Topic 2: Patterns/Sequences

1) Arithmetic Sequences/Series:

a) Linear Sequences:

- A list of numbers where the **difference** between **each term** is the **same** every time.
  E.g. 3, 8, 13, 18, ………
- The **general term of a sequence** (T_n) is a formula that can be used to find the value of any term of the sequence.
- We can also find it by observing the sequence and figuring out the pattern.

Example: Find the general term for the sequence 3, 8, 13, 18…….

Common Difference = +5

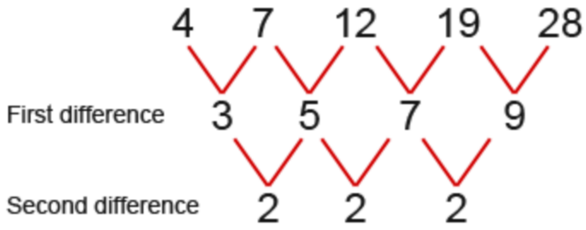
Term Number	Pattern	Term Value
1	5(1)-2	3
2	5(2)-2	8
3	5(3)-2	13
4	5(4)-2	18
n	5(n)-2	5n-2

=> General Term: $T_n = 5n - 2$

- Once we have the General Term, we can find ANY term in the sequence.
  E.g. What is 50th term?
  $T_{50} = 5(50) - 2$
  $= 248$
- The general term also allows us to work back and find what term number a value would be.
  E.g. What term would 458 be?
  $T_n = 458$
  $5n - 2 = 458$
  $5n = 458 + 2$
  $5n = 460$
  $n = 92$ => 92nd term

b) Quadratic Sequences:

- A sequence where the **second difference** is the **same** every time.
  E.g. 4, 7, 12, 19, 28……. (see below)



Steps to find General Term:

1. Let General Term = $T_n = an^2 + bn + c$
2. Find 2nd difference and let = 2a….solve for a.
3. Use any 2 terms to form two equations in b and c.

Example: Find the General Term of the sequence 4, 7, 12, 19, 28

Step 1: Let the General Term $T_n = an^2 + bn + c$.
Step 2: The second difference represented 2a, so halving the second difference gave us a value for a……in the sequence above, the second difference is +2, so 'a' would be +1.
Step 3: Use two of the terms in the sequence to make two simultaneous equations, which we solve to find 'b' and 'c'……
$T_n = an^2 + bn + c$

$T_2 = (2)^2 + b(2) + c = 7$ => $4 + 2b + c = 7$ => $2b + c = 3$…..Eqn 1	$T_3 = (3)^2 + b(3) + c = 12$ => $9 + 3b + c = 12$ => $3b + c = 3$……Eqn 2

Solving Equations 1 and 2 gives b = 0 and c = 3
=> $T_n = n^2 + (0)n + 3$
=> $T_n = n^2 + 3$

c) Exponential Sequences:

- A sequence where each term is found by **multiplying** the previous term by the same number every time.
  E.g. 2, 6, 18, 54, 162………