Topic 3: Patterns/Sequences

1) Arithmetic Sequences/Series:

a) Linear(Arithmetic) Sequences:

- A list of numbers where the **difference** between **each term** is the **same** every time.
  E.g. 3, 8, 13, 18, ………
- In Senior Cycle, we refer to these sequences as **Arithmetic** Sequences.
- The General Term for an Arithmetic sequence can be found using:

$$T_n = a + (n-1)d$$

See Tables pg 22

where 'a' is the first term and 'd' is the common difference between the terms.

Example: i) Find the General Term for the sequence 3, 8, 13, 18…..
ii) Find the 50th term, T_{50}.

a = 3 and d = 5

i) => $T_n = 3 + (n-1)5$	ii) $T_n = 5n - 2$
=> $T_n = 3 + 5n - 5$	=> $T_{50} = 5(50) - 2$
=> $T_n = 5n - 2$	=> $T_{50} = 250 - 2 = 248$

b) Arithmetic Series:

- If we add the terms of an arithmetic sequence together, then we get an arithmetic **series**.
- We need to be able to find the sum of the first n terms of such a series, which we can find using:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

See Tables pg 22

where 'a' is the first term and 'd' is the common difference between the terms of the series.

Example: Find the sum of the first 20 terms of the series 2 + 6 + 10 + 14 + …….

a = 2 and d = 4

=> $S_{20} = \frac{20}{2}\{2(2) + (20-1)4\}$

=> $S_n = 10\{4 + (19)4\}$

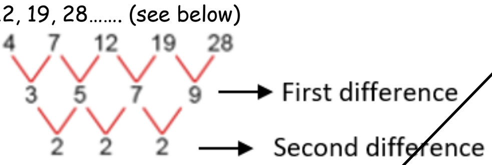
=> $S_n = 10\{80\}$

=> $S_n = 800$

2) Non-Linear Sequences:

a) Quadratic Sequences:

- A sequence where the **second difference** is the **same** every time.
  E.g. 4, 7, 12, 19, 28……. (see below)



Steps to find General Term:

1. Let the General Term $T_n = an^2 + bn + c$.
2. The second difference represents 2a, so halving the second difference gave us a value for a……in the sequence above, the second difference is +2, so 'a' would be 1.
3. Find 'c' by finding Term 0.
4. Use any of the terms to find 'b'.

Example: Find the General Term of the sequence 4, 7, 12, 19, 28
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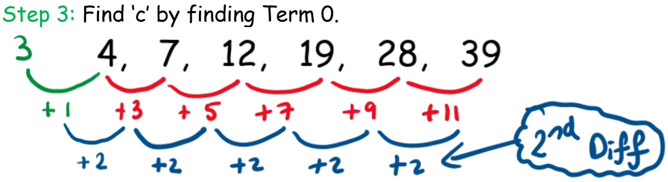
Step 3: Find 'c' by finding Term 0.

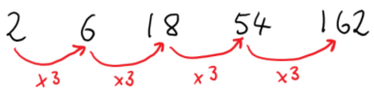


=> c = 3

Step 4: Use any of the terms to find 'b'.
$T_n = 1n^2 + bn + 3$ and $T_1 = 4$
=> $T_1 = 1(1)^2 + b(1) + 3 = 4$ => b = 0
=> $T_n = n^2 + 0n + 3$ => $T_n = n^2 + 3$

b) Exponential Sequences:

- A sequence where each term is found by multiplying the previous term by the same number every time.
  E.g. 2, 6, 18, 54, 162………



c) Cubic Sequences

- A sequence where the **third difference** is the **same** every time.
  E.g. 4, 14, 40, 88, 164……. (see below)