

Worked Solutions

Q1. ($\uparrow\downarrow$) $R = 3g$

($\leftarrow\rightarrow$) $T = m\omega^2 r$

$$T = (3)(3)^2(0.6)$$

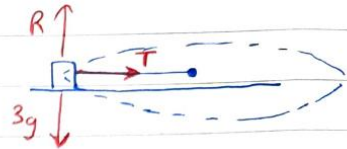
$$= 16.2$$

i) $a = \omega^2 r$

$$= (3)^2(0.6)$$

$$= \boxed{5.4 \text{ m/s}^2}$$

ii) $T = \boxed{16.2 \text{ N}}$



Q2. ($\uparrow\downarrow$) $R = 6g$

($\leftarrow\rightarrow$) $T = \frac{mv^2}{r}$

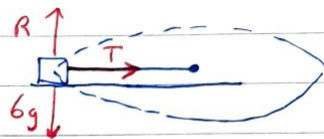
$$62.5 = \frac{6(v^2)}{0.8}$$

$$5v^2 = 62.5(0.8)$$

$$5v^2 = 50$$

$$v^2 = 10$$

$$v = \boxed{\sqrt{10} \text{ m/s}} \text{ or } \boxed{3.16 \text{ m/s}}$$



Q3. ($\uparrow\downarrow$) $R = mg$

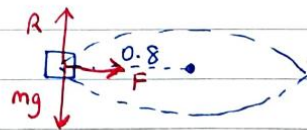
($\leftarrow\rightarrow$) $F = m\omega^2 r$

$$\mu R = m\omega^2 r$$

$$(0.3)(mg) = m\omega^2(0.8)$$

$$\omega^2 = \frac{0.3g}{0.8} = \frac{147}{40}$$

$$\omega = \sqrt{\frac{147}{40}} = \boxed{\frac{7\sqrt{30}}{20} \text{ rad/s}} \text{ or } \boxed{1.92 \text{ rad/s}}$$



Q4.

($\uparrow\downarrow$) $T \sin \alpha = 4g$

$$T \sin 30 = 4g$$

$$T = \boxed{8g \text{ N}}$$

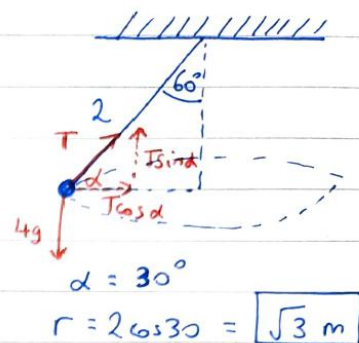
($\leftarrow\rightarrow$) $T \cos \alpha = m\omega^2 r$

$$(8g)\left(\frac{\sqrt{3}}{2}\right) = (4)(\omega^2)(\sqrt{3})$$

$$4g\sqrt{3} = 4\sqrt{3} \omega^2$$

$$\omega^2 = g$$

$$\omega = \boxed{\sqrt{g}} \text{ or } \boxed{\frac{7\sqrt{5}}{5}} \text{ or } \boxed{3.13 \text{ rad/s}}$$



Q5. LCE

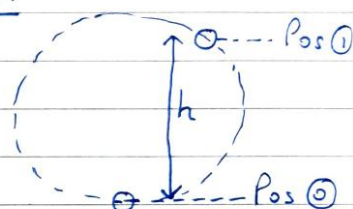
$$i) mgh_0 + \frac{1}{2}mu^2 = mgh_1 + \frac{1}{2}mv^2$$

$$mg(0) + \frac{1}{2}m(\sqrt{3gr})^2 = mgh + \frac{1}{2}m(0)^2$$

$$\frac{1}{2}(3gr) = gh$$

$$\Rightarrow h = \boxed{\frac{3}{2}r}$$

Diag 1:



$$ii) mg(0) + \frac{1}{2}m(\sqrt{3gr})^2 = mgh + \frac{1}{2}m(\sqrt{\frac{3gr}{2}})^2$$

$$\frac{1}{2}(3gr) = gh + \frac{1}{2}\left(\frac{3gr}{2}\right)$$

$$3gr = 4gh + 3gr$$

$$3gr = 4gh$$

$$h = \boxed{\frac{3}{4}r}$$

$$iii) mg(0) + \frac{1}{2}mu^2 = mg(2r) + \frac{1}{2}mv^2$$

$$u^2 = 4gr + v^2 : I$$

$$\text{At posn 1: } mg + R = \frac{mv^2}{r}$$

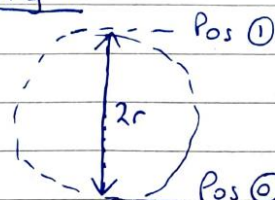
$$\text{Just makes top} \Rightarrow R = 0$$

$$v^2 = gr : II$$

$$\Rightarrow u^2 = 5gr \text{ (using I)}$$

$$\Rightarrow u = \boxed{\sqrt{5gr} \text{ m/s}}$$

Diag 1:



Diag 2: Forces @ top



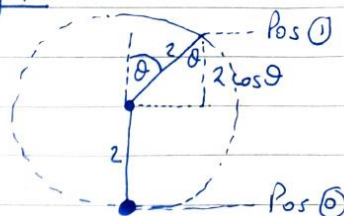
Q6. $mgh_0 + \frac{1}{2}mu^2 = mgh_1 + \frac{1}{2}mv^2$

$$mg(0) + \frac{1}{2}m(8)^2 = mg(2+2\cos\theta) + \frac{1}{2}mv^2$$

$$64 = 4g + 4g\cos\theta + v^2$$

$$v^2 = 24.8 - 4g\cos\theta : I$$

Diag 1:



Forces @ Pos 1:

$$T + mg\cos\theta = \frac{mv^2}{2}$$

$$\text{String slack} \Rightarrow T = 0$$

$$\Rightarrow mg\cos\theta = \frac{mv^2}{2}$$

$$v^2 = 2g\cos\theta : II$$

Equating I & II:

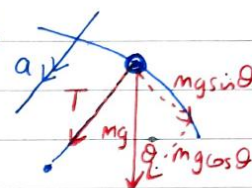
$$2g\cos\theta = 24.8 - 4g\cos\theta$$

$$6g\cos\theta = 24.8$$

$$\cos\theta = \frac{24.8}{6g} = \frac{20}{49}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{20}{49}\right) = \boxed{65.9^\circ}$$

Diag 2:



Q7.

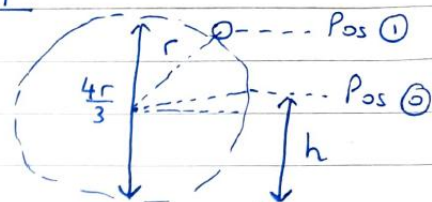
$$mgh_0 + \frac{1}{2}mu^2 = mgh_1 + \frac{1}{2}mv^2$$

$$mg(0) + \frac{1}{2}mv^2 = mg\left(\frac{4r}{3} - h\right) + \frac{1}{2}m(0)^2$$

$$\frac{v^2}{2} = \frac{4gr}{3} - gh$$

$$3v^2 = 8gr - 6gh \quad : I$$

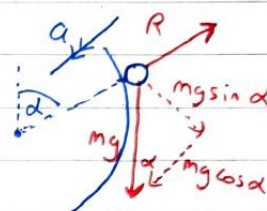
Diag 1:



Forces @ Pos ②:

$$mg \cos \alpha - R = \frac{mv^2}{r}$$

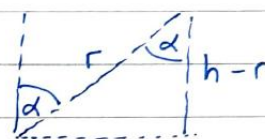
Diag 2:

Loses contact $\Rightarrow R=0$

$$\Rightarrow \frac{v^2}{r} = g \cos \alpha$$

$$v^2 = gr \cos \alpha$$

$$v^2 = gr \left(\frac{h-r}{r} \right)$$



$$\cos \alpha = \frac{h-r}{r}$$

$$v^2 = g(h-r) \quad : II$$

Sub II into I:

$$3(g(h-r)) = 8gr - 6gh$$

$$3gh - 3gr = 8gr - 6gh$$

$$9gh = 11gr$$

$$h = \boxed{\frac{11r}{9} \text{ m}}$$

Q8.

$$F_1 = k(L - l_0) \quad l_0 = 1.5$$

$$= 3.5(x - 0.75) \Rightarrow l_0 = 0.75 \text{ above}$$

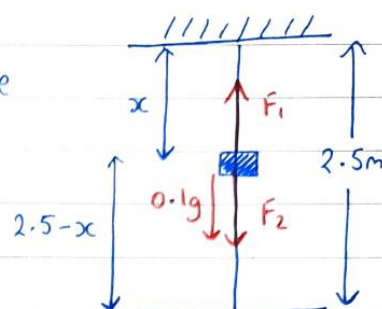
$$= 3.5x - 2.625 \quad \& \text{ below}$$

$$F_2 = k(L - l_0)$$

$$= 3.5(2.5 - x - 0.75)$$

$$= 3.5(1.75 - x)$$

$$= 6.125 - 3.5x$$

In equilibrium $\Rightarrow F_1 = F_2 + 0.1g$

$$3.5x - 2.625 = 6.125 - 3.5x + 0.1(9.8)$$

$$7x = 9.73$$

$$x = \boxed{1.39 \text{ m}}$$

Q9. i) $F = k(L - l_0)$

$$= 3g(L - 1.5)$$

$$= 29.4L - 44.1$$

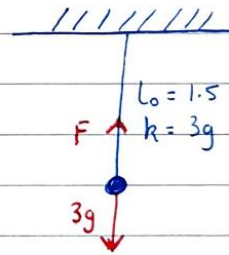
Equilibrium $\Rightarrow F = 3g$

$$\Rightarrow 29.4L - 44.1 = 3g$$

$$29.4L = 3g + 44.1$$

$$29.4L = 73.5$$

$$L = \boxed{2.5\text{m}}$$



ii) $F = k(L - l_0)$

$$= 3g(3 - 1.5)$$

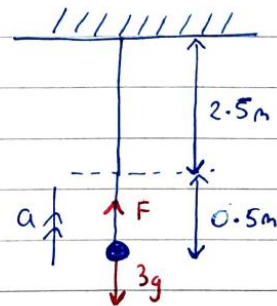
$$= 44.1$$

$$F = ma$$

$$\Rightarrow 44.1 - 3g = 3a$$

$$3a = 14.7$$

$$a = \boxed{4.9\text{ m/s}^2}$$



Q10.

To find x

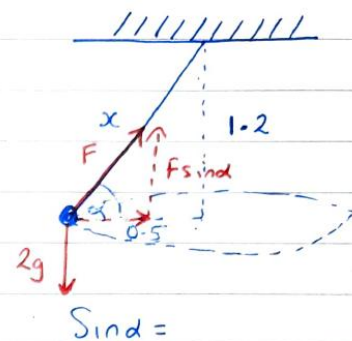
$$x^2 = (1.2)^2 + (0.5)^2$$

$$x^2 = 1.69$$

$$x = \sqrt{1.69} = 1.3$$

$$\cos \alpha = \frac{0.5}{1.3} = \frac{5}{13}$$

$$\sin \alpha = \frac{1.2}{1.3} = \frac{12}{13}$$



i) $(\uparrow\downarrow) F \sin \alpha = 2g$

$$k(1.3 - 1)\left(\frac{12}{13}\right) = 2g$$

$$k\left(\frac{3}{10}\right)\left(\frac{12}{13}\right) = 2g$$

$$k\left(\frac{18}{65}\right) = 2g$$

$$18k = 130g$$

$$k = \frac{130g}{18} = \boxed{\frac{65g}{9}}$$

ii) $(\rightarrow\leftarrow) F \cos \alpha = \frac{mv^2}{r}$

$$k(1.3 - 1)\left(\frac{5}{13}\right) = \frac{2v^2}{0.5}$$

$$\frac{65g}{9}\left(\frac{3}{10}\right)\left(\frac{5}{13}\right) = 4v^2$$

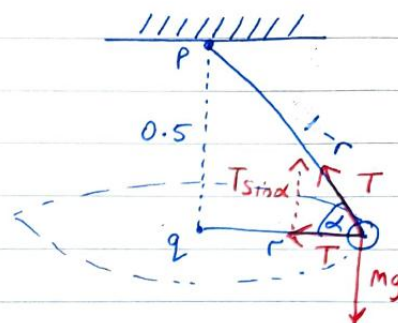
$$4v^2 = \frac{5g}{6}$$

$$v^2 = \frac{5g}{24}$$

$$v = \boxed{\sqrt{5g/24}}$$

Q11.

$$\begin{aligned}
 \text{i)} \quad (1-r)^2 &= r^2 + (0.5)^2 \\
 1 - 2r + r^2 &= r^2 + 0.25 \\
 1 - 0.25 &= 2r \\
 0.75 &= 2r \\
 r &= \boxed{0.375 \text{ m}}
 \end{aligned}$$



$$\begin{aligned}
 \text{ii)} \quad \sin \alpha &= \frac{0.5}{0.625} = \frac{4}{5} \\
 \cos \alpha &= \frac{0.375}{0.625} = \frac{3}{5}
 \end{aligned}$$

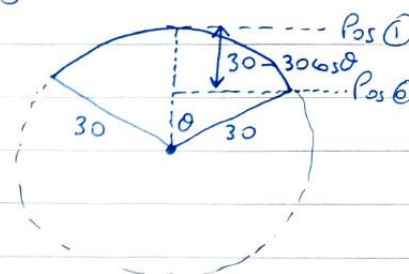
$$\begin{aligned}
 (\uparrow \downarrow) \quad T \sin \alpha &= mg \\
 T \left(\frac{4}{5} \right) &= 1g \\
 T &= \boxed{\frac{5g}{4} \text{ N}} \quad \text{or} \quad \boxed{12.25 \text{ N}}
 \end{aligned}$$

$$\begin{aligned}
 \text{iii)} \quad (\leftarrow \rightarrow) \quad T + T \cos \alpha &= m\omega^2 r \\
 12.25 + 12.25 \left(\frac{3}{5} \right) &= 1(\omega^2)(0.375) \\
 0.375\omega^2 &= 19.6 \\
 \omega^2 &= 52.2666 \\
 \omega &= \sqrt{52.2666} = \boxed{\frac{28\sqrt{15}}{15} \text{ rad/s}} \quad \text{or} \quad \boxed{7.23 \text{ rad/s}}
 \end{aligned}$$

Q12.

$$\begin{aligned}
 mgh_0 + \frac{1}{2}mu^2 &= mgh_1 + \frac{1}{2}mv^2 \\
 mg(30) + \frac{1}{2}mu^2 &= mg(30 - 30\cos\theta) + \frac{1}{2}mv^2 \\
 u^2 &= 2g(30 - 30\cos\theta) + v^2 \\
 u^2 &= 60g - 60g\cos\theta + v^2 : I
 \end{aligned}$$

Diag 1:



Forces @ top:

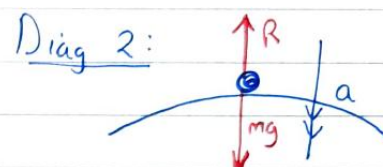
$$mg + R = \frac{mv^2}{r}$$

Leaves surface $\Rightarrow R = 0$

$$mg = \frac{mv^2}{30}$$

$$v^2 = 30g$$

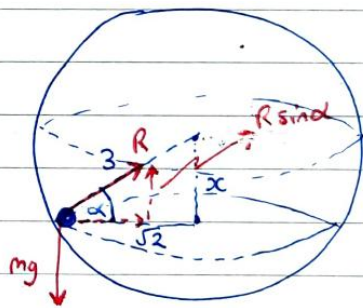
$$v = \sqrt{30g} = \boxed{7\sqrt{6} \text{ m/s}} \quad \text{or} \quad \boxed{17.15 \text{ m/s}}$$



Q13. $(\uparrow\downarrow) R \sin \alpha = mg$
 $R \left(\frac{\sqrt{7}}{3} \right) = mg$

$$R \sqrt{7} = 3mg$$

$$R = \frac{3mg}{\sqrt{7}} = \frac{3\sqrt{7}mg}{7}$$



To find x

$$(3)^2 = x^2 + (\sqrt{7})^2$$

$$x^2 = 7$$

$$x = \sqrt{7}$$

$$(\Rightarrow) R \cos \alpha = m \omega^2 r$$

$$\frac{3\sqrt{7}mg}{7} \left(\frac{\sqrt{2}}{3} \right) = m \omega^2 (\sqrt{2})$$

$$\frac{\sqrt{7}}{7} g = \omega^2$$

$$\omega^2 = \frac{7\sqrt{7}}{5}$$

$$\omega = \sqrt{\frac{7\sqrt{7}}{5}} = 1.92 \text{ rad/s}$$

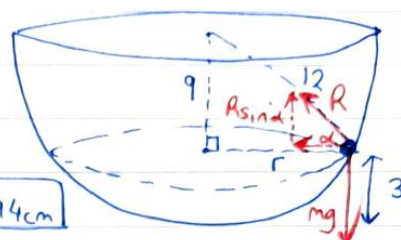
Q14.

i) $(12)^2 = (9)^2 + r^2$

$$144 - 81 = r^2$$

$$r^2 = 63$$

$$r = \sqrt{63} = 3\sqrt{7} \text{ cm} \text{ or } 7.94 \text{ cm}$$



ii) $(\uparrow\downarrow) R \sin \alpha = mg$

$$R \left(\frac{3}{4} \right) = 5g$$

$$3R = 20g$$

$$R = \frac{20g}{3} \text{ N} \text{ or } 65.33 \text{ N}$$

$$\sin \alpha = \frac{9}{12} = \frac{3}{4}$$

$$\cos \alpha = \frac{3\sqrt{7}}{12} = \frac{\sqrt{7}}{4}$$

iii) $(\Rightarrow) R \cos \alpha = m \omega^2 r$

$$\frac{20g}{3} \left(\frac{\sqrt{7}}{4} \right) = 5 \omega^2 (3\sqrt{7})$$

$$15 \omega^2 = \frac{5g}{3}$$

$$\omega^2 = \frac{g}{9}$$

$$\omega = \sqrt{\frac{g}{9}} = \frac{\sqrt{g}}{3} \text{ rad/s}$$

Q15.

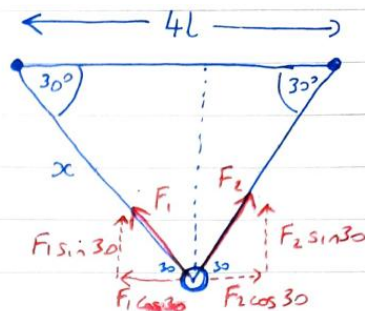
To find x

$$\cos 30^\circ = \frac{2L}{x}$$

$$\frac{\sqrt{3}}{2} = \frac{2L}{x}$$

$$x\sqrt{3} = 4L$$

$$x = \frac{4L}{\sqrt{3}} \quad \text{or} \quad \frac{4\sqrt{3}L}{3}$$



$$(\uparrow\downarrow) F_1 \sin 30^\circ + F_2 \sin 30^\circ = W$$

$$(F_1 + F_2) \sin 30^\circ = W$$

$$k \left(\frac{4L}{\sqrt{3}} - \frac{L}{2} + \frac{4L}{\sqrt{3}} - \frac{L}{2} \right) \left(\frac{1}{2} \right) = W$$

$$k \left(\frac{8L}{\sqrt{3}} - L \right) \left(\frac{1}{2} \right) = W$$

$$k \left(\frac{8L - L\sqrt{3}}{\sqrt{3}} \right) \left(\frac{1}{2} \right) = W$$

$$k(8L - L\sqrt{3}) = 2\sqrt{3}W$$

$$\Rightarrow k = \frac{2\sqrt{3}W}{L(8 - \sqrt{3})} \quad \text{or} \quad k = \frac{0.55W}{L}$$

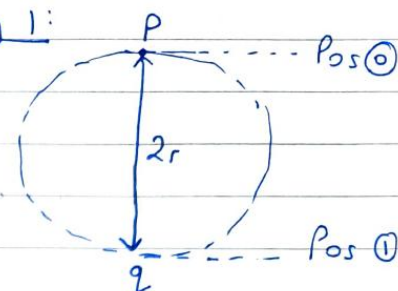
Q16.

$$mgh_0 + \frac{1}{2}mu^2 = mgh_1 + \frac{1}{2}mv^2$$

$$mg(0) + \frac{1}{2}mu^2 = mg(-2r) + \frac{1}{2}mv_1^2$$

$$u^2 = -4gr + v_1^2 \quad : I$$

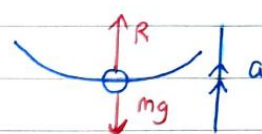
Diag 1:



Forces @ Pos ①:

$$R - mg = \frac{mv_1^2}{r}$$

Diag 2:



Cons of Momentum for collision:

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$mv_1 + m(0) = 2m v_2$$

$$v_2 = \frac{1}{2}v_1$$

Restart using Diag 1:

$$mgh_0 + \frac{1}{2}mu^2 = mgh_1 + \frac{1}{2}mv^2$$

$$2mg(0) + \frac{1}{2}(2m)\left(\frac{v_1}{2}\right)^2 = 2mg(2r) + \frac{1}{2}(2m)(0)^2$$

$$\frac{v_1^2}{4} = 4gr + 0$$

$$v_1^2 = 16gr$$

Subbing into I

$$u^2 = -4gr + 16gr$$

$$u^2 = 12gr$$

$$u = \sqrt{12gr} = \boxed{2\sqrt{3}gr \text{ m/s}}$$

just reaches the top
d stops as u is
a minimum to
make it up there.

Q17. In equilibrium $F_{up} = F_{down}$

$$\Rightarrow F_1 = mg$$

$$k(L - l_0) = mg$$

$$k(0.5) = mg$$

$$\Rightarrow k = 2mg$$

Let x = new extension

$$F_2 = k(L - l_0)$$

$$= 2mg(x)$$

$$(\uparrow\downarrow) F_2 \sin \alpha = mg$$

$$2mgx \sin \alpha = mg$$

$$2x \sin \alpha = 1$$

$$(\leftarrow\rightarrow) F_2 \cos \alpha = m\omega^2 r$$

$$2mgx \left(\frac{r}{L}\right) = m\omega^2 r$$

$$2gx = \omega^2 L$$

$$\text{But } L = 1.2 + x$$

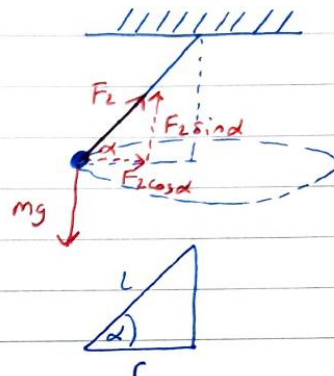
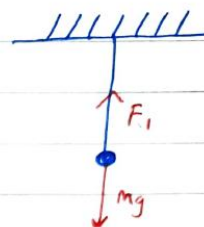
$$\Rightarrow 2gx = \omega^2 (1.2 + x)$$

$$2gx = 1.2\omega^2 + x\omega^2$$

$$2gx - x\omega^2 = 1.2\omega^2$$

$$x(2g - \omega^2) = 1.2\omega^2$$

$$\Rightarrow x = \boxed{\frac{1.2\omega^2}{2g - \omega^2}} \quad \text{or} \quad \boxed{\frac{1.2\omega^2}{19.6 - \omega^2}}$$



Q18.

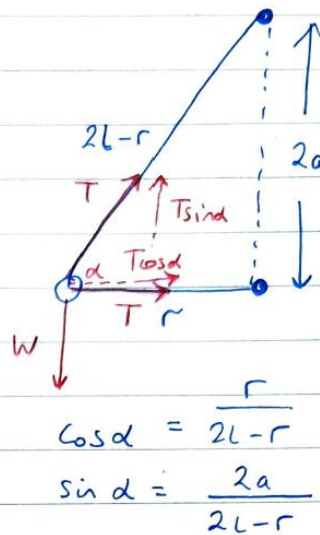
$$(\uparrow\downarrow) T \sin \alpha = W \quad : I$$

$$(\leftarrow\rightarrow) T + T \cos \alpha = \frac{mv^2}{r} \quad : II$$

Using I:

$$T \left(\frac{2a}{2l-r} \right) = W$$

$$\Rightarrow T = \frac{W(2l-r)}{2a} \quad (*)$$



$$\cos \alpha = \frac{r}{2l-r}$$

$$\sin \alpha = \frac{2a}{2l-r}$$

Subbing into II:

$$\frac{W(2l-r)}{2a} + \frac{W(2l-r)}{2a} \left(\frac{r}{2l-r} \right) = \frac{mv^2}{r}$$

$$\frac{2Wl - Wr}{2a} + \frac{Wr}{2a} = \frac{mv^2}{r}$$

$$\frac{2Wl - \cancel{Wr} + \cancel{Wr}}{2a} = \frac{mv^2}{r}$$

$$\frac{Wl}{a} = \frac{mv^2}{r}$$

As $W = mg$

$$\Rightarrow \frac{mgl}{a} = \frac{mv^2}{r}$$

$$v^2 = \frac{rgl}{a}$$

$$\Rightarrow v^2 = \left(\frac{l^2 - a^2}{l} \right) gl$$

$$v^2 = \frac{g(l^2 - a^2)}{a}$$

$$\Rightarrow v = \boxed{\sqrt{\frac{g(l^2 - a^2)}{a}}}$$

To find r in terms of l, a :

$$(2l-r)^2 = r^2 + (2a)^2$$

$$4l^2 + r^2 - 4lr = r^2 + 4a^2$$

$$4lr = 4l^2 - 4a^2$$

$$lr = l^2 - a^2$$

$$r = \frac{l^2 - a^2}{l}$$

Using (*):

$$T = \frac{W(2l - \frac{l^2 - a^2}{l})}{2a}$$

$$= \frac{W \left(\frac{2l^2 - l^2 + a^2}{l} \right)}{2a}$$

$$= \boxed{\frac{W(l^2 + a^2)}{2la}}$$

Q.E.D.

Q19.

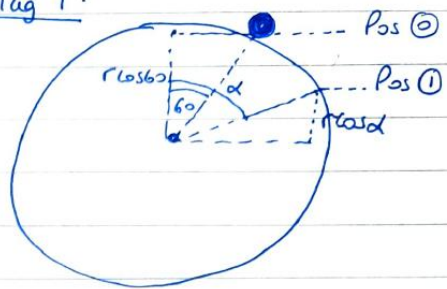
$$mgh_0 + \frac{1}{2}mu^2 = mgh_1 + \frac{1}{2}mv^2$$

$$mg(r\cos 60) + 0 = mg(r\cos \alpha) + \frac{1}{2}mv^2$$

$$gr\left(\frac{1}{2}\right) = gr\cos \alpha + \frac{1}{2}v^2$$

$$v^2 = gr - 2gr\cos \alpha \quad \text{I}$$

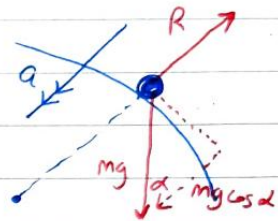
Diag 1:



Forces @ Pos 1:

$$mg\cos \alpha - R = \frac{mv^2}{r}$$

Diag 2:



Leaves surface $\Rightarrow R = 0$

$$mg\cos \alpha = \frac{mv^2}{r}$$

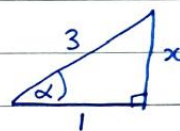
$$v^2 = gr\cos \alpha \quad \text{II}$$

Equating I & II:

$$gr\cos \alpha = gr - 2gr\cos \alpha$$

$$3gr\cos \alpha = gr$$

$$\cos \alpha = \frac{1}{3}$$



$$(3)^2 = (1)^2 + x^2$$

$$x^2 = 8$$

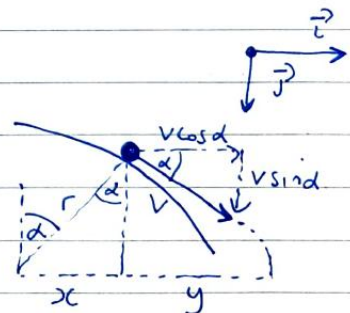
$$x = \sqrt{8} = 2\sqrt{2}$$

$$\sin \alpha = \frac{2\sqrt{2}}{3}$$

$$\text{Height above O} = r\cos \alpha$$

$$= r\left(\frac{1}{3}\right) = \boxed{\frac{r}{3}}$$

Diag 3:



$$\vec{u} = v\cos \alpha \vec{i} + v\sin \alpha \vec{j}$$

$$= \sqrt{\frac{gr}{3}}\left(\frac{1}{3}\right)\vec{i} + \sqrt{\frac{gr}{3}}\left(\frac{2\sqrt{2}}{3}\right)\vec{j}$$

$$= \sqrt{\frac{gr}{27}}\vec{i} + \sqrt{\frac{8gr}{27}}\vec{j}$$

$$V_x = \sqrt{\frac{gr}{27}} \quad V_y = \sqrt{\frac{8gr}{27}} - gt \quad S_x = \sqrt{\frac{gr}{27}}t \quad S_y = \sqrt{\frac{8gr}{27}}t + \frac{1}{2}gt^2$$

To find y:

$$\text{When particle @ O} \quad S_y = r\cos \alpha = \frac{r}{3}$$

$$\Rightarrow \sqrt{\frac{8gr}{27}}t + \frac{1}{2}gt^2 = \frac{r}{3}$$

$$\sqrt{\frac{8gr}{27}}t + \frac{g}{2}t^2 - \frac{r}{3} = 0$$

$$\Rightarrow 4.9t^2 + \sqrt{\frac{8gr}{27}} t - \frac{r}{3} = 0$$

$$\Rightarrow t = \frac{-\sqrt{\frac{8gr}{27}} \pm \sqrt{\left(\sqrt{\frac{8gr}{27}}\right)^2 - 4(4.9)\left(-\frac{r}{3}\right)}}{2(4.9)}$$

$$= \frac{-\sqrt{\frac{8gr}{27}} + \sqrt{\frac{8gr}{27} + \frac{98r}{15}}}{9}$$

$$= \frac{-\sqrt{\frac{8gr}{27}} + \sqrt{\frac{1274r}{135}}}{9}$$

$$= \frac{-\sqrt{\frac{392r}{135}} + \sqrt{\frac{1274r}{135}}}{9}$$

$$= \frac{1.37\sqrt{r}}{9}$$

$$t = 0.14\sqrt{r}$$

$$\Rightarrow S_x = \sqrt{\frac{gr}{27}} (0.14\sqrt{r})$$

$$= 0.08r$$

$$\Rightarrow y = 0.08r$$

$$\text{From Diag 3 } x = r \sin \alpha = r \left(\frac{2\sqrt{2}}{3} \right) = \frac{2r\sqrt{2}}{3}$$

$$\Rightarrow \text{Tot horiz dist} = x + y$$

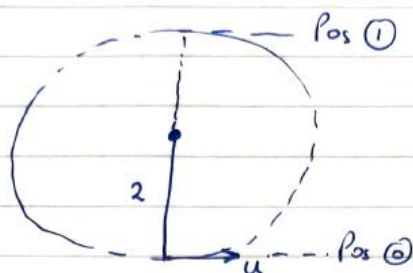
$$= \frac{2r\sqrt{2}}{3} + 0.08r$$

$$= 0.9428r + 0.08r$$

$$= \boxed{1.02r}$$

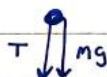
Q6

a) i)

LCE

$$mg(0) + \frac{1}{2}m(u^2) = mg(4) + \frac{1}{2}mv^2$$

$$u^2 = 8g + v^2 \quad \text{I}$$

Forces @ D

$$T + mg = \frac{mv^2}{2}$$

Just reaches D $\Rightarrow T=0$

$$mg = \frac{mv^2}{2}$$

$$\Rightarrow v^2 = 2g \quad \text{II}$$

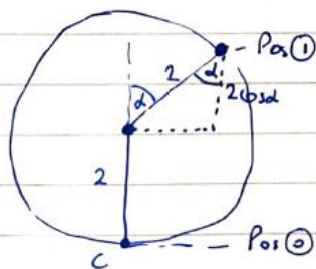
$$\Rightarrow \text{I: } u^2 = 8g + 2g$$

$$u^2 = 10g$$

$$\Rightarrow u = \sqrt{10g}$$

$$= \boxed{7\sqrt{2}}$$

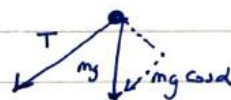
ii)



$$\text{LCE } mgh + \frac{1}{2}mu^2 = mgh + \frac{1}{2}mv^2$$

$$mg(0) + \frac{1}{2}m(7)^2 = mg(2+2\cos\alpha) + \frac{1}{2}mv^2$$

$$49 = 4g + 4g\cos\alpha + v^2 \quad \text{I}$$

Forces @ Pos ①

$$T + mg\cos\alpha = \frac{mv^2}{2}$$

I=0

$$\Rightarrow v^2 = 2g\cos\alpha \quad \text{II}$$

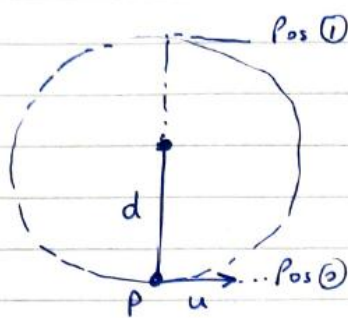
$$\Rightarrow \text{I: } 49 - 4g - 4g\cos\alpha = 2g\cos\alpha$$

$$6g\cos\alpha = 9$$

$$\cos\alpha = \frac{1}{6} \Rightarrow \alpha = \boxed{80.4^\circ}$$

2018 Q 6

Q 6 b)
i)



LCE
 $mg(0) + \frac{1}{2}mu^2 = mg(2d) + \frac{1}{2}mv^2$
 $u^2 = 4gd + v^2$ I

If it just makes top $\Rightarrow T=0$
 $\Rightarrow T + mg = \frac{mv^2}{d}$

$\Rightarrow v^2 = gd$ II

$\Rightarrow$ I: $u^2 = 5gd$

$\Rightarrow u = \sqrt{5gd}$

$\Rightarrow$ to pass the top $\Rightarrow \boxed{u > \sqrt{5gd}}$

ii) Least Tension @ Top = T_1
 Greatest Tension @ Bottom = kT_1

LCE

$$mg(0) + \frac{1}{2}m(\sqrt{6gd})^2 = mg(2d) + \frac{1}{2}mv^2$$

$$3gd = 2gd + \frac{1}{2}v^2$$

$$gd = \frac{1}{2}v^2$$

$$2gd = v^2$$

Forces @ bottom

$$kT_1 - mg = \frac{mv^2}{d}$$

$$kT_1 - mg = \frac{m}{d}(6gd)$$

$$kT_1 = 7mg$$

Forces @ top

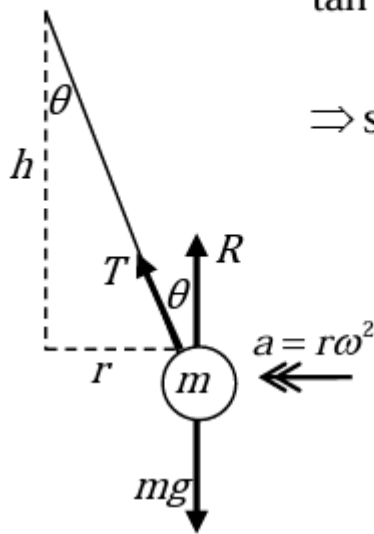
$$T_1 + mg = \frac{mv^2}{d}$$

$$T_1 + mg = \frac{m}{d}(2gd)$$

$$T_1 = mg$$

$\Rightarrow kmg = 7mg$

$\Rightarrow \boxed{k=7}$

Solution: (i)

$$\tan \theta = \frac{r}{h},$$

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$$\Rightarrow \sin \theta = \frac{r}{\sqrt{r^2 + h^2}} \quad \& \quad \cos \theta = \frac{h}{\sqrt{r^2 + h^2}}$$

When particle is about to lose contact

$$\Rightarrow R = 0$$

Resolve $\uparrow$: $T \cos \theta = mg$

$$\Rightarrow T = \frac{mg\sqrt{r^2 + h^2}}{h}$$

$$\vec{F} = m\vec{a} \quad \leftarrow : T \sin \theta = mr\omega^2 \quad \mathbf{A}$$

$$\Rightarrow \frac{mg\sqrt{r^2 + h^2}}{h} \frac{r}{\sqrt{r^2 + h^2}} = mr\omega^2$$

$$\Rightarrow \omega^2 = \frac{g}{h} \quad \text{when } R = 0$$

$$\Rightarrow \omega^2 \leq \frac{g}{h} \quad \text{when } R \geq 0$$

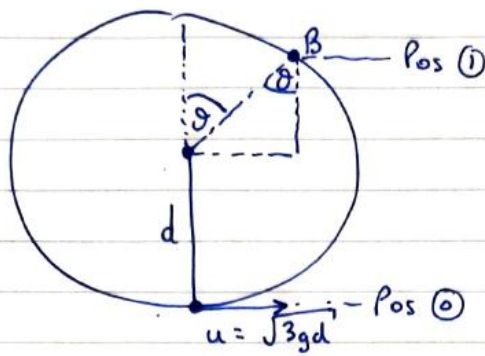
$$(ii) \quad F = k(l - l_0), \quad \Rightarrow T = \frac{2mg}{h}(l - h)$$

$$\text{In } \mathbf{A}: \frac{2mg}{h}(l - h) \frac{r}{l} = mr \left(\frac{2g}{5h} \right)$$

$$\Rightarrow \frac{2(l - h)}{l} = \frac{2}{5}, \quad \Rightarrow l = \frac{5h}{4}$$

$$\Rightarrow \cos \theta = \frac{h}{l} = \frac{h}{\frac{5h}{4}} = \frac{4}{5}, \quad \Rightarrow \theta = 36.9^\circ$$

Q6 b)
i)



LCE

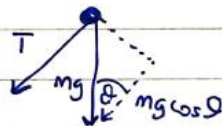
$$mg(0) + \frac{1}{2}m(\sqrt{3gd})^2 = mg(d + d \cos \theta) + \frac{1}{2}mv^2$$

$$3gd = 2gd + 2gd \cos \theta + v^2$$

$$gd - 2gd \cos \theta = v^2 \quad \text{I}$$

Forces @ Pos 1

$$T + mg \cos \theta = \frac{mv^2}{d}$$



$$T=0 \Rightarrow v^2 = gd \cos \theta \quad \text{II}$$

$$\Rightarrow gd - 2gd \cos \theta = gd \cos \theta$$

$$\Rightarrow \boxed{\cos \theta = 1/3} \Rightarrow \sin \theta = \sqrt{8/3}$$

ii) When string goes slack, particle becomes a projectile: with initial velocity $v \cos \theta \hat{i} + v \sin \theta \hat{j}$

$$v^2 = gd \cos \theta$$

$$v^2 = \frac{gd}{3}$$

$$\Rightarrow v = \sqrt{\frac{gd}{3}}$$

$$\Rightarrow \vec{u} = \frac{1}{3} \sqrt{\frac{gd}{3}} \hat{i} + \frac{\sqrt{8}}{3} \sqrt{\frac{gd}{3}} \hat{j}$$

$$\Rightarrow V_y = \frac{\sqrt{8}}{3} \sqrt{\frac{gd}{3}} - gt = 0$$

$$gt = \frac{\sqrt{8}}{3} \sqrt{\frac{gd}{3}}$$

$$t = \frac{\sqrt{8}}{3g} \sqrt{\frac{gd}{3}}$$

$$S_y = \frac{\sqrt{8}}{3} \sqrt{\frac{gd}{3}} t - \frac{1}{2} g t^2$$

$$= \frac{\sqrt{8}}{3} \sqrt{\frac{gd}{3}} \left(\frac{\sqrt{8}}{3g} \right) \left(\sqrt{\frac{gd}{3}} \right) - \frac{g}{2} \left(\frac{8}{9g^2} \right) \left(\frac{gd}{3} \right)$$

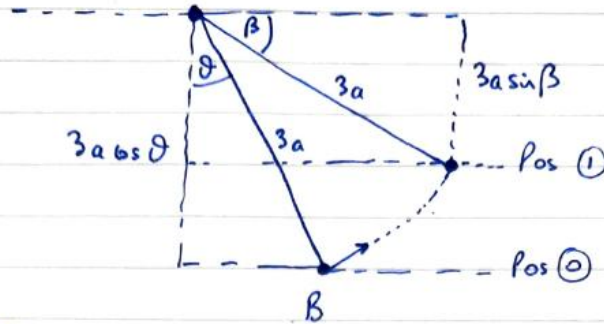
$$= \frac{8\cancel{g}d}{27g} - \frac{8\cancel{g}^2d}{54g^2}$$

$$= \frac{8d}{27} - \frac{8d}{54}$$

$$= \boxed{\frac{4d}{27}}$$

2017 Q6

Q6 b)



$$\cos \theta = \frac{2}{3}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{5}}{3}$$

LCE

$$mg(6) + \frac{1}{2}m(\sqrt{ag})^2 = mg(3a(\frac{2}{3}) - 3a \sin \beta) + \frac{1}{2}mv^2$$

$$\frac{ag}{2} = 2ag - 3ag \sin \beta + \frac{v^2}{2}$$

$$ag = 4ag - 6ag \sin \beta + v^2$$

$$6ag \sin \beta - 3ag = v^2$$

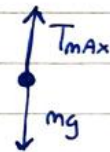
$$\Rightarrow v^2 = 3ag(2 \sin \beta - 1) \quad \text{Q.E.D.}$$

Forces @ bottom for $T_{\max}$

$$\text{Bottom} \Rightarrow \beta = 90^\circ$$

$$\Rightarrow v^2 = 3ag(2(1) - 1)$$

$$v^2 = 3ag$$



$$T_{\max} - mg = \frac{mv^2}{3a}$$

$$T_{\max} = mg + \frac{m}{3a}(3ag)$$

$$= \boxed{2mg}$$

$T_{\min}$ will occur when $v = 0$

$$\Rightarrow 3ag(2 \sin \beta - 1) = 0$$

$$\Rightarrow 2 \sin \beta = 1$$

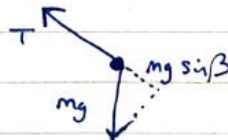
$$\sin \beta = \frac{1}{2} \Rightarrow \beta = 30^\circ$$

Forces @ $\beta = 30^\circ$

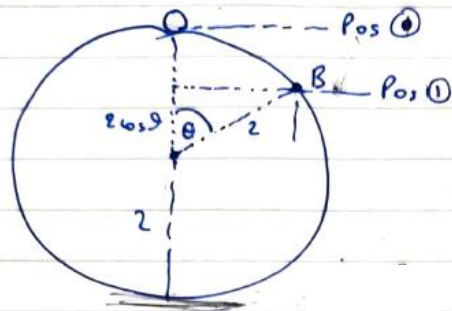
$$T - mg \sin \beta = \frac{mv^2}{3a}$$

$$T - mg/2 = 0$$

$$\Rightarrow T = \boxed{mg/2}$$



Q6 a)



i)

LCE

$$mg(0) + \frac{1}{2}m(0)^2 = mg(2\cos\theta - 2) + \frac{1}{2}mv^2$$

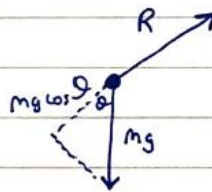
$$v^2 = 4g - 4g\cos\theta \quad \text{I}$$

Forces @ B

$$mg\cos\theta - R = \frac{mv^2}{2}$$

$$R = 0$$

$$v^2 = 2g\cos\theta \quad \text{II}$$



$$\Rightarrow 2g\cos\theta = 4g - 4g\cos\theta$$

$$6g\cos\theta = 4g$$

$$\cos\theta = \frac{2}{3}$$

$$\Rightarrow v^2 = 4g - 4g\left(\frac{2}{3}\right)$$

$$v^2 = \frac{196}{15}$$

$$v = \boxed{\frac{14\sqrt{15}}{15}} \text{ or } \boxed{3.61 \text{ m/s}}$$

ii)

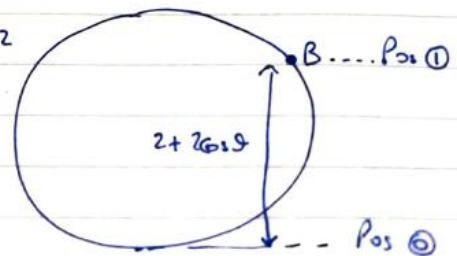
Using LCE

$$mg(2 + 2\cos\theta) + \frac{1}{2}m(3.61)^2 = mg(0) + \frac{1}{2}mv^2$$

$$2g + 2g\left(\frac{2}{3}\right) + \frac{98}{15} = \frac{v^2}{2}$$

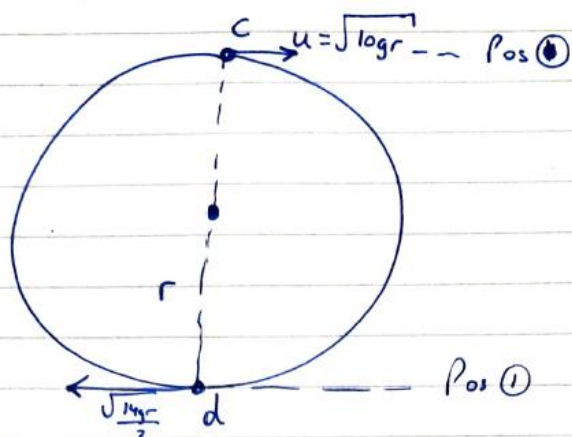
$$v^2 = \frac{392}{5}$$

$$v = \boxed{8.85 \text{ m/s}}$$



2007 Q6

Q6 b)



LCE

$$m'g(2r) + \frac{1}{2}m'(\sqrt{10gr})^2 = m'g(0) + \frac{1}{2}m'v^2$$

$$2gr + 5gr = \frac{1}{2}v^2$$

$$v^2 = 14gr$$

LCM

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$m'(\sqrt{14gr}) + m(0) = (2m')v$$

$$v = \frac{\sqrt{14gr}}{2}$$

LCE

$$2mg(0) + \frac{1}{2}(2m')(\frac{14gr}{4}) = 2m'g(2r) + \frac{1}{2}(2m')v^2$$

$$\frac{7gr}{2} = 4gr + v^2$$

$$v^2 = \frac{7gr}{2} - \frac{8gr}{2}$$

$$v^2 = -\frac{gr}{2}$$

No soln $\Rightarrow$ doesn't make it to top