Leaving Certificate Examination

Sample Paper 1

Applied Mathematics

Higher Level
2 hours and 30 minutes

400 marks

Examination Number

For examiner	
Question	Mark
1	/50
2	/50
3	/50
4	/50
5	/50
6	/50
7	/50
8	/50
~~9~~	~~/50~~
~~10~~	~~/50~~
Written Total	/400
Project	/100
Overall Total	/500
Overall Grade	

Sample Paper 1

Question 1

(a)

A car of mass 1200 kg starts from rest and travels along a straight horizontal road. The engine of the car exerts a constant power of 3000 W.

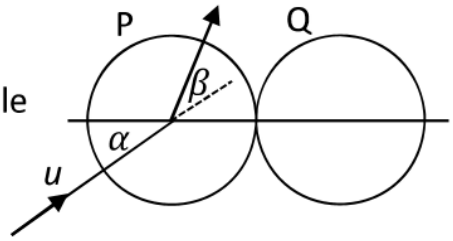
If there is no resistance to the motion of the car, find

(i) the speed of the car after 3 minutes

(ii) the average speed of the car during this time.

(b)

A smooth sphere P has mass m and speed u. It collides obliquely with a smooth sphere Q, of mass m, which is at rest. Before the collision, the direction of P makes an angle α with the line of centres, as shown in the diagram.



The coefficient of restitution between the spheres is $\frac{1}{3}$.

During the impact the direction of motion of P is turned through an angle β.

Show that $\tan\beta = \frac{2\tan\alpha}{1+3\tan^2\alpha}$.

Question 2

(a)

A train takes 40 minutes to travel from rest at station A to rest at station B. The distance between the stations is 20 km. The train left station A at 10:00. At 10:15 the speed of the train was 32 km h^{-1} and at 10:30 the speed was 48 km h^{-1}.

The speed of 48 km h^{-1} was maintained until the brakes were applied, causing a uniform deceleration which brought the train to rest at B.

During the first and second 15-minute intervals the accelerations were constant.

(i) Draw a speed-time graph of the motion.

(ii) Find the time taken for the first 16 km.

(iii) Find the deceleration of the train.

(b)

A woman takes out a loan of €120,000 to build an extension to her house. The bank agrees to a 15-year loan at a monthly percentage rate (MPR) of 0.4%.

(i) What is the annual percentage rate correct to 3 decimal places?

(ii) If D_n is the amount of debt owing after n months and A the amount she pays back each month, write down a difference equation in D_n.

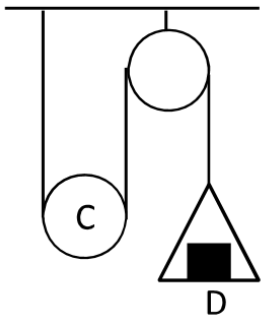
(iii) Solve the difference equation.

(iv) Find, to the nearest cent, the amount she will have to pay back every month.

Question 3

(a)

The diagram shows a light inextensible string having one end fixed, passing under a smooth movable pulley C of mass km kg and then over a fixed smooth pulley. The other end of the string is attached to a light scale pan. A bock D of mass m kg is placed symmetrically on the centre of the scale pan.
The system is released from rest. The scale pan moves upwards.



(i) Show that $k > 2$.

(ii) Find, in terms of k and m, the tension in the string.

(iii) Find, in terms of k and m, the reaction between D and the scale pan.

(b)

(i) Evaluate the following: $\int x^2 \ln x \, . \, dx$

(ii) An elastic constant has natural length 3 m and elastic constant 20 N/m. Find the work fone in stretching the string to a length of 7 m.

Question 4

(a)

A particle is projected from a point on horizontal ground. The speed of projection is 14 m s^{-1} at an angle α to the horizontal.

Find the two values of α that will give a range of 10 m.

(b)

A 60 gram mass is projected vertically upwards with an initial speed of 15 m s^{-1} and half a second later a 40 gram mass is projected vertically upwards from the same point with an initial speed of 22·65 m s^{-1}.

(i) Calculate the height at which the masses will collide.

The masses coalesce on colliding.

(ii) Find the greatest height which the combined mass will reach.

Question 5

(a)

Aoife is a Leaving Cert student. During her exams, she has a break of 5 days. She decides to dedicate three of these days to studying the three subjects she has left (Classical Studies, Economics and Applied Maths). She will not divide up any day between two or three subjects but will spend each day studying one particular subject. She reckons that she will improve her percentage mark by the following amounts:

Number of days	1	2	3
Increase in Classical Studies	11%	21%	26%
Increase in Economics	9%	17%	27%
Increase in Applied Maths	12%	20%	25%

How many days should Aoife allocate to each subject in order to improve her grades by the maximum amount?

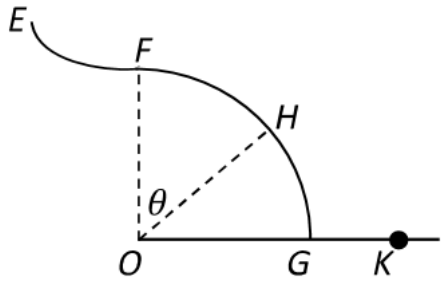
Subject	Days Available	Days Allocated	Days Left	% Increase

(b)

A smooth slide *EFG* is in the shape of two arcs, *EF* and *FG*, each of radius *r*. The centre *O* of arc *FG* is vertically below *F* as shown in the diagram.

Point *E* is at a height $\frac{r}{5}$ above point *F*.

A child starts from rest at *E*, moves along the slide past the point *F* and loses contact with the slide at point *H*. *OH* makes an angle θ with the vertical.



(i) Find the value of θ.

The child lands in a sandpit at point K.

(ii) Find, in terms of r, the speed of the child at K.

Question 6

(a)

A small smooth sphere A, of mass $3m$ moving with speed u, collides directly with a small smooth sphere B, of mass m moving with speed u in the opposite direction. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(i) Find, in terms of u, the speed of each sphere after the collision.

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$.
The first collision between the spheres occurred at a distance 2 metres from the wall.
The spheres collide again 4 seconds after the first collision between them.

(ii) Find the value of u.

(b)

A particle starts from rest and moves in a straight line with acceleration $(25 - 10v)$ m s^{-2}, where v is the speed of the particle.

(i) After time t, find v in terms of t. (Note: $\int \frac{dx}{a+bx} = \frac{1}{b} ln|a + bx| + c$).

(ii) Find the time taken to acquire a speed of 2·25 m s^{-1} and find the distance travelled in this time.

Question 7

(a)

A steamboat of mass m has a power output of $12m$ watts. When the boat is travelling at speed v, the water exerts a drag on the boat of mkv^3 newtons, where k is a constant. The maximum speed of the boat is $6\ m/s$.

(i) Find the value of k.

(ii) The maximum acceleration is $7.5\ m/s^2$ when the speed of the boat is u. Find the value of u.

(b)

A particle P travelling in a straight line has a deceleration of $4v^{n+1}$ m s^{-2}, where n (> 0) is a constant and v is its speed at time t (> 0).

P has an initial speed of u.

(i) Find an expression for v in terms of u, n and t.

(ii) When $n = 3$ obtain an expression for the speed of P when it has travelled a distance of 3 m from its initial position.

Question 8

(a)

A small smooth sphere A, of mass 1·5 kg, moving with speed 6 m s^{-1}, collides directly with a small smooth sphere B, of mass m kg, which is at rest.

After the collision the spheres move in opposite directions with speeds v and $2v$, respectively.

80% of the kinetic energy lost by A as a result of the collision is transferred to B.

The coefficient of restitution between the spheres is e.

(i) Find the value of v

(ii) Find the value of e

(b)

A particle is projected vertically upwards with an initial speed of $2g\ m/s$ in a medium in which there is a resistance of $kv^2\ N$ per unit mass where v is the speed of the particle and k is a constant, where $k > 0$.

Prove that the maximum height reached is $\frac{1}{2k}\ln(1 + 4kg)$.

Page for extra work.

Label any extra work clearly with the question number and part.

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