# Networks & Graphs Worked Solutions

**Q1.** A walk is a succession of edges where the end of one is the start of the next. If all the nodes on a walk are different then the walk is known as a path. A cycle is a path that starts and ends at the same vertex.

**Q2.**

$$\begin{pmatrix} 2 & 0 & 1 \\ 1 & -2 & 0 \\ 4 & -3 & 2 \end{pmatrix}\begin{pmatrix} -1 & 2 & 0 \\ 3 & 1 & -1 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -2+0+0 & 4+0+2 & 0+0+1 \\ -1-6+0 & 2-2+0 & 0+2+0 \\ -4-9+0 & 8-3+4 & 0+3+2 \end{pmatrix}$$

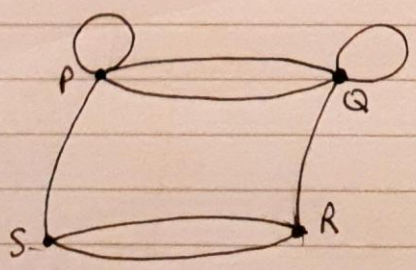
$$= \begin{pmatrix} -2 & 6 & 1 \\ -7 & 0 & 2 \\ -13 & 9 & 5 \end{pmatrix}$$

**Q3.** i)

$$A = \begin{array}{c} \\ P \\ Q \\ R \\ S \end{array}\begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{pmatrix} \end{array}$$

ii) 8

iii)




8 walks of length 4:

RSPPS x2
RQPPS x2
RQQPS x2
RQQRS x2

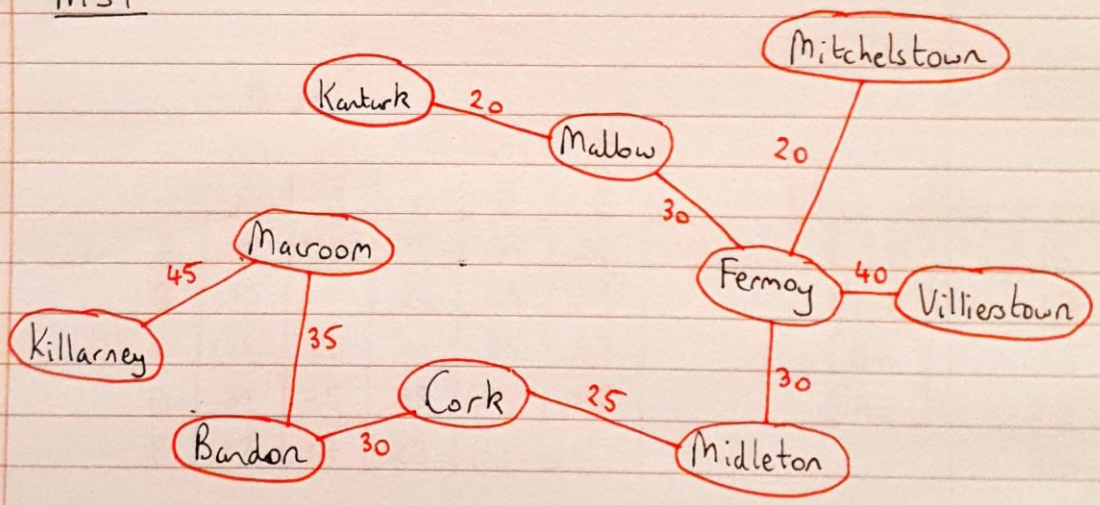
Q4. Edges added in order:

| | Edge | Weight |
|---|---|---|
| ① | Mitchelstown – Fermoy | 20 |
| ② | Kanturk – Mallow | 20 |
| ③ | Cork – Midleton | 25 |
| ④ | Fermoy – Midleton | 30 |
| ⑤ | Fermoy – Mallow | 30 |
| ⑥ | Cork – Bandon | 30 |
| ⑦ | Macroom – Bandon | 35 |
| ⑧ | Fermoy – Villierstown | 40 |
| ⑨ | Macroom – Killarney | 45 |
| | | 275 |

→ Min Weight

* Note: Edge weights are not distinct so minimum spanning tree is not unique but weight would still be 275.

MST



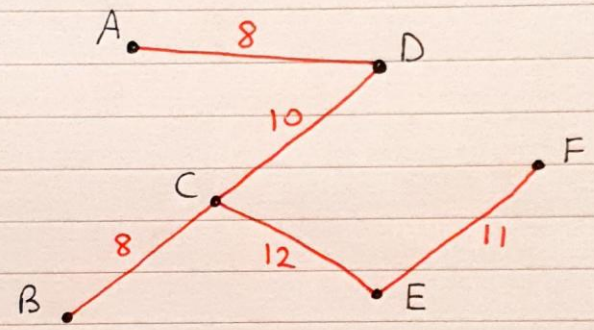
Q5

| | Nodes Visited | Weights |
|---|---|---|
| | C | – |
| (1) | B | 8 |
| (2) | D | 10 |
| (3) | A | 8 |
| (4) | E | 12 |
| (5) | F | 11 |
| | | 49 |

→ Min Weight

* Note: Edge weights are not distinct so minimum sparing tree is not unique.

MST :




Q6.

| | (4) A | (3) B | C | (1) D | (2) E |
|---|---|---|---|---|---|
| A | ~~–~~ | (15) | ~~18~~ | ~~31~~ | ~~22~~ |
| B | ~~15~~ | ~~–~~ | ~~26~~ | ~~19~~ | (17) |
| C | (18) | 26 | – | 35 | 23 |
| D | ~~31~~ | ~~19~~ | ~~35~~ | ~~–~~ | ~~15~~ |
| E | ~~22~~ | ~~17~~ | ~~23~~ | (15) | ~~–~~ |

Edges added in order

[DE] : 15
[EB] : 17
[BA] : 15
[AC] : 18
65

→ Min Weight

MST :