- Chapter 1: Units & Vectors
- Topic 1: Units & Dimensional Analysis

- Units:
  - It's important to be aware of the standard units that are used in the measurements and calculations we will be doing in our course.
  - They are summarised in the table below:

Quantity	SI Unit	Abbreviation	Variables Used	Dimensions using m, kg or s only
Distance / Displacement	metre	m	s, x, h, l	m
Time	second	s	t	s
Mass	kilogram	kg	m	kg
Force	newton	N	F	kg m/s^2
Work, Energy	joule	J	W, E	kg m^2/s^2
Power	watt	W	P	kg m^2/s^3
Velocity	m/s	m/s	u, v	m/s
Acceleration	m/s^2	m/s^2	a, g, f	m/s^2

- Dimensional Analysis:
  - We will be using lots of formulae over the next two years.
  - To evaluate if a formula is a valid one or not, we can perform a dimensional analysis on it.
  - To make it easier to carry out the dimensional analysis, we stick to kg, m and s for all units used.

- Examples: Ex 1.A Pg 7 Qs 6/8

Carry out a dimensional analysis on the following formulae: (i) $s = \frac{1}{2}(u+v)t$ (ii) $E = mgh$

Solution:

(i) From the table above, we start by filling in the correct units for each variable in our formula, and ignore the constant of $\frac{1}{2}$:

$$s = \frac{1}{2}(u+v)t$$
$$m = \left(\frac{m}{s} + \frac{m}{s}\right)s$$

- We can then add the m/s units together in the brackets to get:

$$m = \left(\frac{m}{s}\right)s$$

- And then the 'seconds' cancel out giving:

$$m = m$$

=> Our formula is reasonable

(ii) Again, from the table above, we start by filling in the correct units for each variable in our formula:

$$E = mgh$$
$$kg\,\frac{m^2}{s^2} = (kg)(\frac{m}{s^2})(m)$$

- Multiplying the 'm' by the 'm' on the right gives:

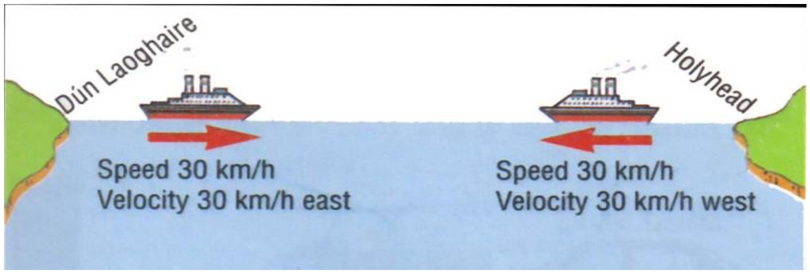
$$kg\,\frac{m^2}{s^2} = kg\,\frac{m^2}{s^2}$$

=> Our formula is reasonable

Classwork Questions: Pg 7 Ex 1A Qs 1/4/5/7/9/11/12/14

Topic 2: Introduction to Vectors and Scalars

- You might recall learning the about speed and velocity in Junior Cert Science. They were both similar but with one key difference.
- Speed was the distance travelled by an object per unit time whereas velocity was the distance travelled by an object per unit time AND the direction in which it is travelling.
- If we look at the example below, both ferries are travelling at the same speed as they're both travelling at 30km/h but they have different velocities as they are moving in different directions.

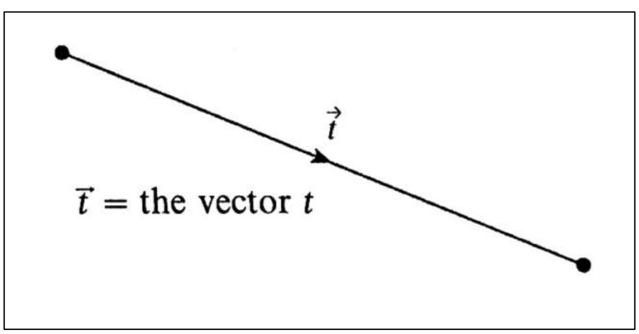


- A quantity that has a magnitude (or size) only is known as a scalar quantity.
  - Examples of scalar quantities would be height, length and speed
- A quantity that has a magnitude (or size) and a direction is known as a vector quantity.
  - Examples of vectors include velocity, acceleration and forces

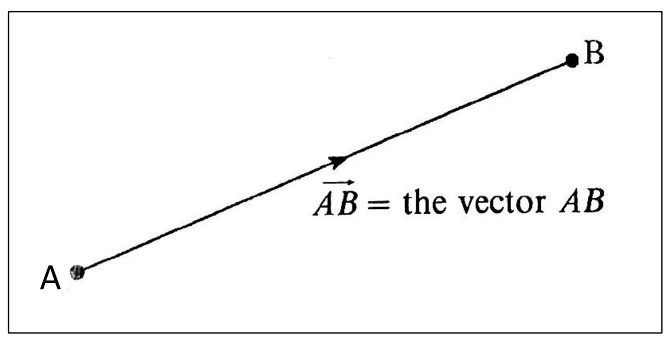
Notation of Vectors:

- We represent vectors using tipped arrows and the length of the arrow usually indicates the "strength" of the quantity i.e. a longer vector represents a stronger force
- We also use small arrows when naming vectors as seen below:

i) Naming a single vector

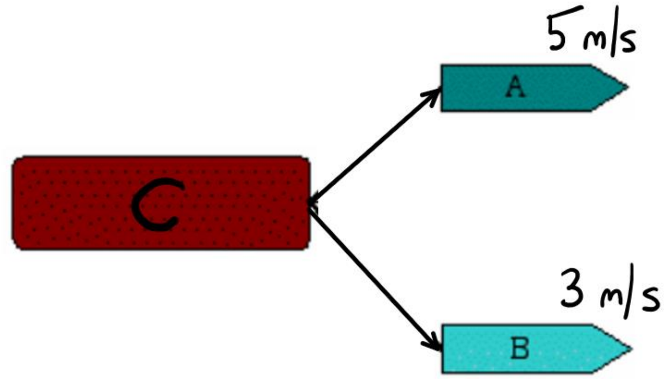


ii) Naming a vector between two points



Note: The arrow on the top of the vector name doesn't indicate direction i.e. $\vec{a}$

- Adding two vectors:
  - Consider the situation shown below where two tug boats A and B are pulling a larger ship C into harbour.
  - If the two tug boats are pulling at different velocities, the question is, what velocity would the ship be moving at?



  - To be able to answer this question, we need to know how to add vectors together.
  - To add vectors, we use the Parallelogram Method.

- Parallelogram Method:
  - If we wanted to add the two vectors $\vec{a}$ and $\vec{b}$ together, in Figure 1 below, we complete the parallelogram made up by the two vectors, as seen in Figure 2 below.
  - We then draw the diagonal of the parallelogram, which is known as the resultant or the sum of $\vec{a}$ and $\vec{b}$, as in Figure 3 below.

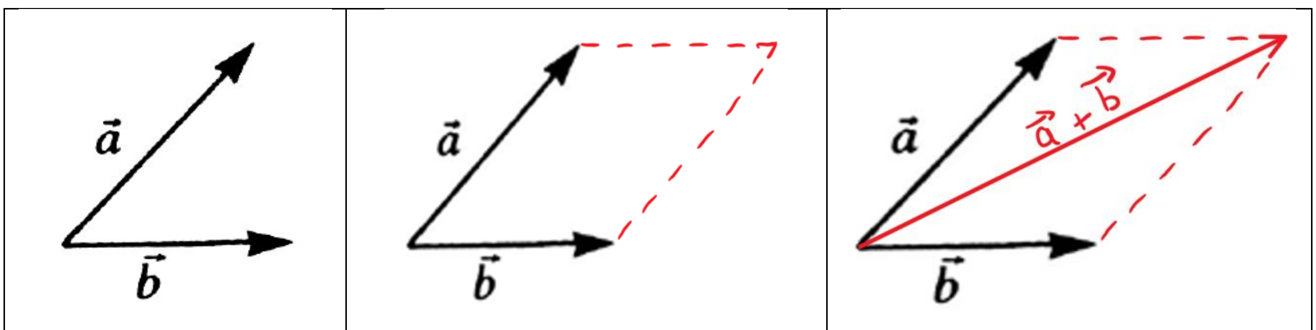
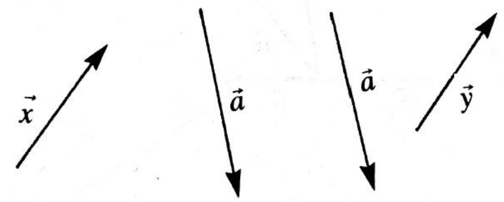


Fig 1 Fig 2 Fig 3

Important Notes:

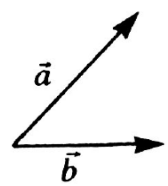
1) If two vectors have the same magnitude the direction, then we say they are equal. In the diagram below, $\vec{x}$ and $\vec{y}$ are equal, so we can call them both $\vec{x}$, or both $\vec{y}$ if we wanted to.



2) If we add two equal vectors together i.e. $\vec{y}$ and $\vec{y}$, then we will get a vector that is twice as long as $\vec{y}$ and in the same direction i.e. $2\vec{y}$

3) If $\vec{y}$ is a vector, then $-\vec{y}$ is a vector with the same magnitude but in the opposite direction.

- **Example:** Copy the diagram below and then construct the vector $2\vec{a} + 3\vec{b}$, if $\vec{a}$ and $\vec{b}$ are as shown.



Show how to do (ii) $3\vec{a} - \vec{b}$ and (iii) $3\vec{a} - 2\vec{b}$ then after.

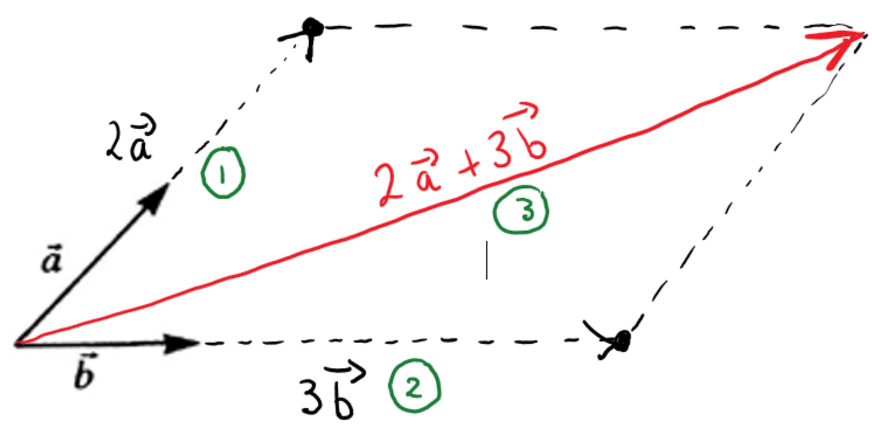
Solution:

Step 1: Draw $2\vec{a}$ by doubling the length of $\vec{a}$.

Step 2: Show $3\vec{b}$ by tripling the length of $\vec{b}$.

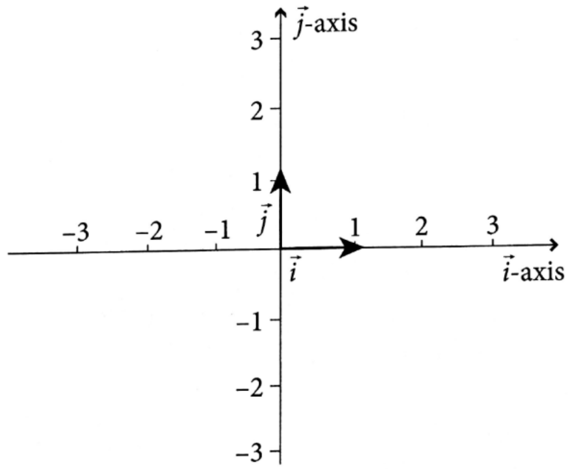
Step 3: Add $2\vec{a}$ and $3\vec{b}$ by the parallelogram method.

Step 4: Label the resultant vector.



Classwork Questions: pg 10 - 12 Ex 1B Qs 1(i) – (iii)/3/4/8/10

- **Topic 3: The $\vec{i}$ – $\vec{j}$ plane**
  - In a similar way to the way we used the cartesian plane in Junior Cert to represent points on a plane, we can use the $\vec{i}$ – $\vec{j}$ plane to represent vectors.

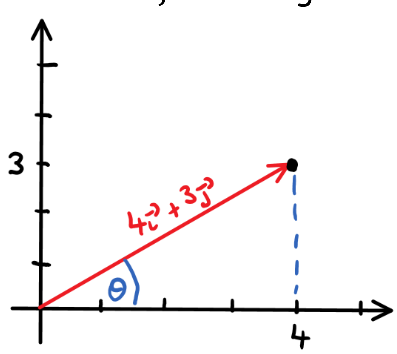


  - The vector $\vec{i}$ is a vector 1 unit long and along the positive i-axis as shown above. The vector $\vec{j}$ is a vector 1 unit long and along the positive j-axis as shown above.
  - The $\vec{i}$ and $\vec{j}$ vectors are both 1 unit long, so we call them unit vectors.
  - They are also at right angles to each other, so we say they are orthogonal.

- Example 1: Show the vectors (i) $4\vec{i} + 3\vec{j}$ and (ii) $-5\vec{i} + 2\vec{j}$ on separate diagrams and find their magnitude and direction.

Solution:

i) The vector $4\vec{i} + 3\vec{j}$ is just the resultant of the vectors $4\vec{i}$ and $3\vec{j}$. See diagram below.



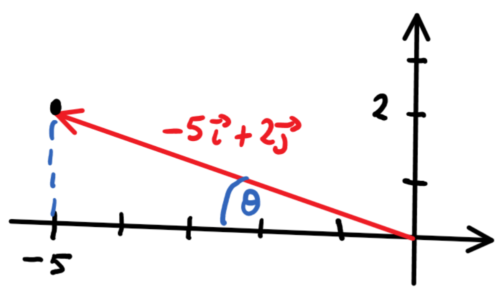
- To find the magnitude of the resultant vector, we need to use Pythagoras' Theorem.
- By letting 'x' be the length of the resultant vector:

$x^2 = 4^2 + 3^2$

$x^2 = 25$

$x = 5$

ii) The vector $-5\vec{i} + 2\vec{j}$ is the resultant of the vectors $-5\vec{i}$ and $2\vec{j}$. See diagram below.



- Again, we can find the magnitude using Pythagoras' Theorem.
- By letting 'x' be the length of the resultant vector:

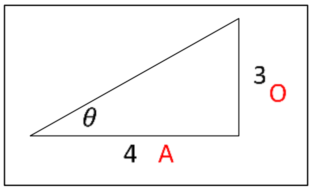
$x^2 = 5^2 + 2^2$

$x^2 = 29$

$x = \sqrt{29}$

$\Rightarrow |-5\vec{i} + 2\vec{j}| = \sqrt{29}$.

- The symbol for magnitude is | | so we would write $|4\vec{i} + 3\vec{j}| = 5$.
- To find the direction, we need to calculate the angle θ in the diagram above.
- To do this we use some Trigonometry we learned in Junior Cert:



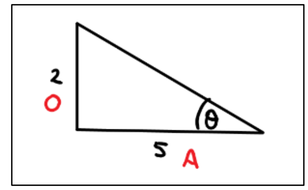
$$\text{Tan } \theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{3}{4}$$

=> $\tan^{-1}\left(\frac{3}{4}\right)$

=> $36.87°$

- So, if the angle $36.87°$, the direction of the resultant vector will be $36.87°$ north of east or E36.87°N.
- We could also write this direction as N53.13°E.

- To find θ :



$$\text{Tan } \theta = \frac{OPP}{ADJ} = \frac{2}{5}$$

=> $\tan^{-1}\left(\frac{2}{5}\right)$

=> $21.8°$

- So, if the angle $21.8°$, the direction of the resultant vector will be $21.8°$ north of west or W21.8°N.
- We could also write this direction as N68.2°W.

Classwork Questions: pg 13 Ex 1C Q1(ii)(iii)(iv)(v)(viii)(x)

- If vectors are in i and j form, we can carry out some calculations with them.

- Example 2: If $\vec{x} = 2\vec{\imath} - 5\vec{\jmath}$ and $\vec{y} = 5\vec{\imath} + 3\vec{\jmath}$
  a) Write in terms of $\vec{\imath}$ and $\vec{\jmath}$.
     i) $\vec{x} + \vec{y}$ ii) $3\vec{x} - \vec{y}$
  b) Find $|3\vec{x} - \vec{y}|$.
  c) Find the values of the scalars p and q such that $p\vec{x} + q\vec{y} = -4\vec{\imath} - 21\vec{\jmath}$
  d) Investigate if $\vec{x}$ is perpendicular to $\vec{y}$.

<u>Solution:</u>

a) i) $\vec{x} + \vec{y} = 2\vec{\imath} - 5\vec{\jmath} + 5\vec{\imath} + 3\vec{\jmath}$

$= 7\vec{\imath} - 2\vec{\jmath}$

ii) $3\vec{x} - \vec{y} = 3(2\vec{\imath} - 5\vec{\jmath}) - (5\vec{\imath} + 3\vec{\jmath})$

$= 6\vec{\imath} - 15\vec{\jmath} - 5\vec{\imath} - 3\vec{\jmath}$

$= \vec{\imath} - 18\vec{\jmath}$

b) We need to tidy up $3\vec{x} - \vec{y}$ into i and j form first, which we did in part (a) above, before finding the magnitude:

=> $|3\vec{x} - \vec{y}| = |\vec{\imath} - 18\vec{\jmath}|$

=> $|\vec{\imath} - 18\vec{\jmath}| = \sqrt{(1)^2 + (-18)^2}$

$= \sqrt{325}$

c) For this question, we begin by substituting in our two vectors $\vec{x}$ and $\vec{y}$:

$$p\vec{x} + q\vec{y} = -4\vec{i} - 21\vec{j}$$
$$p(2\vec{i} - 5\vec{j}) + q(5\vec{i} + 3\vec{j}) = -4\vec{i} - 21\vec{j}$$
$$2p\vec{i} - 5p\vec{j} + 5q\vec{i} + 3q\vec{j} = -4\vec{i} - 21\vec{j}$$

- Now we will tidy up the $\vec{i}$ and $\vec{j}$ components on the left together:

$$(2p + 5q)\,\vec{i} + (-5p + 3q)\,\vec{j} = -4\vec{i} - 21\vec{j}$$

As both sides are equal **the $\vec{i}$ components must be the same** i.e.

$$2p + 5q = -4$$

- And similarly, **the $\vec{j}$ components must be the same** i.e.

$$-5p + 3q = -21$$

- We can now solve these two simultaneous equations to find t and k:

A: $2p + 5q = -4$

B: $-5p + 3q = -21$

A x 5: $10p + 25q = -20$

B x 2: $\underline{-10p + 6q = -42}$

$31q = -62$

$q = -2$

<u>Put q into A:</u>

A: $2p + 5q = -4$

$2(p) + 5(-2) = -4$

$2p - 10 = -4$

$2p = 6$

$p = 3$

d) If we look at the diagram of the vector $\vec{x}$ on the right, we can see that it's slope can be found using Slope = $\frac{RISE}{RUN} = \frac{-5}{2}$

- So, in general, to find the slope of any vector, we can just use:

$$\boxed{\text{Slope} = \frac{RISE}{RUN} = \frac{\vec{j}\ component}{\vec{i}\ component}}$$

- So, to finish this question, the slope of $\vec{x}$ is $\frac{-5}{2}$ and the slope of $\vec{y}$ is $\frac{3}{5}$, so we can now check if these two vectors are perpendicular using a result we used from Junior Cycle i.e. if two lines are perpendicular then their slopes multiply to give -1 or $m_1 \times m_2 = -1$

$$\frac{-5}{2}\ x\ \frac{3}{5} = \frac{-15}{10} = \frac{-3}{2} \text{ which is} \neq -1$$

=> the vectors $\vec{x}$ and $\vec{y}$ are not perpendicular

Classwork Questions: pg 14 Ex 1C Qs 2/3/7 and then try 9/10/12

- Topic 4: Dot Products and Angle between Vectors

a) Dot Product:

- When multiplying two vectors together, we use the following:

$$(a\vec{i} + b\vec{j}) \cdot (c\vec{i} + d\vec{j}) = ac + bd$$

See Pg 17 Tables

- This product is known as the dot product.
- The dot product allows us to measure how much vectors pull together.
  - If the dot product of 2 vectors is positive => they're pulling in roughly the same direction
  - If the dot product of 2 vectors is negative => they're roughly pulling in opposite directions

Note: If two vectors are perpendicular to each other, then the dot product will be 0 i.e.

$$\vec{a} \cdot \vec{b} = 0 <=> \vec{a} \perp \vec{b}$$

Get students to discover this for themselves by giving the vectors $-\vec{i} + 3\vec{j}$ and $9\vec{i} + 3\vec{j}$

b) Angle Between Vectors:

- We can find the angle between two vectors $\vec{a}$ and $\vec{b}$ using the rule:

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

See Pg 17 Tables

- Example: If $\vec{a} = 5\vec{i} - 3\vec{j}$ and $\vec{b} = -2\vec{i} + 4\vec{j}$, find i) $\vec{a} \cdot \vec{b}$ and ii) the angle between the two vectors.

Solution:

i)

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (5\vec{i} - 3\vec{j})(-2\vec{i} + 4\vec{j}) \\ &= (5)(-2) + (-3)(4) \\ &= -10 - 12 \\ &= -22\end{aligned}$$

Get students to quickly calculate $\vec{b} \cdot \vec{a}$ to verify that $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ => commutative

ii) To find the angle between the vectors, we need to find the magnitudes of both vectors:

$$|5\vec{i} - 3\vec{j}| = \sqrt{(5)^2 + (-3)^2} = \sqrt{34}$$

$$-2\vec{i} + 4\vec{j}| = \sqrt{(-2)^2 + (4)^2} = \sqrt{20}$$

- Now, we can sub in these two values and our dot product from part (i) into our rule:

$$\cos\theta = \frac{-22}{\sqrt{34} \cdot \sqrt{20}}$$

$$=> \cos\theta = \frac{-22}{2\sqrt{170}}$$

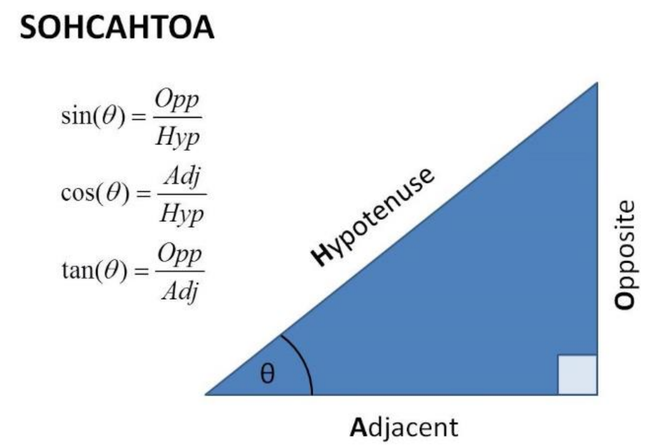
$$=> \cos\theta = -0.84366$$

$$=> \theta = \cos^{-1} -0.84366 = 147.5^0$$

Classwork Questions: pg 16 Ex 1D Qs 1(ii)(iv)/2/3/5/8(ii)(iv)/9/10/16

➢ Topic 5: Junior Cycle Trig Review

- **Sin/Cos/Tan Ratios:**
  - In Junior Cycle, we learned some important rules about right angled triangles:



Pythagoras Theorem: $H^2 = O^2 + A^2$

Useful Shortcut:

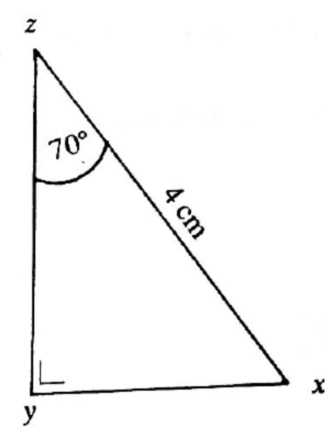
- As we know $\sin\theta = \frac{OPP}{HYP}$, we can cross multiply to get an expression for the opposite side only i.e. $OPP = HYP(\sin\theta)\ or\ h\sin\theta$
- Similarly, is we use our formula for $\cos\theta$ above, we know the adjacent side will be $ADJ = h\cos\theta$
- These two results can speed us up a lot in later sections.

$$\boxed{\begin{aligned} Opposite &= h\sin\theta \\ Adjacent &= h\cos\theta \end{aligned}}$$

- **Example 1:** In the triangle shown on the right below, find the lengths of the missing sides.

Solution:

- If we label the sides of the triangle, |XY| = opposite and |ZY| = adjacent



=> |XY| = $h\sin\theta$	=> |YZ| = $h\cos\theta$
= 4(sin 70)	= 4(cos 70)
= 4(0.9397)	= 4(0.342)
= 3.76cm	= 1.37cm

- <u>Finding sin, cos or tan when given one of them:</u>
  - Another useful trick is to be able to find sin, cos or tan of an angle when given one of them, without actually evaluating the angle and without a calculator.
- **Example 2:** If $\tan A = \frac{3}{4}$, find $\sin A$ and $\cos A$ without using a calculator or the mathematical tables.

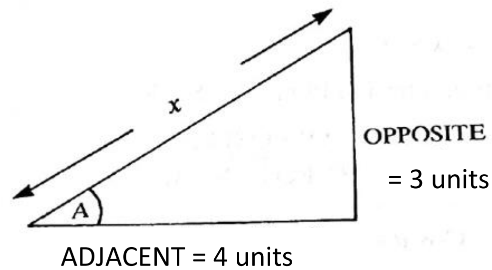
  Sometimes written as $A = \tan^{-1}\frac{3}{4}$

  <u>Solution:</u>
  - First, we will draw a right-angled triangle to match the information we've been given:
  - Now we calculate the length of the missing side, using Pythagoras' Theorem:

    $x^2 = (3)^2 + (4)^2$

    $x^2 = 9 + 16$

    $x = 5$



  - We can now write our other two ratios:

    $\sin A = \frac{3}{5}$ and $\cos A = \frac{4}{5}$

Classwork Questions: pg 18 - 20 Ex 1E Qs 1/2(ii)(iv)/3 (i) – (iv)/(vii)(ix) and then try 5

➢ <u>Topic 6: Writing vectors in terms of $\vec{i}$ and $\vec{j}$ and Polar Form</u>

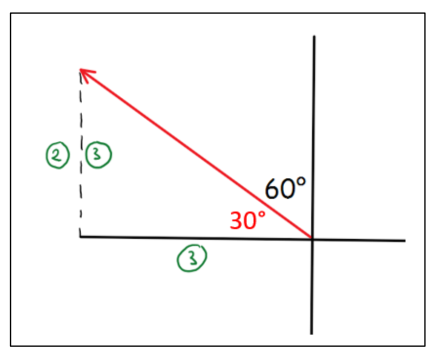
a) <u>Writing vectors in $\vec{i}$ and $\vec{j}$:</u>

- The steps involved are:
  1) Draw a sketch of the vector.
  2) Complete the right-angled triangle.
  3) Calculate the length of the missing sides of the triangle.
  4) Use your diagram to check the correct signs on the $\vec{i}$ and $\vec{j}$ components.

- **Example 1:** $\vec{p}$ is a vector of magnitude 3 in a direction $N60°W$. Write $\vec{p}$ in terms of $\vec{i}$ and $\vec{j}$.

  <u>Solution:</u>

  Step 1: Draw a sketch of the vector $\vec{p}$
  - See diagram on the right



  Step 2: Complete the right-angled triangle and fill in 30°.

  Step 3: Calculate the lengths of the missing sides.
  - The adjacent side to 30° will be $3\cos 30 = \frac{3\sqrt{3}}{2}$ or 2.6
  - The opposite side to 30° will be $3\sin 30 = \frac{3}{2}$ or 1.5

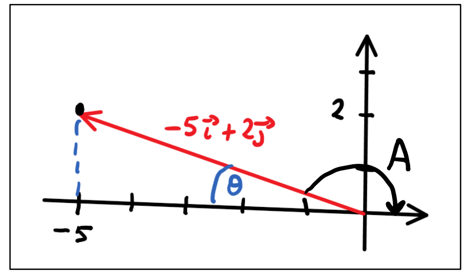
  Step 4: Use your diagram to check the correct signs on the $\vec{i}$ and $\vec{j}$ components.
  - The vector is going to the left and up so the $\vec{i}$ component will be negative and the $\vec{j}$ component will be positive.
  - So, the final solution here is: $-\frac{3\sqrt{3}}{2}\vec{i} + \frac{3}{2}\vec{j}$.

Classwork Questions: pg 13/14 Ex 1D Qs 1(ii)(iv)(v) / 2/3/5

b) Polar Form:

- Another way of writing vectors is to use what's called Polar Form.
- To write vectors in Polar Form, we need to know their magnitude and the anti-clockwise angle that they make with the positive x-axis, which is known as the argument.

• Example: i) Write the vector $\vec{p} = -5\vec{\imath} + 2\vec{\jmath}$ in polar form. ii) Write the vector $5\angle 200°$ in terms of $\vec{\imath}$ and $\vec{\jmath}$.



Solution:

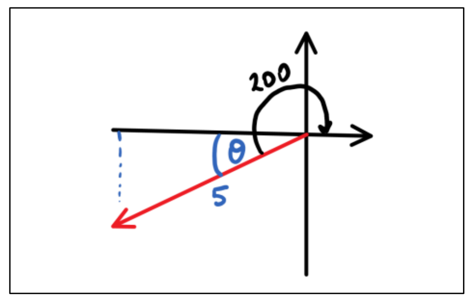
i) A sketch of the vector $\vec{p}$ is shown on the right.

- The magnitude of the vector is: $|-5\vec{\imath} + 2\vec{\jmath}| = \sqrt{29}$
- The angle θ is: $\tan^{-1}\left(\frac{2}{5}\right) = 21.8°$

=> The angle with the positive x-axis (A) will be

= 180 – 21.8 = 158.2°

=> Polar Form = $\sqrt{29}\angle 158.2°$



ii) A sketch of the vector is shown on the right.

- From the diagram, we can see that angle θ will be: 200 – 180 = 20°
- We can now use what we learned above to calculate the $\vec{\imath}$ and $\vec{\jmath}$ components:

$\vec{\imath}$ comp = $5\cos 20 = 4.7$ $\vec{\jmath}$ comp = $5\sin 20 = 1.7$

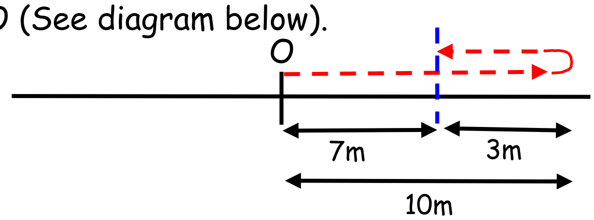
- So, using our diagram to get the correct signs on each component, we get:

$$5\angle 200° = -4.7\vec{\imath} - 1.7\vec{\jmath}$$

Classwork Questions: pg 22/23 Ex 1F Qs 1(i) – (iv)/2(a)(ii)(v)/2(b)(iii)(vi) and then try Q4/6

➢ Topic 7: Distance and Displacement

- An important idea what we will need in future topics is the difference between distance travelled and displacement.
- The easiest way to explain the difference between them is with an example.
- If you started from a point fixed point O, and walked 10m to the right and then walked back 3m to the left, you would have walked a total distance of 13m, but you would only be 7m away from O (See diagram below).

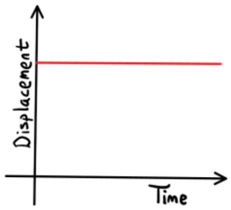


=> Your distance travelled would be 13m but your displacement from O would be 7m.

- If another person walked 4m to the right and then 9m to the left, they would also have walked a distance of 13m, but would end up 5m to the left of O.

- In this case, their displacement from O would be -5m.
- Two important differences between the two are:
    - distance can't be negative but displacement can
    - distance is a scalar and displacement is a vector

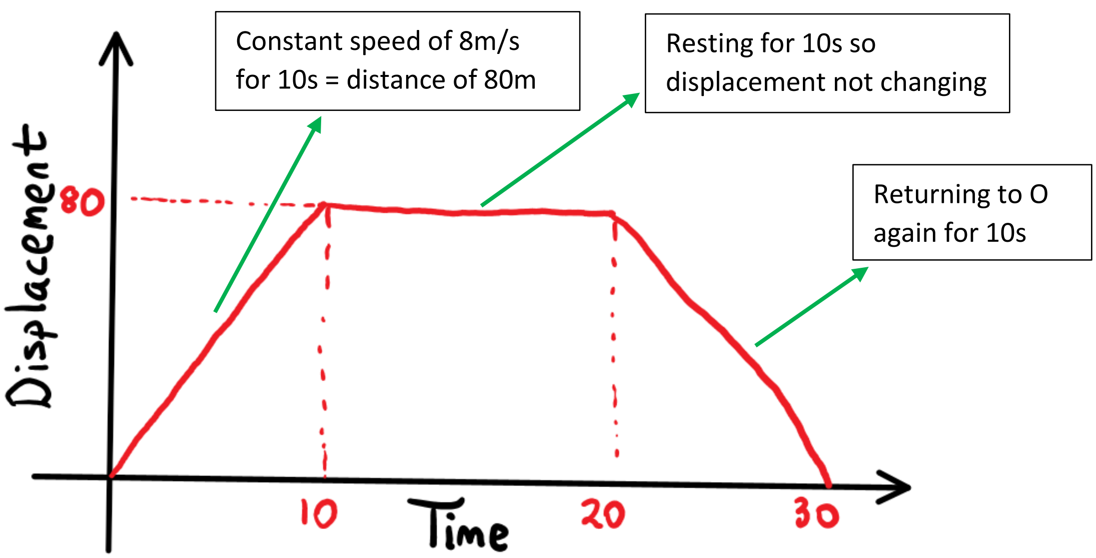
- <u>Time-Displacement Graphs:</u>
  - When displaying time vs displacement on a graph, we normally put time along the x-axis.
  - Some common shapes we see in time-displacement graphs are shown below:

Not moving (Stationary/At rest)	Moving at a steady speed away from O	Moving at a steady speed towards O	Accelerating away from O	Decelerating away from O
				

- <u>Example:</u> Pg 25 Q5

  Sue runs at a constant speed of 8 m/s from a point O, running along a straight road for 10s. She takes a 10 s break and then returns along the same road to O in 10s. Draw a time-displacement graph of her run.

  <u>Solution:</u>



Classwork Questions: Pg 25/26 Ex 1G Qs 2/3/4/7 - 10

Revision Questions and Test