Revision Sheet Worked Solutions

Q1 $3^n + 3^n + 3^n$

$= 3^1 . 3^n$ (3 lots of 3^n)

$= \boxed{3^{n+1}}$ (Law 1)

Q2. $5^x = \frac{1}{5\sqrt{5}}$

$5^x = \frac{1}{5.5^{\frac{1}{2}}}$

$5^x = \frac{1}{5^{3/2}}$ (Law 1)

$5^x = 5^{-3/2}$ (Law 5)

$\Rightarrow \boxed{x = -3/2}$

Q3. $2^{2y+1} - 5(2^y) + 2 = 0$

$2^{2y}.2^1 - 5(2^y) + 2 = 0$

$2.2^{2y} - 5(2^y) + 2 = 0$

$2.(2^y)^2 - 5(2^y) + 2 = 0$

Let $p = 2^y$ ⊛

$2p^2 - 5p + 2 = 0$

$(2p - 1)(p - 2) = 0$

$2p - 1 = 0$ or $p - 2 = 0$

$2p = 1$ $\quad p = 2$

$p = \frac{1}{2}$

Using ⊛

If $p = \frac{1}{2}$: $\Rightarrow 2^y = \frac{1}{2^1}$, $2^y = 2^{-1}$, $\Rightarrow \boxed{y = -1}$

If $p = 2$: $\Rightarrow 2^y = 2^1$, $\Rightarrow \boxed{y = 1}$

Q4. $2^{2x+2} - 33(2^x) + 8 = 0$

$2^{2x}.2^2 - 33(2^x) + 8 = 0$

$4.2^{2x} - 33(2^x) + 8 = 0$

$4.(2^x)^2 - 33(2^x) + 8 = 0$

Let $p = 2^x$ ⊛

$4p^2 - 33p + 8 = 0$

$(4p - 1)(p - 8) = 0$

$4p - 1 = 0$ or $p - 8 = 0$

$4p = 1$ $\quad p = 8$

$p = \frac{1}{4}$

Using ⊛

If $p = \frac{1}{4}$: $\Rightarrow 2^x = \frac{1}{4}$, $2^x = \frac{1}{2^2}$, $2^x = 2^{-2}$, $\Rightarrow \boxed{x = -2}$

If $p = 8$: $\Rightarrow 2^x = 8$, $2^x = 2^3$, $\Rightarrow \boxed{x = 3}$

Q5. $\log_4(6x + 1) - 2 = 2\log_4 x$

$\log_4(6x+1) - 2.1 = \log_4 x^2$ (Law 3)

$\log_4(6x+1) - 2.\log_4 4 = \log_4 x^2$ (Law 6)

$\log_4(6x+1) - \log_4 4^2 = \log_4 x^2$ (Law 3)

$\log_4(6x+1) - \log_4 16 = \log_4 x^2$

$\log_4\left(\frac{6x+1}{16}\right) = \log_4 x^2$ (Law 2)

$\Rightarrow \frac{6x+1}{16} = x^2$

$\Rightarrow 16x^2 = 6x + 1$

$\Rightarrow 16x^2 - 6x - 1 = 0$

$(8x + 1)(2x - 1) = 0$

$x = -\frac{1}{8}$ $\quad x = \frac{1}{2}$

Checking both answers eliminates $x = -\frac{1}{8}$ as can't have $2\log_4\left(-\frac{1}{8}\right)$

$\Rightarrow$ Ans: $\boxed{x = \frac{1}{2}}$

Q6. $\log_2(x+2) + \log_2(x-2) = 5$

$\log_2(x+2)(x-2) = 5$ (Law 1)

$\log_2(x^2-4) = 5$

$\Rightarrow 2^5 = x^2 - 4$ (Log Definition)

$\Rightarrow x^2 - 4 = 32$

$x^2 = 36$

$x = \sqrt{36}$

$x = \pm 6$

Checking both answers eliminates $x = -6$ as can't have $\log_2(-6+2)$

$\Rightarrow$ Ans: $\boxed{x = 6}$

Q7. $\log_2(3x-1) = 3$

$\Rightarrow 2^3 = 3x - 1$ (Log Definition)

$\Rightarrow 8 = 3x - 1$

$\Rightarrow 3x = 9$

$\Rightarrow \boxed{x = 3}$

Q8. $2\log y = \log 2 + \log x$

$\log y^2 = \log 2 + \log x$

$\log y^2 = \log 2x$

$\Rightarrow y^2 = 2x$ I

$2^y = 4^x$

$2^y = (2^2)^x$

$2^y = 2^{2x}$

$\Rightarrow y = 2x$ II

Sub II into I:

$(2x)^2 = 2x$

$4x^2 = 2x$

$4x^2 - 2x = 0$ $(\div 2)$

$2x^2 - x = 0$

$x(2x-1) = 0$

$x = 0$ or $2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

x can't be 0 as $\log(0)$ is undefined

$\Rightarrow x = \frac{1}{2}$

To find y using II

II: $y = 2x$

$y = 2(\frac{1}{2}) = 1$

$\Rightarrow \boxed{x = \frac{1}{2} \quad y = 1}$

Q9. $3^x = 1000$

$\log 3^x = \log 1000$

$\Rightarrow x \cdot \log 3 = \log 1000$

$\Rightarrow x = \dfrac{\log 1000}{\log 3}$

$\Rightarrow x = \boxed{6.29}$

Q10. $2^n = 31$

$\log 2^n = \log 31$

$\Rightarrow n \log 2 = \log 31$

$\Rightarrow n = \dfrac{\log 31}{\log 2}$

$\Rightarrow \boxed{n = 4.954}$

Q11. $\log_3(x^2-10) - \log_3 x = 2\log_3 3$

$\log_3\left(\frac{x^2-10}{x}\right) = 2\log_3 3$ (Law 2)

$\log_3\left(\frac{x^2-10}{x}\right) = \log_3 3^2$ (Law 6)

$\log_3\left(\frac{x^2-10}{x}\right) = \log_3 9$

$\Rightarrow \frac{x^2-10}{x} = 9$

$\Rightarrow x^2 - 10 = 9x$

$\Rightarrow x^2 - 9x - 10 = 0$

$(x+1)(x-10) = 0$

$x+1=0 \quad x-10=0$

$x=-1 \quad x=10$

Checking both answers eliminates $x=-1$ as can't have $\log_3(-1)$

$\Rightarrow$ Ans: $\boxed{x=10}$

Q12. $\log_a(x-6) + \log_a(x-4) = \log_a x$

$\log_a(x-6)(x-4) = \log_a x$

$\log_a(x^2-10x+24) = \log_a x$

$\Rightarrow x^2 - 10x + 24 = x$

$x^2 - 11x + 24 = 0$

$(x-3)(x-8) = 0$

$x=3 \quad x=8$

Checking both answers eliminates $x=3$ as can't have $\log_a(3-4)$

$\Rightarrow$ Ans: $\boxed{x=8}$

Q13. $e^{2x} = 3$

$\ln e^{2x} = \ln 3$

$\Rightarrow 2x \cdot \ln e = \ln 3$

$\Rightarrow 2x = \ln 3$

$\Rightarrow \boxed{x = \frac{\ln 3}{2}}$

Q14.

$Q(t) = Ae^{bt}$

Size of colony doubles

$\Rightarrow Q = 2A$

$\Rightarrow 2A = Ae^{bt}$

$\Rightarrow 2 = e^{bt}$ $(\div A)$

Doubling took 10 mins

$\Rightarrow 2 = e^{b(10)}$

$2 = e^{10b}$

$\ln 2 = \ln e^{10b}$

$\ln 2 = 10b \cdot \ln e$

$\ln 2 = 10b$

$\Rightarrow b = \frac{\ln 2}{10} = \boxed{0.0693}$

Q15.

i) $P = 40000(1.03)^{12}$

$= \boxed{57,030}$

ii) @ $t=0$ $P = \boxed{40000}$

iii) P doubled $\Rightarrow P = 80000$

$\Rightarrow \frac{80000}{40000} = \frac{40000(1.03)^t}{40000}$

$2 = (1.03)^t$

$\log 2 = \log(1.03)^t$

$\Rightarrow t = \frac{\log 2}{\log 1.03} \doteq \boxed{23.5 \text{ yrs}}$