## Leaving Cert Higher \& Ordinary Level Constructions:

## Higher \& Ordinary:

1. Bisector of an angle, using only a compass and a straight edge.
2. Perpendicular bisector of a segment, using only a compass and a straight edge.
3. Line perpendicular to a given line 1 , passing through a given point not on 1 .
4. Line perpendicular to a given line 1 , passing through a given point on 1 .
5. Line parallel to a given line, through a given point.
6. Division of a line segment into 2 or 3 equal parts, without measuring it.
7. Division of a line segment into n equal segments, without measurement.
8. Line segment of a given length on a given ray.
9. Angle of a given number of degrees with a given ray as one arm.
10. Triangle, given the length of three sides. (SSS)
11. Triangle, given two sides and the included angle. (SAS)
12. Triangle, given two angles and the common side. (ASA)
13. Right-angled triangle, given length of hypotenuse and one other side. (RHS)
14. Right-angled triangle, given one side and one of the acute angles.
15. Rectangle, given side lengths.
16. Circumcentre and circumcircle of a given triangle, using ruler and compass.
17. Incentre and incircle of a given triangle, using ruler and compass.
18. Angle of $60^{\circ}$, without using a protractor or set square.
19. Tangent to a given circle at a given point on it.
20. Parallelogram, given the length of the sides and the measure of the angles.
21. Centroid of a triangle.

## Higher Level Only:

22. Orthocentre of a triangle

## 1. Bisect an angle:

| After doing this |
| :--- | :--- |
| Start with angle PQR that we will bisect. |
| 1. Place the compass point on the angle's vertex Q . |
| 4. The compass width can be changed here if desired. |
| Recommended: leave it the same. |
| 2. Adjust the compass to a medium wide setting. The exact |
| width is not important. |
| each leg of the angle. |

After doing this
5. Place the compass on the point where one arc crosses a leg
and draw an arc in the interior of the angle.
6. Without changing the compass setting repeat for the other
leg so that the two arcs cross.
Done. This is the bisector of the angle $\angle P Q R$.
7he point where the arcs cross

## 2. Perpendicular bisector of a line segment:

|  | After doing this | Start with a line segment PQ. |
| :--- | :--- | :--- |
| 1 | Place the compass on one end <br> of the line segment. |  |
| 2 | Set the compass width to a <br> approximately two thirds the line <br> length. The actual width does not <br> matter. | Without changing the compass look like this <br> width, draw an arc above and <br> below the line. |
| 4 |  |  |


| 5 | Using a straightedge, draw a line between the points where the arcs intersect. |  |
| :---: | :---: | :---: |
| 6 | Done. This line is perpendicular to the first line and bisects it (cuts it at the exact midpoint of the line). |  |

3. Perpendicular to a line through a point: (Higher Level)

|  | After doing this | Start with a line and point R which is not on that line. |
| :--- | :--- | :--- |


|  | After doing this |  |
| :--- | :--- | :--- |
| Step 5 | Place a straightedge between R and the point where <br> the arcs intersect. Draw the perpendicular line from R to <br> the line, or beyond if you wish. |  |
| Step 6 | Done. This line is perpendicular to the first line and <br> passes through the point R . It also bisects the segment <br> PQ (divides it into two equal parts) |  |

## 4. Perpendicular at a point on a line:

|  | After doing this | Start with a line and point K on that line. |
| :--- | :--- | :--- | :--- | :--- |



## 5. Line Parallel to a given line through a given point:

| After doing this |  |
| :--- | :--- |
| Start with a line PQ and a point R off the |  |
| line. |  |
| 1. Draw a transverse line through R and |  |
| across the line PQ at an angle, forming |  |
| the point J where it intersects the line PQ. |  |
| The exact angle is not important. |  |


| After doing this |
| :--- | :--- |
| 4. Set compass width to the distance |
| where the lower arc crosses the two lines. |

6. Divide a line segment into 2 or 3 equal parts:

|  | After doing this | Your work should look like this |
| :---: | :---: | :---: |
|  | Start with a line segment $A B$ that we will divide up into 2 or 3 (in this case) equal parts. | $\stackrel{\text { A }}{\square}$ |
| Step 1 | From point A , draw a line segment at an angle to the given line, and about the same length. The exact length is not important. |  |
| Step 2 | Set the compass on A , and set its width to a bit less than one third of the length of the new line. |  |
| Step 3 | Step the compass along the line, marking off $2 / 3$ arcs (depending on how many segments you want). Label the last one C . |  |
| Step 4 | Using a ruler, join C to B. |  |
| Step 5 | Using a set square and a ruler, construct two more lines that are parallel to CB from each of the other two arcs. |  |
| Step 6 | Line $A B$ should now be split into 2 or 3 equal parts. |  |

7. Divide a line segment into n equal parts: (Higher Level)

|  | After doing this | Your work should look like this |
| :---: | :---: | :---: |
|  | Start with a line segment AB that we will divide up into 5 (in this case) equal parts. | ${ }^{\text {a }}$ |
| Step 1 | From point A, draw a line segment at an angle to the given line, and about the same length. The exact length is not important. |  |
| Step 2 | Set the compass on $A$, and set its width to a bit less than one fifth of the length of the new line. |  |
| Step 3 | Step the compass along the line, marking off 5 arcs. Label the last one C . |  |
| Step 4 | With the compass width set to CB, draw an arc from $A$ just below it. |  |
| Step 5 | With the compass width set to AC, draw an arc from $B$ crossing the one drawn in step 4 . This intersection is point $D$. |  |


| Step 6 | After doing this | Draw a line from D to B . |
| :--- | :--- | :--- |
| Step 7 | Using the same compass width as used to step <br> along AC, step the compass from D along DB <br> making 4 new arcs across the line |  |
| Step 8 | Draw lines between the corresponding points along <br> AC and DB. |  |
| Step 9 | Done. The lines divide the given line segment AB <br> in to 5 congruent parts. |  |

8. Line segment of a given length on a given ray:

|  | After doing this | Your work should look like this |
| :---: | :---: | :---: |
|  | Draw a line segment $\|A B\|$ of the required length with a ruler. | $\mathrm{A} \frac{4.5 \mathrm{~cm}}{\mathrm{~B}}$ |
| 1 | Draw a ray [OK], the length of which should be greater than $4.5 \mathrm{~cm} \ldots$...about 8 cm . |  |
| 2 | Mark off a point P on the ray [OK]. |  |
| 3 | Using the line segment $[A B]$, set your compass width to the length of the line $[A B]$. <br> Place your compass needle at P and swing an arc on the ray [OK]. <br> Label the intersection of the arc and the ray as C . |  |
| 4 | Join P to C. |  |

9. Angle of a given number of degrees with a given ray as one arm:

|  | After doing this | Your work should look like this |
| :--- | :--- | :--- |
| 1 | Draw a line m. |  |
| 2 |  |  |

## 10. Triangle given three sides (SSS):

| After doing this | Start with three line segments that will be |
| :--- | :--- |
| the three sides of the triangle ABC . |  |

After doing this

| 4. Mark a point B on this arc. This will |
| :--- |
| become the next vertex of the new triangle. |


| 5. Set the compass width to the length of |
| :--- |
| the line segment AC. |

7. Use the compass to measure the length
of the segment BC , the length of the third
side of the triangle.
8. Place the compass point on A and make
an are in the vicinity of where the third
vertex of the triangle (C) will be. All points
along this arc are the distance AC from A,
but we do not yet quite know exactly where
the vertex C is.

| After doing this | Your work should look like this |
| :---: | :---: |
| 8. From point $B$, draw an arc crossing the first. Where these intersect is the vertex $C$ of the triangle |  |
| 9. Finally, draw the three sides $A B, A C$, and $B C$ of the new triangle. |  |
| 10. Done. The blue triangle $A B C$ has each side congruent to the the corresponding line segment. |  |

11. Triangle given two sides and included angle (SAS):

| After doing this | Your work should look like this |
| :---: | :---: |
| Start with two line segments and the included angle. | A. <br> A. $\qquad$ <br> $\rightarrow$ B |
| 1. Mark a point $A$ that will be one vertex of the new triangle. |  |
| 2. Draw a ray from point $A$. This will become the side $A B$ of the new triangle, so make it longer than $A B$. | A |
| 3. Set the compass width to the length of the given side $A B$. | A |


| After doing this | Your work should look like this |
| :---: | :---: |
| 4. Set the compass on $A$, and mark a point $B$ on the ray just drawn. | $\qquad$ |
| 5. With the compass set to any convenient width, from the point A on the given angle, draw an arc across both lines.. |  |
| 6. Without changing the compass width, draw a similar sized arc at point A on the new triangle. | $A=C$ |


| After doing this | Your work should look like this |
| :---: | :---: |
| 7. Set the compass to the arc width at the given angle A. This the distance between the points where the arc intersects the sides of the angle. |  |
| 8. Make a similar arc on the new triangle so it crosses the previous arc. |  |
| 9. Draw a ray from A , through where the arcs intersect and onwards. This will become side $A C$ of the triangle so make it longer than $A C$. |  |

After doing this

| After doing this | Your work should look like this |
| :--- | :--- | :--- |
| Done, the triangle ABC has the desired two side <br> lengths and included angle. |  |

## 12. Triangle given two angles and included side (ASA):

| After doing this |  |
| :--- | :--- |
| Start with the given line segment <br> and two angles. | Nour work should look like this <br> two angles. The lines making up the given angles have random lengths that <br> have no significance in the construction. |

The first part of this construction (steps $1-4$ ) is to copy a line segment to form one side of the new triangle. (See Copying a Line Segment).

1. Mark a point A that will be one
vertex of the new triangle.
2. Set the compass width to the
length of the segment AB .
3. With the compass point on A,
make an arc near the future vertex
B of the triangle.
After doing this

| 4. Mark a point B on this arc. Then |
| :--- |
| draw the line AB . This will be one |
| side of the new triangle. |
| What we do now is |

5. With the compass at any
convenient width, draw an arc
across both lines of the given angle
A.
6. Set the compass to the arc width
at the given angle A. This the
distance between the points where
the arc intersects the sides of the
angle.

| After doing this |  |
| :--- | :--- |
| 8. Near point A draw an arc in a <br> similar position so it crosses the arc <br> drawn earlier. This, in effect, <br> 'copies' the measure of the angle at <br> P to the angle at A . |  |
| 9. Draw a line from A through the <br> point where the arcs intersect. This <br> will become the second side of the <br> triangle. Draw it long. |  |
| 10. Repeat this process at B . <br> Copying the angle measure from <br> the given angle B to the new <br> triangle at B . The point where the <br> lines intersect is C , the third vertex <br> of the triangle. |  |


| After doing this | Draw a rough sketch of the triangle first. |
| :--- | :--- |
| 1. Draw a line segment to represent the base, using a ruler. |  |
| Label the left most point of the line segment A . |  | 2. Place your protractor at point A and draw a line at an angle

## 14.

 Right-angled Triangle given one side and one of the acute angles:| After doing this | Draw a rough sketch of the triangle first. |
| :--- | :--- |
| 1. Draw a line segment to represent the base, using a ruler. |  |
| Label the leftmost point A . |  | 2. Place your protractor at point A and draw a line at an angle

## 15. Rectangle:

| After doing this |
| :--- | :--- |
| Draw a rough sketch of the rectangle first. |
| 1. Using a ruler, draw a horizontal line longer than 7 cm <br> in length. <br> Using a ruler, mark off a line segment [AB] measuring <br> 7cm. Label the end points A and B . |
| 2. Set your compass to a small radius and using A as a <br> centre, draw a circle as shown. Label the two points <br> where the circle crosses the line as P and Q as shown. |
| 3. Place your compass needle at points P and Q and <br> swing two arcs as shown. |

## 16. Circumcircle of a triangle:

| After doing this |
| :--- | :--- |
| We start with a triangle ABC. |
| 1. Find the bisector of one of the triangle sides. Any one will do. |
| 2. Repeat for another side. Any one will do. |
| 3. The point where these two perpendiculars intersect is the |
| triangle's circumcentre, the center of the circle we desire. Note: |
| This point may lie outside the triangle. This is normal. |


| After doing this |
| :--- |
| 4. Place the compass point on the intersection of the |
| perpendiculars and set the compass width to one of the points |
| A,B or C. Draw a circle that will pass through all three. |
| 5. Done. The circle drawn is the triangle's circumcircle, the only |
| circle that will pass through all three of its vertices. |

## 17. Incircle of a triangle:

| After doing this |
| :--- | :--- |
| We start with the given triangle. |
| 1. Place the compass point on any of the triangle's |
| vertices. Adjust the compass to a medium width |
| setting. The exact width is not important. |
| 3. Change the compass width if desired, then from |
| the point where each arc crosses the side, draw two |
| arcs inside the triangle so that they cross each |
| other, using the same compass width for each. |
| 2. Without changing the compass width, strike an |
| arc across each adjacent side. |


| After doing this |
| :--- | :--- |
| 4. Using the straightedge, draw a line from the |
| vertex of the triangle to where the last two arcs |
| cross. |
| 5. Repeat all of the above at any other vertex of the |
| triangle. You will now have two new lines drawn. |
| 6. Where the two new lines intersect, mark a point |
| as the incentre of the triangle. |
| 7. Draw the perpendicular from the incentre to a |
| side of the triangle. Label the point where it meets |
| the side M. |


| After doing this | Your work should look like this |
| :--- | :--- |
|  |  |
| 8. Place the compass on the incentre and set the |  |
| width to point M . This is the radius of the incircle. |  |

18. $\quad \underline{0^{\circ}}$ angle:

| After doing this |
| :--- | :--- |
| 1. Draw a line segment which will become one side of the angle. |
| (Skip this step if you are given this line.) The exact length is not |
| important. Label it PQ . P will be the angle's vertex. |
| 2. Set the compass on P , and set its width to any convenient |
| setting. |
| 3. Draw an arc across PQ and up over above the point P . |
| 4. Without changing the compass width, move the compass to the |
| point where the arc crosses PQ , and make an arc that crosses the |
| first one. |
| 6. Draw a line from P , through the intersection of the two arcs. |

19. Tangent at a point on a circle:
After doing this
20. Draw a circle with centre o and mark a point $p$ on the
circumference.
21. Join o to $p$ to form a radius.
22. Use a ruler to extend the tangent to a point $S$ as shown.
23. Place one edge of a set square along the radius [op] and draw a
tangent along the second edge as shown.

## 20. Parallelogram:

| After doing this | Draw a rough sketch of the parallelogram first. |
| :--- | :--- |
| 1. Using a ruler, draw a horizontal line segment 8 cm in |  |
| length. Label the end points A and B . |  |
| 2. Place your protractor on point A . Draw an angle of |  |
| 70 |  |

## 21. Centroid of a triangle:

| After doing this | We start with a triangle PQR. |
| :--- | :--- |
| 1. Construct the bisector of the line segment PQ. Label the |  |
| midpoint of the line S . |  |


| After doing this |
| :--- | :--- |
| 4. Draw the median from the midpoint T to the opposite |
| vertex P |

## 22. Orthocentre of a triangle:

| After doing this | We start with the triangle ABC . |
| :--- | :--- |
| 1. Construct a perpendicular line to [BC] passing through the opposite vertex |  |
| A. |  |
| 3. Mark the point where the two lines you drew overlap. This is the |  |
| 2. Repeat step 1 for another side and opposite vertex e.g. side [AC] and |  |
| through vertex B . |  |

