Topic: Arithmetic/Financial Maths (Topics 73 to 76)

Q1. A car loan is €20,000 is to be repaid in 25 equal instalments. If the effective interest rate is 2%, calculate the amount of each instalment. **Ans:** €1024

Q2. A sum of money is invested at an AER of 6%. Calculate how many years it will take to double in value. **Ans:** 12

Q3. Silvia is planning an overseas trip lasting 3 years and she estimates that she will need €600 per month for expenses. How much money does she need to have saved to fund this trip? Assume an average rate of interest of 4% over the period of the trip.
Ans: €20,344.29

Q4. A credit card company offers clients an introductory interest rate on outstanding balances of 1.25% per month, and a regular rate of 2.5% per month after 1 year. Find the equivalent interest rates (AER) per annum.
Ans: Introductory: 16.1% Regular: 34.5%

Q5. A bank offers you a rate of 10% on a 20-year mortgage to be paid in monthly repayments. If the most you can afford to pay in monthly repayments is €700, find the value of the biggest mortgage you can afford.
Ans: €74.736

Q6. An annuity involves saving €3000 per year at 7.3% per annum for 8 years.
(i) Calculate the single amount of money which could be invested at the same rate and for the same amount of time to give the same final amount.
(ii) Find the final amount of the investment.
Ans: (i) €19,000.13 (ii) €33,385.23

Q7. Which is the better result at the end of 20 years?
(i) An investment of €100,000 at 12% per annum compounded monthly or (ii) €1000 invested monthly at 12% per annum compounded monthly.
Ans: (i) amounts to €964,629.31 and (ii) amounts to €919,857.64 => Option 2

Q8. You are 35 years-old today and you are planning for your retirement needs. You expect to retire at the age of 65 years and actuarial studies suggest that you will live to be a 100 year-old. You want to move to a country location when you retire. You estimate that it will cost you €300,000 to move (on your 65th birthday) and your living expenses will be €20,000 a year, starting at the end of the first year after retirement. Assuming an average annual rate of 4% over the lifetime of the plan,
(i) how much will you need to have saved on your retirement to afford this plan?
(ii) You have €40,000 in savings now. If you can invest this money (tax-free) at 5% per year, how much money do you need to save each year in order to afford your retirement plan?
(iii) If you have no savings and could not start saving for another 5 years, how much would you then have to set aside each year to afford this plan? **Ans:** (i) €673,292.26 (ii) €7173.30 (iii) P = €13,435.36

Q9. Your company has an expected pension liability of €500,000 in 10 years time. (i) What amount of money would you now require to cover this expected liability? Assume an annual rate of 9%. (ii) How much would you need to set aside at the end of each year for the next 10 years to cover the liability (assuming the same rate applies)?
Ans: (i) €211,205.40 (ii) €32,910

Q10. Assume that you are going to retire in 25 years time. You want a mortgage of €100,000 now to extend and renovate your house but want to have it paid in full before you retire. The maximum repayment per month your budget will allow is €800. Using trial and error, what is the rate of interest you need from your bank to have the loan repaid in 300 monthly repayments (i.e. 25 years)?
Ans: 8.75%

Q11. A sum of money is invested at an AER of i. Show that the number of years it takes to double in value can be written $\log_{(1+i)} 2$.

Q12. A pupil saves money each day in the month of November. The pupil saves 10c on the first day of November, and every day after that he saves 5c more than the previous day. How much does he save in total in the 30 days of November? **Ans:** €24.75

Q13. A person saves €x at the beginning of each year for 4 consecutive years at an effective annual rate of 10%. The total value of the investments at the end of the fourth year was €51,051. Find the value of x.
Ans: 1000

Q14. A company invested €100,000 in new machinery at the beginning of each year for three consecutive years. The machinery depreciated at the rate of 10% per annum. (i) Find the value of the first investment of €100,000 at the end of the third year (ii) Find the total value of all the investments at the end of the third year.
Ans: (i) €72900 (ii) €243900

Q15. Eugene invested €2500 for three years at compound interest. The effective rate of annual interest was 4% for the first year and 3% for the second year. (i) Calculate the value of the investment after two years. (ii) If the investment amounted to €2744.95 after three years, calculate the rate of interest in the third year.
Ans: (i) €2678 (ii) 2.5%

Q16. Shane takes out a car loan of €15,000, at an AER of 9.5%. The loan is to be repaid by equal monthly payments starting one month after he draws down the loan, and is to continue for 3 years i.e. 36 repayments (i) Find the monthly interest rate, correct to six decimal places, that is equivalent to an AER of 9.5% (ii) When he draws down the loan, find the amount of his monthly repayments (iii) On the date of his last repayment, find the amount of his monthly repayments. What do you notice?
Ans: (i) 0.007592 (ii) €477.76 (iii) €477.76

Q17. Leaky Homes Ltd is a development company. Five years ago it borrowed €23 million from a bank and three years ago it borrowed a further €18 million. Four years ago it repayed €7.2 million and six months ago it repayed a further €9.3 million. It wants to borrow €8.4 million more now, and promises to pay back all its debts with two equal repayments, one in eighteen months' time and the other in three years' time. If the AER for the transactions is 4%, find the values of the two equal repayments, correct to six significant figures. **Ans:** €21.1388 million

Q18. Nora invests €600 at the beginning of each year for three consecutive years at an AER of 4.5%. (i) Find the value of all her investments at the end of the third year (ii) Instead, she decides to invest €50 a month, starting now and continuing for a total of 36 months. The AER of interest is the same. How much less will her investments be worth at the end of the third year?
Ans: (i) €1966.91 (ii) €39.12

Q19. A company invest €25,000 in machinery at the beginning of each year for twelve consecutive years. The machinery depreciates at the rate of 15% per annum compound depreciation. Find the total value of all the machinery at the end of twelve years, correct to the nearest euro. **Ans:** €121515.75

Q20. Lucy is due to retire in exactly 10 years. She wants to boost her pension fund as much as possible between now and then. She cannot afford to save any more than €800 a month between now and the date of her retirement. The AER for pension fund contributions is 7%. (i) If she invests €800 a month, starting now, for the next 120 months, what will be the value of these investments on the date she retires? (ii) From other sources, she has an additional €300,000 in her pension fund on the date she retires. With her total pension fund she wishes to buy an annuity, starting on the date she retires and continuing each year for a total of 25 years. If these payments are equal, and the AER is 4% after her retirement, find the amount she will receive each year for 25 years.
Ans: (i) €137615.11 (ii) €26935.19

Q21. Tony saves €25 from his wages each week, putting it in a savings account that gives an AER of 3.75%. (i) What weekly rate of interest is equivalent to an AER of 3.75%? Give your answer to six decimal places, taking 1 year = 52 weeks. (ii) What is the total value of Tony's savings at the end of 10 years, i.e. one week after he has made his 520^{th} saving?
Ans: (i) 0.000708 (ii) €15720.34

Q22. Helen is due to pay Joe €25000 in two years' time and €30000 in four years' time. She offers to pay him €47000 now instead. Taking the annual rate of discount to be 5%, determine if Joe should accept her offer.
Ans: No

Q23. A company invests €P in new machinery. The machinery depreciates at the rate of i per annum. If the machinery depreciates to one quarter of its original value after 8 years, find i, correct to 3 decimal place. **Ans:** 0.159

Q24. Today is John's birthday. He plans to save an amount on his birthday each year, starting today and going 20 years. He plans to save €200 today but to increase the amount he saves by 5% each year. The AER of interest throughout is 4%. (i) Calculate the value of all John's savings 20 years from now. (ii) If tax at 21% is deducted from his interest each year, find the value of all his savings after 20 years. Give your answers correct to the nearest euro.
Ans: (i) €9613 (ii) €8861

Q25. Yvonne earns €61,000 in a year. Her standard rate cut off point is €32,700 and tax credit are €3250 for the year. The standard rate of tax is 20% and the higher rate is 41%, The first €127 per week of her income is exempt from PRSI, but she pays PRSI at the rate of 4% on all her income above this. She also pays USC at the rate of 2% on the first €10036 of her income, 4% on the next €5980, and 7% on all income above this. (i) Calculate Yvonne's take home pay for the year and her monthly take home pay (ii) She is offered the opportunity to earn an extra €6000 a year by working overtime for three nights a week. If she takes the offer, she will have to pay extra childcare costs of €220 a month. Do you think she should take the offer? Give reasons.
Ans: (i) €40342.36, €3361.86