

Revision Sheet 4 Worked Solutions

Q1. $(1,1) \xrightarrow{-1} (0,0)$
 $(8,-5) \xrightarrow{-1} (7,-6) = (x_1, y_1)$
 $(5,-2) \xrightarrow{-1} (4,-3) = (x_2, y_2)$

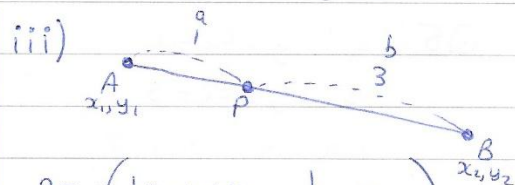
$$\begin{aligned} \text{Area} &= \frac{1}{2} |x_1 y_2 - x_2 y_1| \\ &= \frac{1}{2} |(7)(-3) - (-6)(4)| \\ &= \frac{1}{2} |-21 + 24| \\ &= \frac{1}{2} |3| \\ &= \boxed{1.5 \text{ units}^2} \end{aligned}$$

Q2. i) m: $7x - y + 7 = 0$
 Point on line
 $\Rightarrow 7(k-2) - (7k-7) + 7 \stackrel{?}{=} 0$
 $7k - 14 - 7k + 7 + 7 \stackrel{?}{=} 0$
 $14 - 14 = 0$
 $0 = 0$
 $\Rightarrow$ ON Line Q.E.D.

ii) Can't be done

Q3. $A = (5, -9)$ $B = (-3, 3)$
 i) mp. $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
 $= \left(\frac{5 + (-3)}{2}, \frac{-9 + 3}{2} \right)$
 $= \boxed{(1, -3)}$

ii) mp $= \left(\frac{5 + 1}{2}, \frac{-9 - 3}{2} \right)$
 $= \boxed{(3, -6)}$

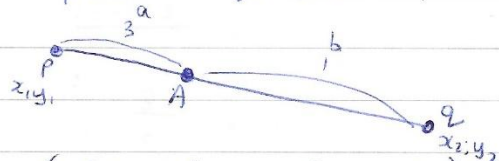


$$P = \left(\frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a} \right)$$

$$\begin{aligned} P &= \left(\frac{3(5) + 1(-3)}{1+3}, \frac{3(-9) + 1(3)}{1+3} \right) \\ &= \left(\frac{15-3}{4}, \frac{-27+3}{4} \right) \\ &= \boxed{(3, -6)} \end{aligned}$$

iv) As N is the midpoint of [AM] it is the point that is $\frac{1}{4}$ of the way along the line, which is P.

Q4 $p(4, -3)$ $q(-4, 9)$



$$\begin{aligned} A &= \left(\frac{1(4) + 3(-4)}{3+1}, \frac{1(-3) + 3(9)}{3+1} \right) \\ &= \left(\frac{4-12}{4}, \frac{-3+27}{4} \right) \\ &= \boxed{(-2, 6)} \end{aligned}$$

Q5. $L_1: y = 4x - 1$
 $L_2: y = 2x + 3$
 $\Rightarrow m_1 = 4 \quad m_2 = 2$

$$\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \pm \frac{4 - 2}{1 + (4)(2)}$$

$$= \pm \frac{2}{9}$$

Acute angle $\Rightarrow \tan \theta = \frac{2}{9}$
 $\Rightarrow \theta = \tan^{-1} \frac{2}{9}$
 $= 12.53$
 $= \boxed{13^\circ}$

Q6. To find slope of line joining $(3, -6)$ and $(-7, 12)$

$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{12 - (-6)}{-7 - 3}$$

$$= \frac{18}{-10}$$

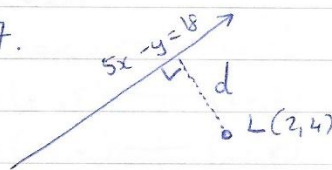
$$= -\frac{9}{5}$$

Line 2 has slope of $-\frac{x \text{ no.}}{y \text{ no.}}$
 $= -\frac{5}{9}$
 $= \frac{5}{9}$

If $\perp m_1 \times m_2 = -1$
 $-\frac{9}{5} \times \frac{5}{9} = -1$

$-1 = -1$
 $\Rightarrow \perp$ Q.E.D.

Q7.



$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|5(2) + 1(4) - 18|}{\sqrt{5^2 + (-1)^2}}$$

$$= \frac{|10 - 4 - 18|}{\sqrt{26}}$$

$$= \frac{|-12|}{\sqrt{26}} = \frac{12}{\sqrt{26}} = \boxed{2.4}$$

Q8. To find pt. of intersection:

A: $2x - 3y = -1$

B: $x + 2y = 3$

A x 2: $4x - 6y = -2$

B x 3: $3x + 6y = 9$
 $7x = 7$
 $x = 1 \Rightarrow y = 1$ (Sub x into B)

i) Eqn of L $\leftarrow$ PT. ON LINE $(2, -1)$
 slope?

To find slope

$(x_1, y_1) = (2, -1)$

$(x_2, y_2) = (1, 1)$

$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{1 - 2} = -2$$

$\Rightarrow$ Eqn: $y - (-1) = -2(x - 2)$

$y + 1 = -2x + 4$

$2x + y - 3 = 0$

Q9. To find pt of intersection:

$$A: 5x + y + 8 = 0$$

$$B: x + 2y - 2 = 0$$

$$A \times 2: 10x + 2y + 16 = 0$$

$$B: (-1)x + 2y - 2 = 0$$

$$9x + 18 = 0$$

$$9x = -18$$

$$x = -2$$

$$A: 5x + y + 8 = 0$$

$$5(-2) + y + 8 = 0$$

$$-10 + y + 8 = 0$$

$$y = 2$$

$$\Rightarrow (-2, 2)$$

PT. $(-2, 2)$
EQN $\swarrow$ Slope?

To find slope

$$(x_1, y_1) = (-2, 2)$$

$$(x_2, y_2) = (0, 0)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2}{0 - (-2)} = -1$$

To find eqn:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -1(x - (-2))$$

$$y - 2 = -1(x + 2)$$

$$y - 2 = -x - 2$$

$$\boxed{x + y = 0}$$

Q10.

$$(x_1, y_1) = (5, 6)$$

$$(x_2, y_2) = (k, 2)$$

$$\begin{aligned} \text{Dist} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(k - 5)^2 + (2 - 6)^2} \\ &= \sqrt{k^2 - 10k + 25 + 16} \\ &= \sqrt{k^2 - 10k + 41} \end{aligned}$$

$$(\sqrt{k^2 - 10k + 41})^2 = (2\sqrt{5})^2$$

$$k^2 - 10k + 41 = 20$$

$$k^2 - 10k + 21 = 0$$

$$(k - 7)(k - 3) = 0$$

$$k - 7 = 0 \quad \text{or} \quad k - 3 = 0$$

$$\boxed{k = 7} \quad \text{or} \quad \boxed{k = 3}$$

Q11. Slope = $-\frac{a}{b}$

$$\text{Slope line 1} = \frac{-1}{b-4}$$

$$\text{Slope line 2} = \frac{-b}{1}$$

$$\perp \Rightarrow m_1 \times m_2 = -1$$

$$\left(\frac{-1}{b-4}\right)\left(\frac{-b}{1}\right) = -1$$

$$\frac{b}{b-4} = -1$$

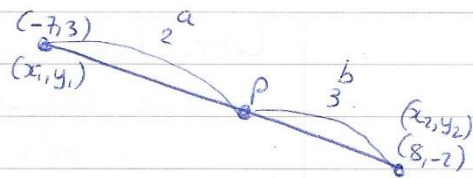
$$b = -1(b-4)$$

$$b = -b + 4$$

$$2b = 4$$

$$\boxed{b = 2}$$

Q12.



$$P = \left(\frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a} \right)$$

$$= \left(\frac{3(-7) + 2(8)}{2+3}, \frac{3(3) + 2(-2)}{3+2} \right)$$

$$= \left(\frac{-21+16}{5}, \frac{9-4}{5} \right)$$

$$= \boxed{(-1, 1)}$$

Q13. i) $x + 2y = -4$

$$2y = -x - 4$$

$$y = -\frac{1}{2}x - 2 \Rightarrow \boxed{L}$$

ii) $2x - y = -4$

$$y = 2x + 4 \Rightarrow \boxed{M}$$

iii) $x + 2y = 8$

$$2y = -x + 8$$

$$y = -\frac{1}{2}x + 4 \Rightarrow \boxed{J}$$

iv) $2x - y = 2$

$$y = 2x - 2 \Rightarrow \boxed{N}$$

$$\begin{aligned} \text{Q14. Slope of } L &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - (-1)}{0 - 5} \\ &= -4/5 \end{aligned}$$

$$\text{Slope of } K = \frac{-x_{no}}{y_{no}} = -25/k$$

$$\Rightarrow -25/k = 5/4$$

$$\Rightarrow 5k = -100$$

$$\Rightarrow \boxed{k = -20}$$

Q15. Equations of any line through (4, 3) is:

$$y - 3 = m(x - 4)$$

$$y - 3 = mx - 4m$$

$$mx - y + 3 - 4m = 0 \quad *$$

Slope of $6x + y - 5 = 0$ given

$$\text{by: } \frac{-x_{no}}{y_{no}} = -6/1 = -6$$

Angle between lines = 45°

$$\Rightarrow \left| \frac{m_1 - (-6)}{1 + (m_1)(-6)} \right| = \tan 45$$

$$\Rightarrow 1 = \left| \frac{m_1 + 6}{1 - 6m_1} \right|$$

$$\Rightarrow 1(1 - 6m_1) = m_1 + 6$$

$$1 - 6m_1 = m_1 + 6$$

$$-5 = 7m_1$$

$$\Rightarrow m_1 = -5/7$$

OR

$$-1(1 - 6m_1) = m_1 + 6$$

$$-1 + 6m_1 = m_1 + 6$$

$$5m_1 = 7$$

$$m_1 = 7/5$$

Put two m values into *

$$-\frac{5}{7}x - y + 3 - 4(-5/7) = 0$$

$$-5x - 7y + 21 + 20 = 0$$

$$\boxed{5x + 7y - 41 = 0}$$

$$\frac{7}{5}x - y + 3 - 4(\frac{7}{5}) = 0$$

$$7x - 5y + 15 - 28 = 0$$

$$\boxed{7x - 5y - 13 = 0}$$

Q16. Distance from $(-2, a)$ to both lines is the same

$$\left| \frac{4(-2) + 3(a) - 3}{\sqrt{(4)^2 + (3)^2}} \right| = \left| \frac{12(-2) + 5(a) - 13}{\sqrt{(12)^2 + (5)^2}} \right|$$

$$\left| \frac{-11 + 3a}{5} \right| = \left| \frac{5a - 37}{13} \right|$$

Square both sides:

$$\left(\frac{-11 + 3a}{5} \right)^2 = \left(\frac{5a - 37}{13} \right)^2$$

$$\frac{9a^2 - 66a + 121}{25} = \frac{25a^2 - 370a + 1369}{169}$$

$$1521a^2 - 11154a + 20449 = 625a^2 - 9250a + 34225$$

$$896a^2 - 1904a - 13776 = 0$$

$$224a^2 - 476a - 3444 = 0$$

$$32a^2 - 68a - 492 = 0$$

$$8a^2 - 17a - 123 = 0$$

$$a = \frac{17 \pm \sqrt{(-17)^2 - 4(8)(-123)}}{2(8)}$$

$$= \frac{17 \pm \sqrt{289 + 3936}}{16}$$

$$= \frac{17 \pm 65}{16}$$

$$= \boxed{-3 \text{ or } 5.125}$$

Q17. Equation of any line through $(4, 1)$ is:

$$y - 1 = m(x - 4)$$

$$y - 1 = mx - 4m$$

$$mx - y + 1 - 4m = 0 \quad *$$

Distance to $(1, 2) = 2\sqrt{2}$

$$\frac{|m(1) - (2) + 1 - 4m|}{\sqrt{(m)^2 + (-1)^2}} = 2\sqrt{2}$$

$$\frac{|-3m - 1|}{\sqrt{m^2 + 1}} = 2\sqrt{2}$$

$$|-3m - 1| = 2\sqrt{2}\sqrt{m^2 + 1}$$

$$|-3m - 1| = 2\sqrt{2m^2 + 2}$$

Square both sides:

$$(-3m - 1)^2 = (2\sqrt{2m^2 + 2})^2$$

$$9m^2 + 6m + 1 = 4(2m^2 + 2)$$

$$9m^2 + 6m + 1 = 8m^2 + 8$$

$$m^2 + 6m - 7 = 0$$

$$(m + 7)(m - 1) = 0$$

$$m = -7 \quad m = 1$$

Put m values into $*$:

$$m = -7$$

$$-7x - y + 1 - 4(-7) = 0$$

$$-7x - y + 29 = 0$$

$$\boxed{7x + y - 29 = 0}$$

$$m = 1$$

$$1x - y + 1 - 4(1) = 0$$

$$\boxed{x - y - 3 = 0}$$

Q18. $3x + 2y = c$

Cuts x-axis ($y=0$)

$$3x + 2(0) = c$$

$$3x = c$$

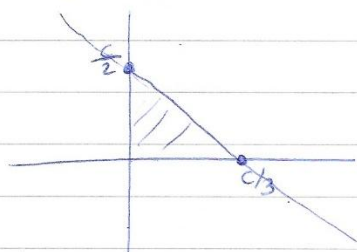
$$x = c/3$$

Cuts y-axis ($x=0$)

$$3(0) + 2y = c$$

$$2y = c$$

$$y = c/2$$



$$\text{Area} = 24$$

$$\Rightarrow \frac{1}{2}bh = 24$$

$$\frac{1}{2}\left(\frac{c}{3}\right)\left(\frac{c}{2}\right) = 24$$

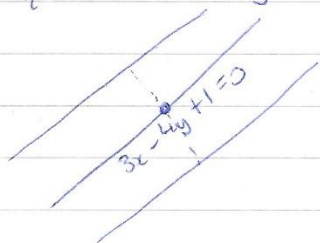
$$\frac{c^2}{12} = 24$$

$$c^2 = 288$$

$$c = \sqrt{288}$$

$$\Rightarrow c = \boxed{12\sqrt{2}}$$

Q19. Lines parallel to $3x - 4y + 1 = 0$ have equation $3x - 4y + d = 0$ *



To find a point on $3x - 4y + 1 = 0$

$$\text{Let } x = 1$$

$$\Rightarrow 3(1) - 4y + 1 = 0$$

$$-4y = -4$$

$$y = 1$$

$$\Rightarrow (1, 1) \text{ is on } 3x - 4y + 1 = 0$$

Dist from $(1, 1)$ to parallel lines is 2

$$\Rightarrow \frac{|3(1) - 4(1) + d|}{\sqrt{(3)^2 + (4)^2}} = 2$$

$$\frac{|d - 1|}{5} = 2$$

$$|d - 1| = 10$$

$$\Rightarrow d - 1 = 10 \quad \text{or} \quad d - 1 = -10$$

$$d = 11 \quad \text{or} \quad d = -9$$

Put d values into *

$$\boxed{3x - 4y + 11 = 0}$$

or

$$\boxed{3x - 4y - 9 = 0}$$