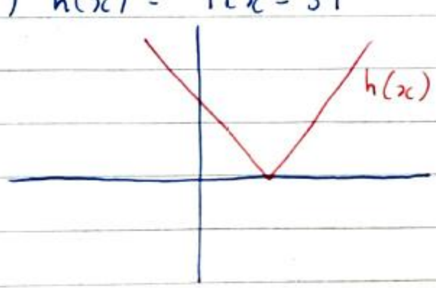
iv) $f(x) = \sin x$



v) $g(x) = \ln x$



vi) $h(x) = |2x - 3|$



Q2.

i) $f(x) = x^3 - 2x^2 + 3x, \ -3 \le x \le 4$

$\boxed{(-3,-54), (-2,-22), (-1,-6), (0,0), (1,2), (2,6), (3,18), (4,44)}$

ii) $g(x) = \ln(2x-3), \ 3 \le x \le 6$

$\boxed{(3, 1.0986), (4, 1.61), (5, 1.95), (6, 2.2)}$

iii) $h(\theta) = 2\sin 3\theta, \ -180 \le \theta \le 180°$

As its 3θ, we "step" in $30°$ as the period will be $\frac{360}{3} = 120° \div 4 = 30°$

$\boxed{(-180,0), (-150,-2), (-120,0), (-90,2), (-60,0), (-30,-2), (0,0), (30,2), (60,0), (90,-2), (120,0), (150,2), (180,0)}$

Q3. $p(x) = 5x + 4 \quad g(x) = 3x - 2$

i) $pg(x) \Rightarrow g(x)$ first

$p(3x-2) = 5(3x-2) + 4$

$= 15x - 10 + 4$

$= \boxed{15x - 6}$

ii) $g^2(x) = gg(x) \Rightarrow g(x)$ first

$g(x) = 3x - 2$

$g(3x-2) = 3(3x-2) - 2$

$= 9x - 6 - 2$

$= \boxed{9x - 8}$

Q4. $f(x) = x - \frac{3}{2}$ $\quad g(x) = \frac{2x-5}{3}$

i) $fg(4) \Rightarrow g(4)$ first

$g(4) = \frac{2(4)-5}{3} = \frac{3}{3} = 1$

$f(1) = 1 - \frac{3}{2} = \boxed{-\frac{1}{2}}$

ii) $gf(x) \Rightarrow f(x)$ first

$f(x) = x - \frac{3}{2}$

$g(x - \frac{3}{2}) = \frac{2(x-\frac{3}{2})-5}{3}$

$= \frac{2x-3-5}{3}$

$= \boxed{\frac{2x-8}{3}}$

iii) $f^2(x) = ff(x) \Rightarrow f(x)$ first

$f(x) = x - \frac{3}{2}$

$f(x-\frac{3}{2}) = (x - \frac{3}{2}) - \frac{3}{2}$

$= \boxed{x - 3}$

Q5. $f(x) = e^{2x-1}$ $\quad g(x) = \sqrt{x}$

i) $fg(9) \Rightarrow g(9)$ first

$g(9) = \sqrt{9} = 3$ as $x > 0$

$f(9) = e^{2(9)-1}$

$= \boxed{e^{17}}$

ii) $gf(x) \Rightarrow f(x)$ first

$f(x) = e^{2x-1}$

$g(e^{2x-1}) = \boxed{\sqrt{e^{2x-1}}}$

iii) $f^{-1}g(x) \Rightarrow g(x)$ first

$g(x) = \sqrt{x}$

To find $f^{-1}(x)$:

$y = e^{2x-1}$

$\ln y = \ln e^{2x-1}$

$\ln y = 2x - 1$

$1 + \ln y = 2x$

$\frac{1+\ln y}{2} = x$

$\Rightarrow y = \frac{1+\ln x}{2}$ (Swap x & y)

$\Rightarrow f^{-1}(x) = \frac{1+\ln x}{2}$

$f^{-1}(\sqrt{x}) = \boxed{\frac{1+\ln\sqrt{x}}{2}}$

Q6. $f(x) = 6x - 3$ $\quad g(x) = 5x + q$

$fg(x)$: $\Rightarrow g(x)$ first

$g(x) = 5x + q$

$f(5x+q) = 6(5x+q) - 3$

$= 30x + 6q - 3$

$gf(x)$: $\Rightarrow f(x)$ first

$f(x) = 6x - 3$

$g(6x-3) = 5(6x-3) + q$

$= 30x - 15 + q$

$fg(x) = gf(x)$

$\cancel{30x} + 6q - 3 = \cancel{30x} - 15 + q$

$5q = -12$

$q = \boxed{\frac{-12}{5}}$

Q7. $f(x) = 6x - 1$

L.H.S : $f[(x + f(x))]$?

$x + f(x) = x + 6x - 1$

$= 7x - 1$

$f(7x-1) = 6(7x-1) - 1$

$= 42x - 6 - 1$

$= 42x - 7$

R.H.S : $p[f(x)]$

$f(x) = 6x - 1$

$p[f(x)] = p(6x-1)$

$= 6xp - p$

LHS = RHS

$42x - 7 = 6xp - p$

Comparing x terms:

$\Rightarrow 42x = 6xp$

$\boxed{p = 7}$

Q8 $f(x) = 3x - 5 \quad g(x) = 2x + 4$

$fg(x) \Rightarrow g(x)$ first

$g(x) = 2x + 4$

$f(2x+4) = 3(2x+4) - 5$

$= 6x + 12 - 5$

$= 6x + 7$

$fg(x) = 24$

$\Rightarrow 6x + 7 = 24$

$6x = 17$

$x = \boxed{17/6}$

Q9. $f(x) = 3^x \quad g(x) = x + 2$

$fg(x)$: $\Rightarrow g(x)$ first

$g(x) = x + 2$

$f(x+2) = 3^{x+2}$

$gf(x)$: $\Rightarrow f(x)$ first

$f(x) = 3^x$

$g(3^x) = (3^x) + 2 = 3^x + 2$

$fg(x) = gf(x)$

$\Rightarrow 3^{x+2} = 3^x + 2$

$3^x . 3^2 = 3^x + 2$

$9.3^x = 3^x + 2$

$9.3^x - 1.3^x = 2$

$8.3^x = 2$

$3^x = 2/8 = 1/4$

$\log 3^x = \log 1/4$

$x . \log 3 = \log 1/4$

$\Rightarrow x = \frac{\log 1/4}{\log 3} = \boxed{-1.262}$

Q10.

i) $f(x) = x^2 + 8x - 3$

$= \boxed{(x+4)^2 - 19}$

ii) $g(x) = x^2 - 10x + 8$

$= \boxed{(x-5)^2 - 17}$

iii) $h(x) = x^2 + 12x + 43$

$= \boxed{(x+6)^2 + 7}$

Q11. i) $f(x) = 9x^2 + 36x - 4$

$= 9\left[x^2 + 4x - \frac{4}{9}\right]$

$= 9\left[(x+2)^2 - \frac{40}{9}\right]$

$= \boxed{9(x+2)^2 - 40}$

ii) $g(x) = 4x^2 - 20x + 31$

$= 4\left[x^2 - 5x + \frac{31}{4}\right]$

$= 4\left[\left(x - \frac{5}{2}\right)^2 + \frac{3}{2}\right]$

$= \boxed{4\left(x - \frac{5}{2}\right)^2 + 6}$

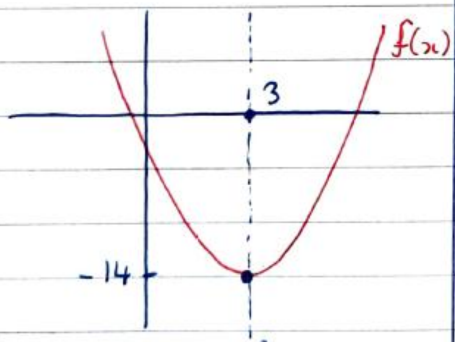
Q12. i) $f(x) = x^2 - 6x - 5$

$= \boxed{(x-3)^2 - 14}$

ii) Min pt $= \boxed{(3, -14)}$

Axis of sym: $\boxed{x = 3}$

iii)



iv) $y = (x-3)^2 - 14$

$y + 14 = (x-3)^2$

$x - 3 = \sqrt{y+14}$

$x = \sqrt{y+14} + 3$

Swap x and y:

$y = \sqrt{x+14} + 3$

$\Rightarrow f^{-1}(x) = \boxed{\sqrt{x+14} + 3}$

Q13. i) $f(x) = x^2 - 6x + 13$

$= \boxed{(x-3)^2 + 4}$

ii) $y = (x-3)^2 + 4$

$y - 4 = (x-3)^2$

$x - 3 = \sqrt{y-4}$

$x = \sqrt{y-4} + 3$

Swap x and y:

$y = \sqrt{x-3} + 3$

$\Rightarrow f^{-1}(x) = \boxed{\sqrt{x-3} + 3}$

$\sqrt{}$ can't be negative

$\Rightarrow x - 3 \geq 0$

$\boxed{x \geq 3}$ = Domain

iii) $p \circ h \Rightarrow h$ first

$h(x) = x - 3$

$p(x-3) = f(x)$

$p(x-3) = (x-3)^2 + 4$

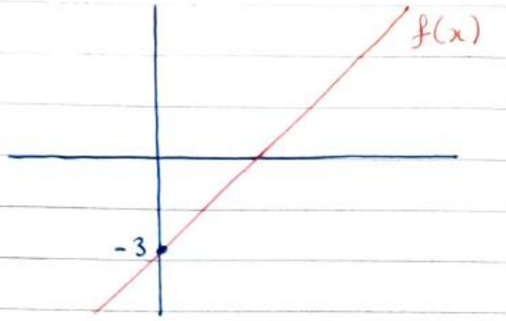
$x - 3 \rightarrow \boxed{} \rightarrow (x-3)^2 + 4$

$\Rightarrow$ function must square the input and add 4

$\Rightarrow p(x) = \boxed{x^2 + 4}$

Q14. $f(x) = 5x - 3$ $\mathbb{R} \to \mathbb{R}$

i)



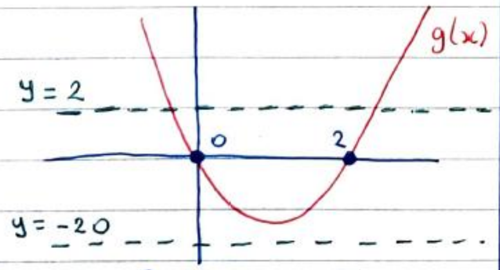
ii) Injective? Yes as any horizontal line in graph above will cut at most once

Surjective? Yes as any horizontal line in graph above will cut at least once.

Bijective? Yes as it is both injective and surjective

Q15. $g(x) = x^2 - 2x$ $\mathbb{R} \to \mathbb{R}$

i)

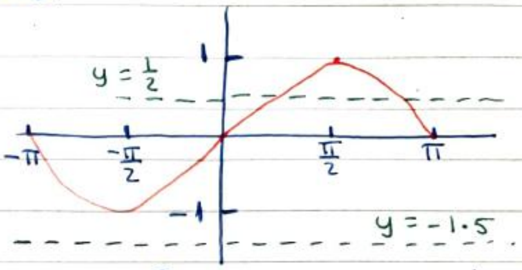


ii) Injective? No as line $y = 2$ cuts more than once.

Surjective? No as line $y = -20$ never cuts the graph.

Bijective? No as it's neither injective nor surjective.

Q16.



Injective? No as line $y = \frac{1}{2}$ cuts twice in diag above.

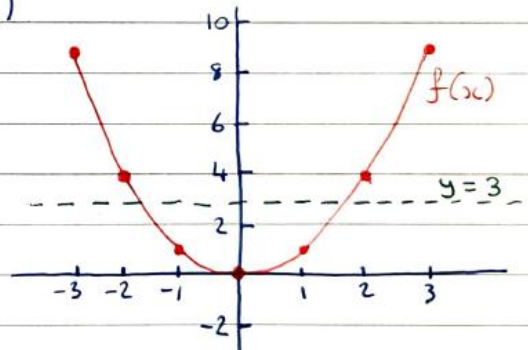
Surjective? No as line $y = -1.5$ never cuts the graph.

Bijective? No as it's neither injective nor surjective.

Q17.

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9

i)



ii) Not injective as line $y = 3$ crosses twice.

As codomain is positive real numbers, it is surjective as all horizontal lines above the x-axis will cut the graph at least once.

iii) Domain changed to $\boxed{x \geq 0}$ or $\boxed{x \leq 0}$

Q18. i) $f(x) = 5x - 2$

$y = 5x - 2$

$5x = y + 2$

$x = \dfrac{y+2}{5}$

Swap x and y:

$y = \boxed{\dfrac{x+2}{5}}$

ii) $g(x) = x^2 - 6x$

$y = x^2 - 6x$

$y = (x-3)^2 - 9$

$9 + y = (x-3)^2$

$x = 3 + \sqrt{y+9}$

Swap x and y:

$y = \boxed{3 + \sqrt{x+9}}$

iii) $h(x) = 2(3^x)$

$y = 2.(3^x)$

$\dfrac{y}{2} = 3^x$

$\log \dfrac{y}{2} = \log 3^x$

$x.\log 3 = \log y/2$

$x = \dfrac{\log y/2}{\log 3}$

Swap x and y:

$y = \boxed{\dfrac{\log x/2}{\log 3}}$

iv) $P(x) = \sqrt{2x-4}$

$(y)^2 = (\sqrt{2x-4})^2$

$2x - 4 = y^2$

$2x = y^2 + 4$

$x = \dfrac{y^2+4}{2}$

Swap x and y:

$y = \boxed{\dfrac{x^2+4}{2}}$

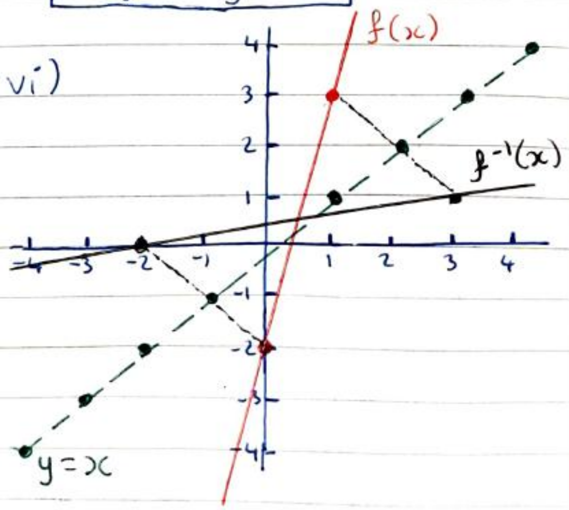
v) Domain of $p(x)$ was $x \geq 2$ as $\sqrt{\ }$ can't be negative.

Range of $p(x)$ was $y \geq 0$ as you can't get a negative number out of a square root

$\Rightarrow$ Domain and range of $p^{-1}(x)$ will be the opposite

$\Rightarrow$ $\boxed{\begin{array}{l}\text{Domain}: x \geq 0 \\ \text{Range}: y \geq 2\end{array}}$ and

vi)



Q19. i) $f(x) = \frac{x-6}{4}$

$y = \frac{x-6}{4}$

$4y = x - 6$

$x = 4y + 6$

Swap x and y:

$y = \boxed{4x + 6}$

ii) $g(x) = 4x + \frac{2}{3}$

$y = 4x + \frac{2}{3}$

$y - \frac{2}{3} = 4x$

$x = \frac{y - \frac{2}{3}}{4}$

$= \frac{\frac{3y-2}{3}}{4}$

$x = \frac{3y-2}{12}$

Swap x and y:

$y = \boxed{\frac{3x-2}{12}}$ or $\boxed{\frac{x-\frac{2}{3}}{4}}$

iii) $h(x) = \sqrt{x+4}$

$(y)^2 = (\sqrt{x+4})^2$

$y^2 = x + 4$

$x = y^2 - 4$

Swap x and y:

$y = \boxed{x^2 - 4}$

Q20.

i) $(y)^2 = (\sqrt{x-3})^2$

$y^2 = x - 3$

$x = y^2 + 3$

Swap x and y:

$y = \boxed{x^2 + 3}$

Domain of $f(x) = x \geq 3$

$\Rightarrow$ Range of $f^{-1}(x) = \boxed{x \geq 3}$

ii) $y = x^3$

$x = \sqrt[3]{y}$

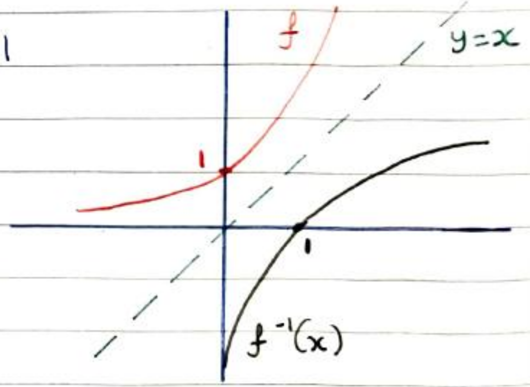
Swap x and y:

$y = \boxed{\sqrt[3]{x}}$

Domain of $g(x) = x \in \mathbb{R}$

$\Rightarrow$ Range of $g^{-1}(x) = \boxed{x \in \mathbb{R}}$

Q21



Q22