- Chapter 5: Work, Power, Energy and Momentum
- Topic 26: Work and Power
  - We already looked at the definition of a Force in the last chapter:

    A Force is something that causes an object to speed up or slow down i.e. to accelerate.

  - Anytime a force moves something, we say work has been done.
  - In fact, we calculate the amount of work done by multiplying the force applied, by the distance the object moves in the direction of the force:

    Work = Force x Distance

  - We measure work in Joules (or kilojoules).
  - If we measure how quickly work is being done, we are measuring the Power applied.

    Power = $\frac{Work}{Time}$

  - We measure Power in Watts (or kilowatts).
  - If an object is moving at a constant speed v, driven by some tractive force F, it can be shown that:

    Power Output = Fv

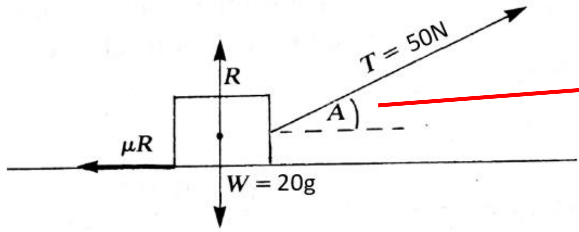
- Example 1: Pg 93 Ex 5A Q6

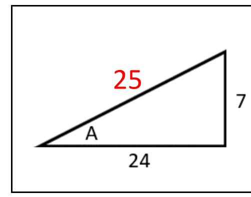
  An Inuit loads some provisions on a toboggan. He drags the toboggan across horizontal snowy ground, by means of a light rope. The loaded toboggan has a mass of 20 kg. The coefficient of friction between the toboggan and the ground is $\frac{1}{7}$. The tension in the rope is 50 N and the rope makes an angle A, where $\tan A = \frac{7}{24}$, with the ground. If the toboggan moves a distance of 30 m, find:

  i) the magnitude of the normal reaction at the ground
  ii) the magnitude of the friction force
  iii) the work done by the tension force
  iv) the acceleration
  v) the power output of the man if all this took 20 seconds.

Solution:

i) A diagram showing the forces on the block is shown below:



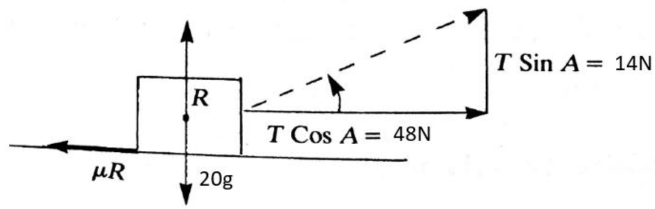


$tanA = \frac{7}{24}$

=> $cosA = \frac{24}{25}$

and $sinA = \frac{7}{25}$

- To do this part, we have to resolve the Tension force first:



- We can now look at forces perpendicular to the ground

$$T \sin A + R = 20g$$
$$14 + R = 20(9.8)$$
$$14 + R = 196$$
$$R = 196 - 14$$
$$R = 182 \text{ N}.$$

ii) So, the friction force will be:

Friction = μR

=> Friction = $(\frac{1}{7})(182)$

=> Friction = 26 N.

iii) The pulling force along the horizontal is 48 N, so the work done will be:

Work = Force x Distance

= (48)(30)

= 1440 J or 1.44 kJ.

iv) The only forces along the horizontal are the 48N and the friction force so the overall force will be:

48 - 26 = 22N

- But F = ma, so:

22 = 20(a)

=> a = $\frac{22}{20} = \frac{11}{10}$ or 1.1 m/s^2

v) The power output can be found using the formula above:

Power = $\frac{Work}{Time}$

=> Power = $\frac{1440}{20}$ = 72W.

- Example 2: Car engine has a power output of 200 kW. Car moving at speed of 25 m/s. What is the tractive force of the engine?
- Solution:

Power Output = Fv

=> 200,000 = F(25)

=> F = $\frac{200000}{25}$ = 8000 N or 8 kN

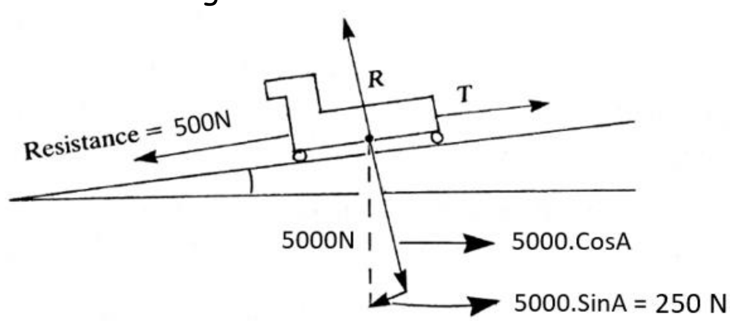
Classwork Questions: Pg 92/93 Ex 5A Qs 2/3/5/11/

- Example 3: Pg 93 Ex 5A Q7

A car whose weight is 5000 N climbs a hill of incline one in twenty at a steady speed of 12 m/s. If the resistance to motion is 500 N, find the power output of the car's engine.

Solution:

- An incline of one in twenty means $\sin A = \frac{1}{20}$, where A is the angle of inclination of the hill.
- We will let T = the tractive effort of the car (the pulling force)
- A diagram of the forces acting on the car is shown below:



- If the car is moving at a "steady speed", then there is no acceleration.
- That means the forces in any line must be zero:

=> T = 500 + 250

= 750 N

- The Power Output = Fv

=> P = (750)(12) = 9000 W or 9 kW

- Drag Forces:
  - When an object is moving through any medium, it generally experiences some sort of drag or resistance on it, caused by the medium it's moving through.
  - Examples include a swimmer experiencing water resistance when moving through the water, or a plane experiencing air resistance when flying
  - The drag usually increases as the speed increases.
  - This drag can usually be modelled by:

$$D = kv^n$$ where n is some positive real number

- Example 4: Pg 93 Ex 5A Q14

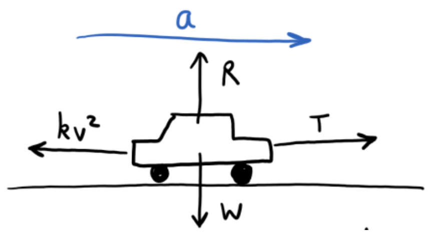
A car of mass 1 tonne has a maximum power output of 24 kW. When the car is driving at v m/s, there is a drag of kv^2 newtons where k is a constant. When the car travels at 10 m/s along a horizontal road, its maximum acceleration is 2.1 m/s^2.

i) Find the value of k.

ii) Find the maximum speed of the car. (Hint: when travelling at maximum speed, the acceleration has to be zero)

Solution:

- First we will draw a diagram showing all forces on the car:



i) The maximum power output is 24 kW, so we can use our formula from above to calculate the tractive force T, when the speed is 10 m/s:

Power = Tv

=> 24,000 = T(10)

=> T = $\frac{24,000}{10}$ = 2400 N (at a speed of 10 m/s)

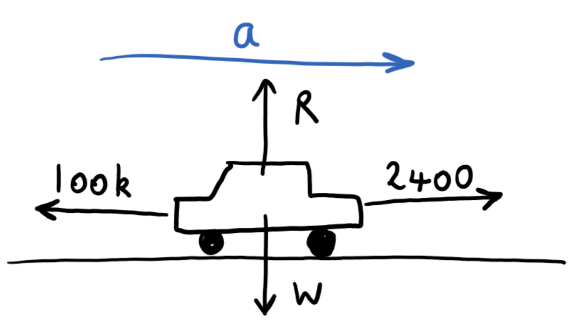
- At a speed of 10 m/s, the drag will be:

$D = kv^2$

=> $D = k(10)^2$

=> $D = 100k$ Newtons

- So, we can update our diagram above now to show the forces acting on the car, when it's travelling at 10 m/s:



- We also were given the acceleration of the car at this speed, so using F = ma:

F = ma

=> 2400 – 100k = (1000)(2.1)

=> 2400 – 100k = 2100

=> 100k = 300

=> k = 3

ii) For this part, we will now let 'v' be the maximum speed.

- So, when the car is moving at speed v, the tractive force can be found using:

$$P = Tv$$
$$\Rightarrow 24{,}000 = Tv$$
$$\Rightarrow T = \frac{24{,}000}{v}$$

- The drag force will be:

$$D = 3v^2$$

- As the car is moving at maximum speed, and hence there must be no acceleration, the tractive force and the drag must be equal in magnitude to each other:

$$\Rightarrow 3v^2 = \frac{24{,}000}{v}$$
$$\Rightarrow 3v^3 = 24{,}000$$
$$\Rightarrow v^3 = 8{,}000$$
$$\Rightarrow v = \sqrt[3]{8000}$$
$$\Rightarrow v = 20 \text{ m/s}$$

Classwork Questions: Pg 93/94 Ex 5A Qs 8/10/13/15/17

Topic 27: Energy and Conservation of Energy

- You might recall learning about Energy in Junior Cert Science.

Energy is the ability to do work.

- Energy is measured in Joules (or kilojoules).
- You also might remember learning about some forms of energy, namely, potential and kinetic energy:
  a. Potential Energy is energy an object has due to its position.
  b. Kinetic Energy is energy a moving object has.
- To step on a bit, we have two formulae that we can use to calculate both of these energies:

Potential Energy = mgh (m = mass, g = 9.8, h = height)

Kinetic Energy = $\frac{1}{2}mv^2$ (m = mass, v = velocity)

- The Law of Conservation of Energy was something you also covered in JC Science.

Energy cannot be created or destroyed; only changed from one form to another.

- On a roller coaster, the carriage has a lot of potential energy, when it's at the top of a hill, and low kinetic energy as it's moving very slowly.
- As it moves down the hill, the kinetic energy rapidly increases, as its speed increases.

- At the bottom of the hill, the potential energy is very low, and the kinetic energy is at a maximum, as the carriage is at its top speed.
- At any point along the roller coaster, the overall energy in the system remains constant i.e.

Kinetic Energy + Potential Energy = Constant

- And it also follows that:

Work Done = Energy Gained

e.g. lifting 5kg bucket up 10m

- Example 1: A body of mass 2 kg fires rockets which speed it up from 40m/s to 60m/s. Find the work done.

  Solution:

  - The work done is measured by the gain in energy and in this example, the energy is kinetic energy, so:

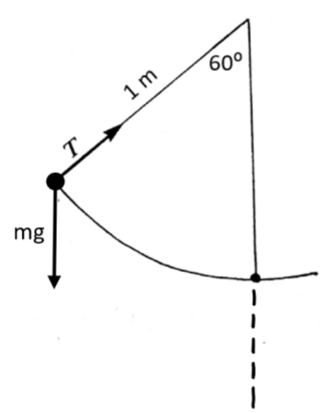
Energy Before = $\frac{1}{2}mu^2$ $= \frac{1}{2}(2)(40)^2$ = 1600J = 1.6 kJ	Energy After = $\frac{1}{2}mv^2$ $= \frac{1}{2}(2)(60)^2$ = 3600J = 3.6 kJ
So, the energy gained is: 3600 – 1600 = 2000 J. => Work Done = 2000 J or 2 kJ.	

- Example 2: Pg 96 Ex 5B Q3

  A pendulum is made of a mass at the end of a light string. The pendulum, which is 1 m long, makes an angle of 60° with the vertical when at its greatest height. Find its speed (in m/s) when at its lowest point, correct to 2 decimal places.

  Solution:

  - First, let's look at a diagram of the situation:



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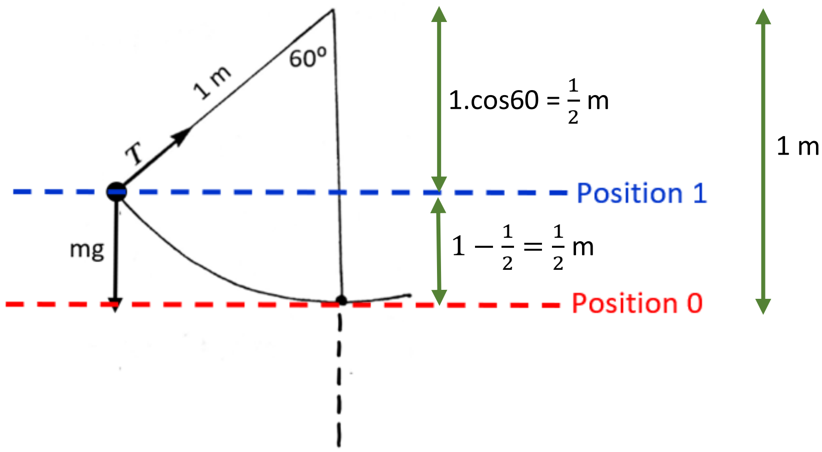
- We start by defining a "standard position" or "Position 0" that we measure all our distances off.
- We can put "Position 0" anywhere we want that suits us, but we'll put it at the lowest point that the mass reaches, in this question.
- We will also define a second position "Position 1" at the point where the particle is at its greatest highest.



Tip: Use some Trigonometry, to calculate distance between Positions 0 and 1 before starting the question.

- The tension in the string acts perpendicularly to the direction of motion, so it has no effect on the particle's speed and hence, does no work.
- The only force that does work and affects the speed of the particle is gravity, so we can use the Law of Conservation of Energy here.
- We will now use the Law of Conservation of Energy between positions 0 and 1:

=> $KE_0 + PE_0 = KE_1 + PE_1$

=> $\frac{1}{2}m(v)^2 + mg(0) = \frac{1}{2}m(0)^2 + mg(\frac{1}{2})$

=> $\frac{1}{2}m(v)^2 = (\frac{1}{2})mg$

=> $v^2 = g$ (dividing both sides by m and multiplying both sides by 2)

=> $v = \sqrt{9.8}$

=> $v = 3.13m/s$.

Classwork Questions: pg 96/97 Ex 5B Qs 2/4/8/9/10/11

Topic 28: Conservation of Momentum

- Principle of Conservation of Momentum:
  - We came across momentum in a previous chapter.

Momentum = Mass x Velocity

  - The units we use for momentum are kg m/s.
  - Newton's Second Law stated that "the rate of change of momentum is proportional to the applied force".
  - This means that, in the absence of external forces, the momentum of a system doesn't change.
  - This is called the Principle of Conservation of Momentum.
  - The change in momentum of a body is known as its Impulse.
  - We can calculate Impulse by: $\vec{I} = m\vec{v} - m\vec{u}$
  - Impulse is measured in newton-seconds, Ns.

- Example 1: Pg 99 Ex 5C Q3

A hammer of mass 4 kg is used to drive a 1 kg wooden stake into the ground. The hammer, immediately before impact, has a downward velocity of 2 m/s. Find the speed of the stake immediately after the impact, when the hammer comes to rest. Find, also, the impulse imparted to (i) the stake (ii) the hammer

Solution:

- The total momentum in the system must be conserved, so:

Momentum Hammer Before + Momentum Stake Before = Momentum Hammer After + Momentum Stake After

- We will work out the momentum of both objects before and after the hit first:

Momentum Hammer Before = $m\vec{u}$ $= (4)(-2\vec{j}) = -8\vec{j}$ Momentum Stake Before = $m\vec{u}$ $= (1)(0\vec{j}) = 0\vec{j}$	Momentum Hammer After = $m\vec{v}$ $= (4)(0\vec{j}) = 0\vec{j}$ Momentum Stake After = $m\vec{v}$ $= (1)(v\vec{j}) = v\vec{j}$

- So, from the Law of Conservation of Momentum:

$$-8\vec{j} + 0\vec{j} = 0\vec{j} + v\vec{j}$$

=> $v = -8$ => the speed of the stake will be 8 m/s in a downward direction

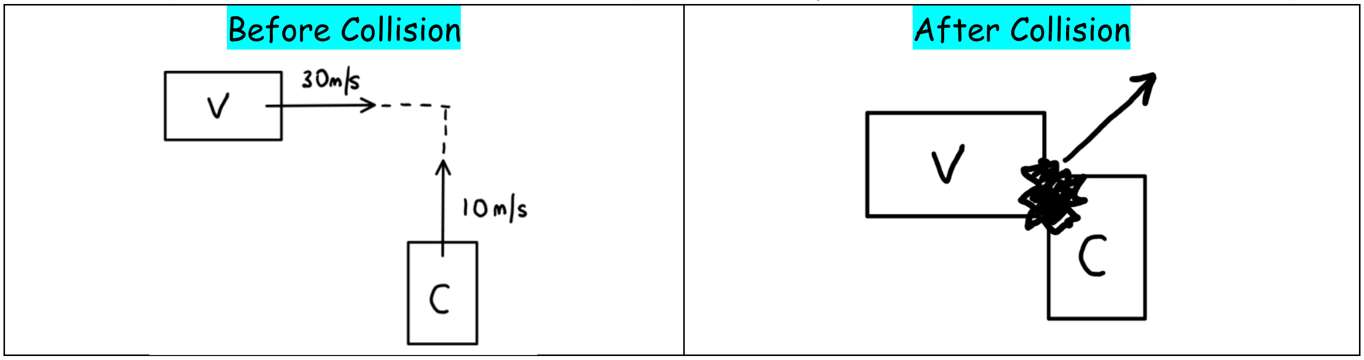
- We now calculate the Impulse imparted to both bodies:

Hammer	Stake
$\vec{I} = m\vec{v} - m\vec{u}$ $\vec{I} = 4(0\vec{j}) - 4(-2)\vec{j}$ $= 8\vec{j}$ Ns	$\vec{I} = m\vec{v} - m\vec{u}$ $\vec{I} = 1(-8\vec{j}) - 0\vec{j}$ $= -8\vec{j}$ Ns

Classwork Questions: Pg 99 Ex 5C Qs 2/4/7/9

- Principle of Conservation of Momentum in 2 Dimensions:
- Example: Pg 101 Ex 5D Q4

A van of mass 800 kg, moving with speed 30 m/s along a straight road, collides with a car of mass 700 kg, moving at 10 m/s along a road at right angles to the first road. When the two collide, they become entangled. Find the speed of the joint mass immediately after impact.



Solution :

- We will let $\vec{i}$ direction be along the line of motion of the van and $\vec{j}$ be along the line of motion of the car
- Let 'v' be the speed of the entangled mass immediately after the collision, so:

Momentum Van Before + Momentum Car Before = Momentum Van After + Momentum Car After

=> $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

=> $(800)(30\vec{i}) + (700)(10\vec{j}) = (800 + 700)v$

=> $24000\vec{i} + 7000\vec{j} = (1500)v$

=> $15v = 240\vec{i} + 70\vec{j}$

=> $v = \frac{240}{15}\vec{i} + \frac{70}{15}\vec{j}$

=> $v = 16\vec{i} + 4.7\vec{j}$

As they become entangled, there is a single body of mass 1500kg moving after the collision.

- We can now calculate the speed of the entangled mass:

Speed = $\sqrt{(16)^2 + (4.7)^2}$

= $\sqrt{278.09}$

= 16.67 m/s

Classwork Questions: Pg 101 Ex 5D Qs 2/3/5/7

- Principle of Conservation of Momentum and Strings:
- Example: Pg 103 Ex 5E Q3

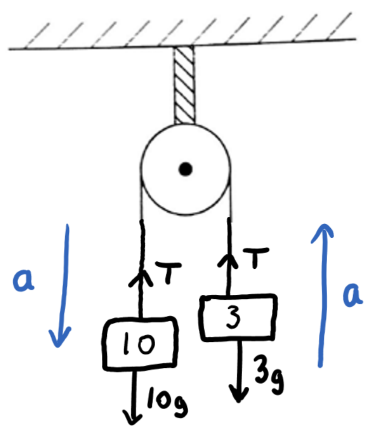
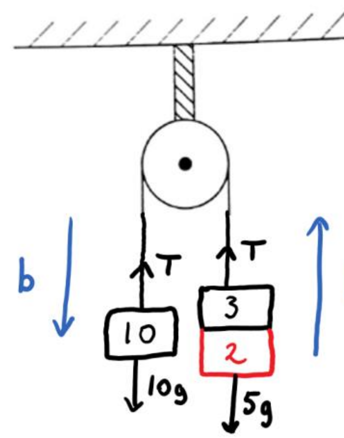
Two particles of masses 10kg and 3kg hang are at the ends of a light inextensible string. The system is released from rest. After 2 seconds, the 3kg mass picks up a particle of mass 2kg.

i) What distance will the 10 kg particle fall in the first 2-second period?

ii) What distance will the 10 kg particle fall in the second 2-second period?

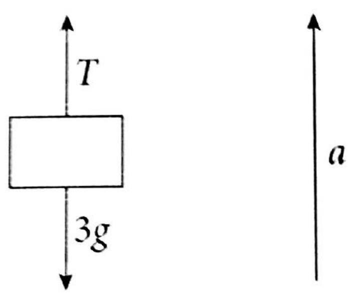
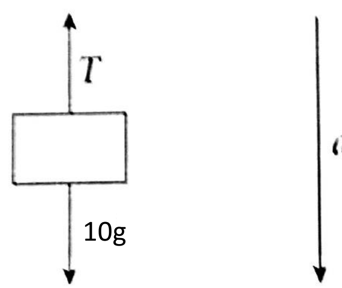
Solution:

- As always, let's begin with a diagram of the situation:

Before picking up 2kg	After picking up 2kg
	

i) We will let a = common acceleration of the particles during the first 2 seconds.

- Let's look at the forces on the 10kg and 3kg particles:

3kg Particle	10kg Particle
 Using F = ma: $T - 3g = 3a$ => $T - 3a = 3g$.......Eqn 1	 Using F = ma: $10g - T = 10a$ => $-T - 10a = -10g$.........Eqn 2

We can now solve equations 1 and 2 together:

Eqn 1: $T - 3a = 3g$

Eqn 2: $-T - 10a = -10g$

=> $-13a = -7g$

=> $a = \frac{-7g}{-13} = \frac{7g}{13}$ m/s^2

- So, to find how far the 10kg will fall:

$u = 0,\ t = 2,\ a = \frac{7g}{13},\ s = ?$

$s = ut + \frac{1}{2}at^2$

=> $s = (0)(2) + \frac{1}{2}\left(\frac{7g}{13}\right)(2)^2$

=> $s = \frac{14g}{13}$ m

ii)

- To tackle this part, we need to know what speed the system was travelling at right before the 3kg mass picks up the 2kg mass:

$v = u + at$

=> $v = 0 + (\frac{7g}{13})(2)$

=> $v = \frac{14g}{13}$ m/s

- At this point, the 3kg mass picks up a 2kg mass, to become a new mass of 5kg.
- The system will immediately be jolted to a steady speed.
- The mass of the whole system was 13kg before the jolt and 15kg after the jolt.
- We will let v = the new speed, and now we can use the Principle of Conservation of Momentum:

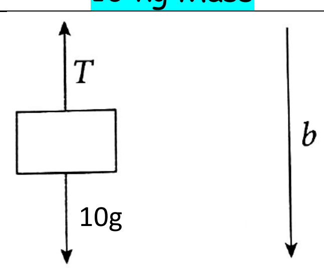
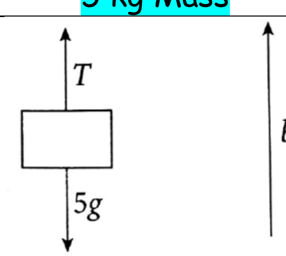
$m_1u = m_2v$

=> $(13)(\frac{14g}{13}) = (15)v$

=> $14g = 15v$

=> $v = \frac{14g}{15}$ m/s

- We now let b = the new common acceleration of the particles, and look at the forces on the 10 kg and 5 kg masses:

10 kg Mass	5 kg Mass
 Using F = ma: $10g - T = 10b$ => $-T - 10b = -10g$........Eqn 3	 Using F = ma: $T - 5g = 5b$ => $T - 5b = 5g$........Eqn 4
We can now solve equations 3 and 4 together: Eqn 3: $-T - 10b = -10g$ Eqn 4: $T - 5b = 5g$ => $-15b = -5g$ => $b = \frac{5g}{15} = \frac{g}{3}$ m/s^2	

- To finish, we now look at the 10 kg mass, after the jolt.
- We know that a = $\frac{g}{3}$, u = $\frac{14g}{15}$, t = 2 and s = ?

$$s = ut + \frac{1}{2}at^2$$

=> $s = \left(\frac{14g}{15}\right)(2) + \frac{1}{2}\left(\frac{g}{3}\right)(2)^2$

=> $s = \frac{28g}{15} + \frac{2g}{3}$

=> $s = \frac{84g}{45} + \frac{30g}{45}$

=> $s = \frac{114g}{45}$

=> $s = \frac{38g}{15}$ m

Classwork Questions: Pg 103 Ex 5E Qs 1/2/5

Revision Questions and Test