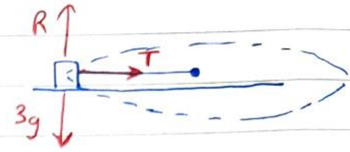
Worked Solutions

Q1. $(\uparrow\downarrow)$ $R = 3g$



$(\rightleftarrows)$ $T = m\omega^2 r$

$T = (3)(3)^2(0.6)$

$= 16.2$

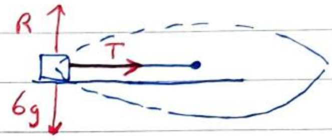
i) $a = \omega^2 r$

$= (3)^2(0.6)$

$= \boxed{5.4 \text{ m/s}^2}$

ii) $T = \boxed{16.2\text{N}}$

Q2. $(\uparrow\downarrow)$ $R = 6g$



$(\rightleftarrows)$ $T = \frac{mv^2}{r}$

$62.5 = \frac{6(v^2)}{0.8}$

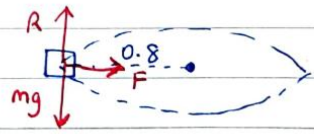
$5v^2 = 62.5(0.8)$

$5v^2 = 50$

$v^2 = 10$

$v = \boxed{\sqrt{10} \text{ m/s}}$ or $\boxed{3.16 \text{ m/s}}$

Q3. $(\uparrow\downarrow)$ $R = mg$



$(\rightleftarrows)$ $F = m\omega^2 r$

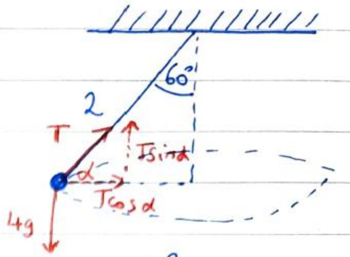
$\mu R = m\omega^2 r$

$(0.3)(\cancel{m}g) = \cancel{m}\omega^2(0.8)$

$\omega^2 = \frac{0.3g}{0.8} = \frac{147}{40}$

$\omega = \sqrt{\frac{147}{40}} = \boxed{\frac{7\sqrt{30}}{20} \text{ rad/s}}$ or $\boxed{1.92 \text{ rad/s}}$

Q4.



$(\uparrow\downarrow)$ $T\sin\alpha = 4g$

$T\sin 30 = 4g$

$T = \boxed{8g \text{ N}}$

$\alpha = 30°$

$r = 2\cos 30 = \boxed{\sqrt{3} \text{ m}}$

$(\rightleftarrows)$ $T\cos\alpha = m\omega^2 r$

$(8g)\left(\frac{\sqrt{3}}{2}\right) = (4)(\omega^2)(\sqrt{3})$

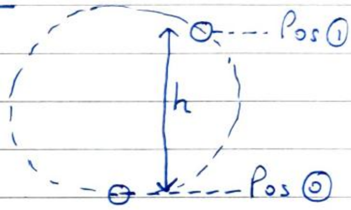
$4g\cancel{\sqrt{3}} = \cancel{4\sqrt{3}}\,\omega^2$

$\omega^2 = g$

$\omega = \boxed{\sqrt{g}}$ or $\boxed{\frac{7\sqrt{5}}{5}}$ or $\boxed{3.13 \text{ rad/s}}$

Q5. LCE

Diag 1:



i) $mgh_0 + \frac{1}{2}mu^2 = mgh_1 + \frac{1}{2}mv^2$

$\cancel{m}g(0) + \frac{1}{2}\cancel{m}(\sqrt{3gr})^2 = \cancel{m}gh + \frac{1}{2}\cancel{m}(0)^2$

$\frac{1}{2}(3\cancel{g}r) = \cancel{g}h$

$\Rightarrow h = \boxed{\frac{3}{2}r}$

ii) $\cancel{m}g(0) + \frac{1}{2}\cancel{m}(\sqrt{3gr})^2 = \cancel{m}gh + \frac{1}{2}\cancel{m}\left(\sqrt{\frac{3gr}{2}}\right)^2$

$\frac{1}{2}(3gr) = gh + \frac{1}{2}\left(\frac{3gr}{2}\right)$

$6gr = 4gh + 3gr$

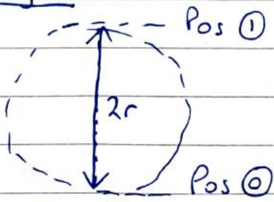
$3\cancel{g}r = 4\cancel{g}h$

$h = \boxed{\frac{3}{4}r}$

iii) $\cancel{m}g(0) + \frac{1}{2}\cancel{m}u^2 = \cancel{m}g(2r) + \frac{1}{2}\cancel{m}v^2$

$u^2 = 4gr + v^2 \quad : \mathrm{I}$

Diag 1:

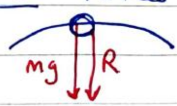


At posn 1: $mg + R = \frac{mv^2}{r}$

Just makes top $\Rightarrow R = 0$

$v^2 = gr \quad : \mathrm{II}$

Diag 2: Forces @ top

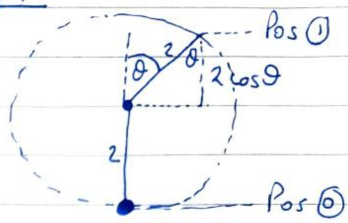


$\Rightarrow u^2 = 5gr$ (using I)

$\Rightarrow u = \boxed{\sqrt{5gr}\ \text{m/s}}$

Q6. $mgh_0 + \frac{1}{2}mu^2 = mgh_1 + \frac{1}{2}mv^2$

Diag 1:



$\cancel{m}g(0) + \frac{1}{2}\cancel{m}(8)^2 = \cancel{m}g(2 + 2\cos\theta) + \frac{1}{2}\cancel{m}v^2$

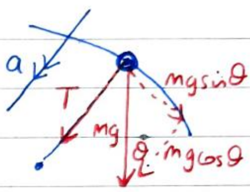
$64 = 4g + 4g\cos\theta + v^2$

$v^2 = 24.8 - 4g\cos\theta \quad : \mathrm{I}$

Forces @ Pos ①:

$T + mg\cos\theta = \frac{mv^2}{2}$

Diag 2:



String slack $\Rightarrow T = 0$

$\Rightarrow \cancel{m}g\cos\theta = \frac{\cancel{m}v^2}{2}$

$v^2 = 2g\cos\theta \quad : \mathrm{II}$

Equating I & II:

$2g\cos\theta = 24.8 - 4g\cos\theta$

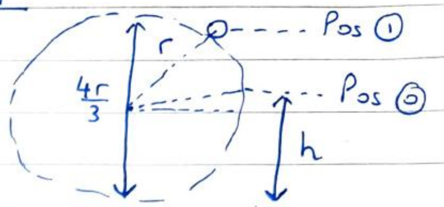
$6g\cos\theta = 24.8$

$\cos\theta = \frac{24.8}{6g} = \frac{20}{49}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{20}{49}\right) = \boxed{65.9^\circ}$

Q7.

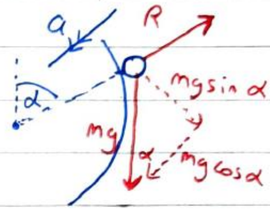
Diag 1:



$$mgh_0 + \tfrac{1}{2}mu^2 = mgh_1 + \tfrac{1}{2}mv^2$$
$$\cancel{m}g(0) + \tfrac{1}{2}\cancel{m}v^2 = \cancel{m}g\left(\tfrac{4r}{3} - h\right) + \tfrac{1}{2}\cancel{m}(0)^2$$

$$\frac{v^2}{2} = \frac{4gr}{3} - gh$$
$$3v^2 = 8gr - 6gh \quad : \mathrm{I}$$

Diag 2:



Forces @ Pos ⓪:

$$mg\cos\alpha - R = \frac{mv^2}{r}$$

Loses contact $\Rightarrow R = 0$

$$\Rightarrow \frac{v^2}{r} = g\cos\alpha$$

$$\cos\alpha = \frac{h-r}{r}$$

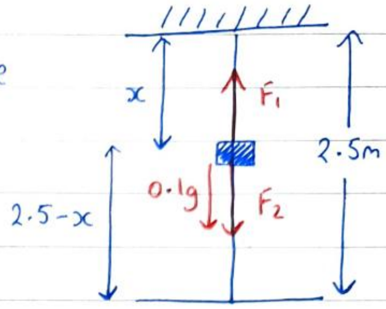
$$v^2 = gr\cos\alpha$$
$$v^2 = gr\left(\frac{h-r}{r}\right)$$

$$v^2 = g(h-r) \quad : \mathrm{II}$$

Sub II into I:

$$3(g(h-r)) = 8gr - 6gh$$
$$3gh - 3gr = 8gr - 6gh$$
$$9\cancel{g}h = 11\cancel{g}r$$
$$h = \boxed{\frac{11r}{9}\ m}$$

Q8.



$F_1 = k(l - l_0) \qquad l_0 = 1.5$

$= 3.5(x - 0.75) \Rightarrow l_0 = 0.75$ above

$= 3.5x - 2.625$ & below

$$F_2 = k(l - l_0)$$
$$= 3.5(2.5 - x - 0.75)$$
$$= 3.5(1.75 - x)$$
$$= 6.125 - 3.5x$$

In equilibrium $\Rightarrow F_1 = F_2 + 0.1g$

$$3.5x - 2.625 = 6.125 - 3.5x + 0.1(9.8)$$
$$7x = 9.73$$
$$x = \boxed{1.39\ m}$$

Q9. i) $F = k(l - l_0)$

$= 3g(l - 1.5)$

$= 29.4l - 44.1$

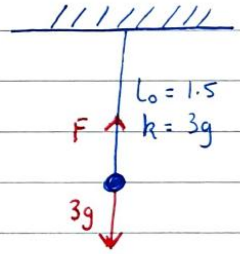
Equilibrium $\Rightarrow$ $F = 3g$

$\Rightarrow 29.4l - 44.1 = 3g$

$29.4l = 3g + 44.1$

$29.4l = 73.5$

$l = \boxed{2.5m}$



ii) $F = k(l - l_0)$

$= 3g(3 - 1.5)$

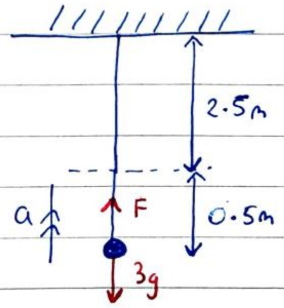
$= 44.1$

$F = ma$

$\Rightarrow 44.1 - 3g = 3a$

$3a = 14.7$

$a = \boxed{4.9 \, m/s^2}$



Q10. <u>To find x</u>

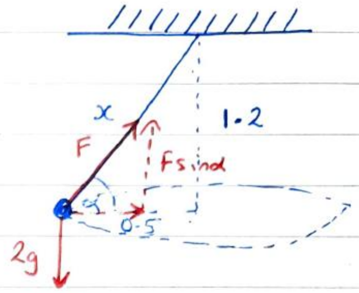
$x^2 = (1.2)^2 + (0.5)^2$

$x^2 = 1.69$

$x = \sqrt{1.69} = 1.3$

$\cos\alpha = \frac{0.5}{1.3} = \frac{5}{13}$

$\sin\alpha = \frac{1.2}{1.3} = \frac{12}{13}$



Sinα =

i) $(\uparrow\downarrow)$ $F\sin\alpha = 2g$

$k(1.3 - 1)\left(\frac{12}{13}\right) = 2g$

$k\left(\frac{3}{10}\right)\left(\frac{12}{13}\right) = 2g$

$k\left(\frac{18}{65}\right) = 2g$

$18k = 130g$

$k = \frac{130g}{18} = \boxed{\frac{65g}{9}}$

ii) $(\rightleftarrows)$ $F\cos\alpha = \frac{mv^2}{r}$

$k(1.3 - 1)\left(\frac{12}{13}\right) = \frac{2v^2}{0.5}$

$\frac{65g}{9}\left(\frac{3}{10}\right)\left(\frac{12}{13}\right) = 4v^2$

$4v^2 = 2g$

$v^2 = \frac{g}{2}$

$v = \boxed{\sqrt{g/2}}$

Q11.

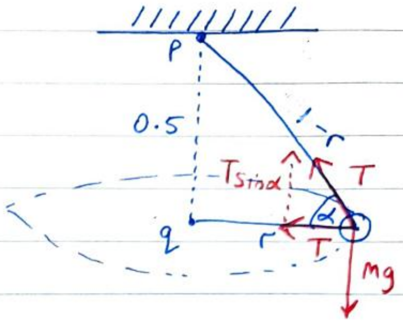
i) $(1-r)^2 = r^2 + (0.5)^2$

$1 - 2r + \cancel{r^2} = \cancel{r^2} + 0.25$

$1 - 0.25 = 2r$

$0.75 = 2r$

$r = \boxed{0.375\,m}$



ii) $\sin\alpha = \frac{0.5}{0.625} = \frac{4}{5}$

$\cos\alpha = \frac{0.375}{0.625} = \frac{3}{5}$

$(\uparrow\downarrow)\ T\sin\alpha = mg$

$T\left(\frac{4}{5}\right) = 1g$

$T = \boxed{\frac{5g}{4}\ N}$ or $\boxed{12.25\,N}$

iii) $(\rightleftarrows)\ T + T\cos\alpha = m\omega^2 r$

$12.25 + 12.25\left(\frac{3}{5}\right) = 1(\omega^2)(0.375)$

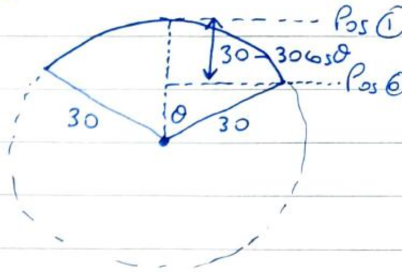
$0.375\omega^2 = 19.6$

$\omega^2 = 52.2666$

$\omega = \sqrt{52.266} = \boxed{\frac{28\sqrt{15}}{15}\ rad/s}$ or $\boxed{7.23\,rad/s}$

Q12.

Diag 1:



$mgh_0 + \frac{1}{2}mu^2 = mgh_1 + \frac{1}{2}mv^2$

$\cancel{m}g(0) + \frac{1}{2}\cancel{m}u^2 = \cancel{m}g(30 - 30\cos\theta) + \frac{1}{2}\cancel{m}v^2$

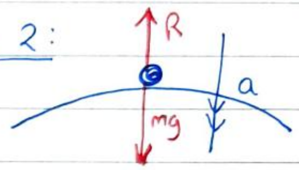
$u^2 = 2g(30 - 30\cos\theta) + v^2$

$u^2 = 60g - 60g\cos\theta + v^2 : \mathrm{I}$

Forces @ top:

$mg - R = \frac{mv^2}{r}$

Diag 2:



Leaves surface $\Rightarrow R = 0$

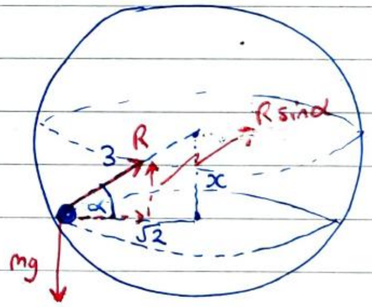
$\cancel{m}g = \frac{\cancel{m}v^2}{30}$

$v^2 = 30g$

$v = \sqrt{30g} = \boxed{7\sqrt{6}\ m/s}$ or $\boxed{17.15\ m/s}$

Q13. $(\uparrow\downarrow)$ $R\sin\alpha = mg$

$R\left(\frac{\sqrt{7}}{3}\right) = mg$



$R\sqrt{7} = 3mg$

$R = \boxed{\frac{3mg}{\sqrt{7}}} = \boxed{\frac{3\sqrt{7}\,mg}{7}}$

To find x

$(3)^2 = x^2 + (\sqrt{2})^2$

$x^2 = 7$

$x = \sqrt{7}$

$(\rightleftarrows)$ $R\cos\alpha = m\omega^2 r$

$\cancel{3}\frac{\sqrt{7}\cancel{m}g}{7}\left(\frac{\cancel{\sqrt{2}}}{\cancel{3}}\right) = \cancel{m}\omega^2(\cancel{\sqrt{2}})$

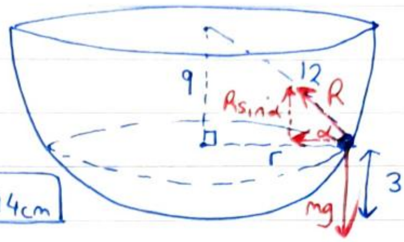
$\frac{\sqrt{7}}{7}g = \omega^2$

$\omega^2 = \frac{7\sqrt{7}}{5}$

$\omega = \sqrt{\frac{7\sqrt{7}}{5}} = \boxed{1.92 \text{ rad/s}}$

Q14.

i) $(12)^2 = (9)^2 + r^2$



$144 - 81 = r^2$

$r^2 = 63$

$r = \sqrt{63} = \boxed{3\sqrt{7} \text{ cm}}$ or $\boxed{7.94 \text{cm}}$

ii) $(\uparrow\downarrow)$ $R\sin\alpha = mg$

$\sin\alpha = 9/12 = 3/4$

$\cos\alpha = 3\sqrt{7}/12 = \sqrt{7}/4$

$R(3/4) = 5g$

$3R = 20g$

$R = \boxed{\frac{20g}{3} \text{ N}}$ or $\boxed{65.33 \text{ N}}$

iii) $(\rightleftarrows)$ $R\cos\alpha = m\omega^2 r$

$\frac{20g}{3}\left(\frac{\cancel{\sqrt{7}}}{4}\right) = 5\omega^2(3\cancel{\sqrt{7}})$

$15\omega^2 = \frac{5g}{3}$

$\omega^2 = \frac{g}{9}$

$\omega = \sqrt{\frac{g}{9}} = \boxed{\frac{\sqrt{g}}{3}}$ rad/s

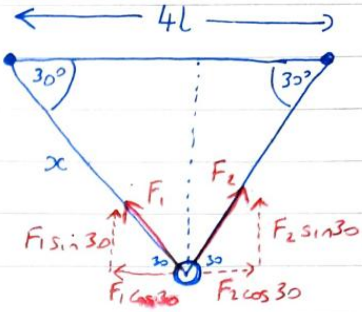
Q15. To find x

$$\cos 30 = \frac{2l}{x}$$

$$\frac{\sqrt{3}}{2} = \frac{2l}{x}$$

$$x\sqrt{3} = 4l$$

$$x = \frac{4l}{\sqrt{3}} \quad \text{or} \quad \frac{4\sqrt{3}\,l}{3}$$



$$(\uparrow\downarrow) \quad F_1 \sin 30 + F_2 \sin 30 = W$$

$$(F_1 + F_2)\sin 30 = W$$

$$k\left(\frac{4l}{\sqrt{3}} - \frac{l}{2} + \frac{4l}{\sqrt{3}} - \frac{l}{2}\right)\left(\frac{1}{2}\right) = W$$

$$k\left(\frac{8l}{\sqrt{3}} - l\right)\left(\frac{1}{2}\right) = W$$

$$k\left(\frac{8l - l\sqrt{3}}{\sqrt{3}}\right)\left(\frac{1}{2}\right) = W$$

$$k(8l - l\sqrt{3}) = 2\sqrt{3}\,W$$

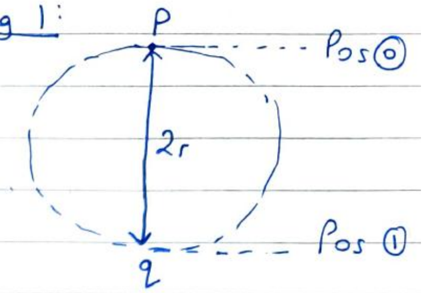
$$\Rightarrow k = \boxed{\frac{2\sqrt{3}\,W}{l(8-\sqrt{3})}} \quad \text{or} \quad k = \boxed{\frac{0.55W}{l}}$$

Q16. $mgh_0 + \frac{1}{2}mu^2 = mgh_1 + \frac{1}{2}mv^2$

$\cancel{m}g(0) + \frac{1}{2}\cancel{m}u^2 = \cancel{m}g(-2r) + \frac{1}{2}\cancel{m}v_1^2$

$u^2 = -4gr + v_1^2 \quad : \mathrm{I}$

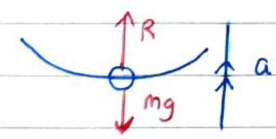
Diag 1:



Forces @ Pos 1:

$$R - mg = \frac{mv_1^2}{r}$$

Diag 2:



Cons of Momentum for collision:

$m_1u_1 + m_2u_2 = (m_1+m_2)v$

$\cancel{m}v_1 + m(0) = 2\cancel{m}\,v_2$

$v_2 = \frac{1}{2}v_1$

Restart using Diag 1:

$mgh_0 + \frac{1}{2}mu^2 = mgh_1 + \frac{1}{2}mv^2$

$2mg(0) + \frac{1}{2}(2\cancel{m})\left(\frac{v_1}{2}\right)^2 = 2\cancel{m}g(2r) + \frac{1}{2}(2\cancel{m})(0)^2$

just reaches the top & stops as u is a minimum to make it up there.

$\frac{v_1^2}{4} = 4gr + 0$

$v_1^2 = 16gr$

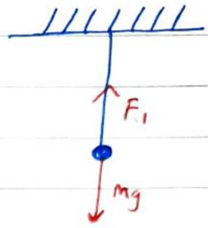
Subbing into I

$u^2 = -4gr + 16gr$

$u^2 = 12gr$

$u = \sqrt{12gr} = \boxed{2\sqrt{3gr} \text{ m/s}}$

Q17. In equilibrium $F_{UP} = F_{DOWN}$



$\Rightarrow F_1 = mg$

$k(l - l_0) = mg$

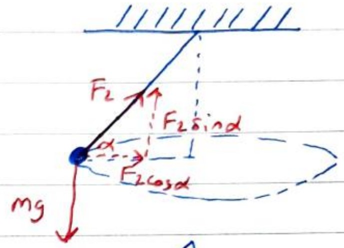
$k(0.5) = mg$

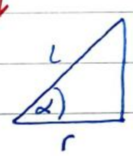
$\Rightarrow k = 2mg$

Let x = new extension

$F_2 = k(l - l_0)$

$= 2mg(x)$





$(\uparrow\downarrow)$ $F_2 \sin\alpha = mg$

$2\cancel{m}\cancel{g}x \sin\alpha = \cancel{mg}$

$2x\sin\alpha = 1$

$(\rightleftarrows)$ $F_2\cos\alpha = m\omega^2 r$

$2\cancel{m}gx\left(\frac{\cancel{r}}{l}\right) = \cancel{m}\omega^2\cancel{r}$

$2gx = \omega^2 l$

But $l = 1.2 + x$

$\Rightarrow 2gx = \omega^2(1.2 + x)$

$2gx = 1.2\omega^2 + x\omega^2$

$2gx - x\omega^2 = 1.2\omega^2$

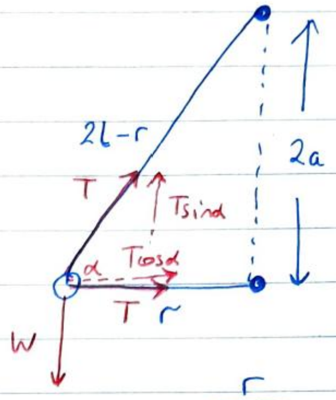
$x(2g - \omega^2) = 1.2\omega^2$

$\Rightarrow x = \boxed{\dfrac{1.2\omega^2}{2g - \omega^2}}$ or $\boxed{\dfrac{1.2\omega^2}{19.6 - \omega^2}}$

Q18.

$(\uparrow\downarrow)\quad T\sin\alpha = W \quad : \mathrm{I}$

$(\rightleftarrows)\quad T + T\cos\alpha = \frac{mv^2}{r} \quad : \mathrm{II}$



$\cos\alpha = \frac{r}{2l-r}$

$\sin\alpha = \frac{2a}{2l-r}$

Using I:

$T\left(\frac{2a}{2l-r}\right) = W$

$\Rightarrow T = \frac{W(2l-r)}{2a} \quad (*)$

Subbing into II:

$$\frac{W(2l-r)}{2a} + \frac{W\cancel{(2l-r)}}{2a}\left(\frac{r}{\cancel{2l-r}}\right) = \frac{mv^2}{r}$$

$$\frac{2Wl - Wr}{2a} + \frac{Wr}{2a} = \frac{mv^2}{r}$$

$$\frac{2Wl - \cancel{Wr} + \cancel{Wr}}{2a} = \frac{mv^2}{r}$$

$$\frac{Wl}{a} = \frac{mv^2}{r}$$

As $W = mg$

$$\Rightarrow \frac{\cancel{m}gl}{a} = \frac{\cancel{m}v^2}{r}$$

$$v^2 = \frac{rgl}{a}$$

$$\Rightarrow v^2 = \frac{\left(\frac{l^2-a^2}{l}\right)gl}{a}$$

$$v^2 = \frac{g(l^2-a^2)}{a}$$

$$\Rightarrow v = \boxed{\sqrt{\frac{g(l^2-a^2)}{a}}}$$

To find r in terms of l, a:

$$(2l-r)^2 = r^2 + (2a)^2$$

$$4l^2 + \cancel{r^2} - 4lr = \cancel{r^2} + 4a^2$$

$$4lr = 4l^2 - 4a^2$$

$$lr = l^2 - a^2$$

$$r = \frac{l^2-a^2}{l}$$

Using $(*)$:

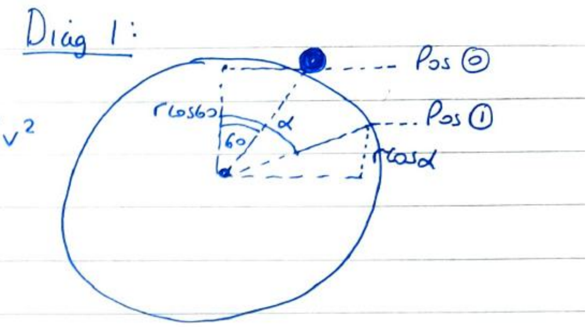
$$T = \frac{W\left(2l - \frac{l^2-a^2}{l}\right)}{2a}$$

$$= \frac{W\left(\frac{2l^2-l^2+a^2}{l}\right)}{2a}$$

$$= \boxed{\frac{W(l^2+a^2)}{2la}}$$

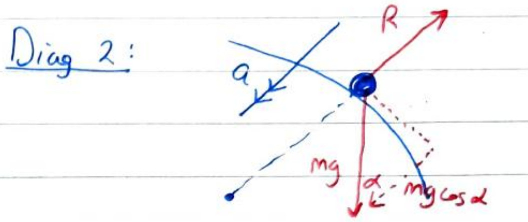
Q.E.D.

Q19.



$$mgh_0 + \tfrac{1}{2}mu^2 = mgh_1 + \tfrac{1}{2}mv^2$$
$$\cancel{m}g(r\cos 60) + 0 = \cancel{m}g(r\cos\alpha) + \tfrac{1}{2}\cancel{m}v^2$$
$$gr\left(\tfrac{1}{2}\right) = gr\cos\alpha + \tfrac{1}{2}v^2$$
$$v^2 = gr - 2gr\cos\alpha \quad : \mathrm{I}$$

Forces @ Pos ①:



$$mg\cos\alpha - R = \frac{mv^2}{r}$$

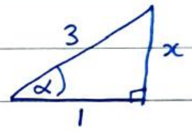
Leaves surface $\Rightarrow R = 0$

$$\cancel{m}g\cos\alpha = \frac{\cancel{m}v^2}{r}$$

$$v^2 = gr\cos\alpha \qquad \mathrm{II}$$

Equating I & II:

$$gr\cos\alpha = gr - 2gr\cos\alpha$$
$$3\cancel{gr}\cos\alpha = \cancel{gr}$$
$$\cos\alpha = \tfrac{1}{3}$$



$(3)^2 = (1)^2 + x^2$

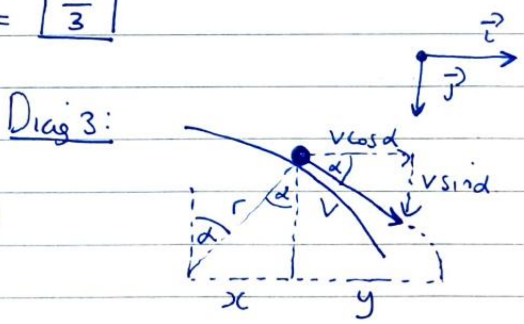
$x^2 = 8$

$x = \sqrt{8} = 2\sqrt{2}$

$\sin\alpha = \frac{2\sqrt{2}}{3}$

Height above $O = r\cos\alpha$

$= r\left(\tfrac{1}{3}\right) = \boxed{\tfrac{r}{3}}$



$$\vec{u} = v\cos\alpha\,\vec{i} + v\sin\alpha\,\vec{j}$$
$$= \sqrt{\frac{gr}{3}}\left(\frac{1}{3}\right)\vec{i} + \sqrt{\frac{gr}{3}}\left(\frac{2\sqrt{2}}{3}\right)\vec{j}$$
$$= \sqrt{\frac{gr}{27}}\,\vec{i} + \sqrt{\frac{8gr}{27}}\,\vec{j}$$

$V_x = \sqrt{\frac{gr}{27}} \quad V_y = \sqrt{\frac{8gr}{27}} - gt \quad S_x = \sqrt{\frac{gr}{27}}\,t \quad S_y = \sqrt{\frac{8gr}{27}}\,t + \frac{1}{2}gt^2$

To find y:

When particle @ O $\quad S_y = r\cos\alpha = \frac{r}{3}$

$$\Rightarrow \sqrt{\frac{8gr}{27}}\,t + \frac{1}{2}gt^2 = \frac{r}{3}$$

$$\sqrt{\frac{8gr}{27}}\,t + \frac{g}{2}t^2 - \frac{r}{3} = 0$$

$$\Rightarrow \quad 4.9t^2 + \sqrt{\frac{8gr}{27}}\, t - \frac{r}{3} = 0$$

$$\Rightarrow \quad t = \frac{-\sqrt{\frac{8gr}{27}} \pm \sqrt{\left(\sqrt{\frac{8gr}{27}}\right)^2 - 4(4.9)\left(\frac{-r}{3}\right)}}{2(4.9)}$$

$$= \frac{-\sqrt{\frac{8gr}{27}} + \sqrt{\frac{8gr}{27} + \frac{98r}{15}}}{9}$$

$$= \frac{-\sqrt{\frac{8gr}{27}} + \sqrt{\frac{1274r}{135}}}{9}$$

$$= \frac{-\sqrt{\frac{392r}{135}} + \sqrt{\frac{1274r}{135}}}{9}$$

$$= \frac{1.37\sqrt{r}}{9}$$

$$t = 0.14\sqrt{r}$$

$$\Rightarrow \quad S_x = \sqrt{\frac{gr}{27}}\left(0.14\sqrt{r}\right)$$

$$= 0.08r$$

$$\Rightarrow \quad y = 0.08r$$

From Diag 3 $\quad x = r\sin\alpha = r\left(\frac{2\sqrt{2}}{3}\right) = \frac{2r\sqrt{2}}{3}$

$\Rightarrow$ Tot horiz dist $= x + y$

$$= \frac{2r\sqrt{2}}{3} + 0.08r$$

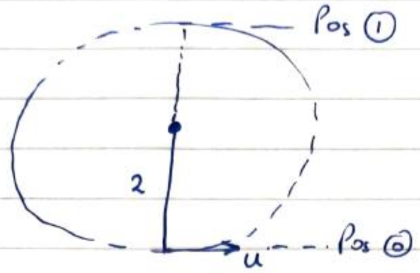
$$= 0.9428r + 0.08r$$

$$= \boxed{1.02r}$$

2016 Q6

Q6

a) i)

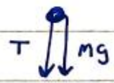


<u>LCE</u>

$$\cancel{m}g(0) + \tfrac{1}{2}\cancel{m}(u^2) = \cancel{m}g(4) + \tfrac{1}{2}\cancel{m}v^2$$

$$u^2 = 8g + v^2 \quad \text{I}$$

<u>Forces @ D</u>



$$T + mg = \frac{mv^2}{2}$$

Just reaches D $\Rightarrow T = 0$

$$\cancel{m}g = \frac{\cancel{m}v^2}{2}$$

$$\Rightarrow v^2 = 2g \quad \text{II}$$

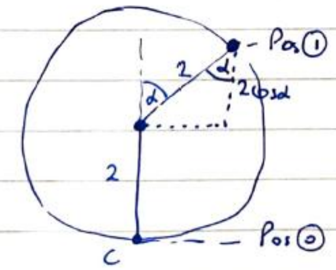
$$\Rightarrow \text{I}: \quad u^2 = 8g + 2g$$

$$u^2 = 10g$$

$$\Rightarrow u = \sqrt{10g}$$

$$= \boxed{7\sqrt{2}}$$

ii)

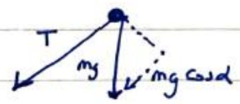


<u>LCE</u> $\quad mgh + \tfrac{1}{2}mu^2 = mgh + \tfrac{1}{2}mv^2$

$$\cancel{m}g(0) + \tfrac{1}{2}\cancel{m}(7)^2 = \cancel{m}g(2 + 2\cos\alpha) + \tfrac{1}{2}\cancel{m}v^2$$

$$49 = 4g + 4g\cos\alpha + v^2 \quad \text{I}$$

<u>Forces @ Pos ①</u>



$$T + \cancel{m}g\cos\alpha = \frac{\cancel{m}v^2}{2}$$

$$T = 0$$

$$\Rightarrow v^2 = 2g\cos\alpha \quad \text{II}$$

$$\Rightarrow \text{I}: \quad 49 - 4g - 4g\cos\alpha = 2g\cos\alpha$$

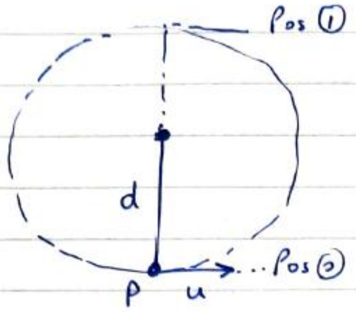
$$6g\cos\alpha = 9$$

$$\cos\alpha = \tfrac{1}{6} \quad \Rightarrow \alpha = \boxed{80.4^\circ}$$

2018 Q 6

Q 6 b)

i)



<u>LCE</u>

$$mg(0) + \tfrac{1}{2}\cancel{m}u^2 = \cancel{m}g(2d) + \tfrac{1}{2}\cancel{m}v^2$$

$$u^2 = 4gd + v^2 \quad \text{I}$$

If it just makes top $\Rightarrow T = 0$

$$\Rightarrow T + \cancel{m}g = \frac{\cancel{m}v^2}{d}$$

$$\Rightarrow v^2 = gd \quad \text{II}$$

$$\Rightarrow \text{I}: \; u^2 = 5gd$$

$$\Rightarrow u = \sqrt{5gd}$$

$\Rightarrow$ to pass the top $\Rightarrow \boxed{u \geqslant \sqrt{5gd}}$

ii) Least Tension @ Top $= T_1$

Greatest Tension @ Bottom $= kT_1$

<u>LCE</u>

$$mg(0) + \tfrac{1}{2}\cancel{m}(\sqrt{6gd})^2 = \cancel{m}g(2d) + \tfrac{1}{2}\cancel{m}v^2$$

$$3gd = 2gd + \tfrac{1}{2}v^2$$

$$gd = \tfrac{1}{2}v^2$$

$$2gd = v^2$$

<u>Forces @ bottom</u>

$$kT_1 - mg = \frac{mu^2}{d}$$

$$kT_1 - mg = \frac{m}{\cancel{d}}(6g\cancel{d})$$

$$kT_1 = 7mg$$

<u>Forces @ top</u>

$$T_1 + mg = \frac{mv^2}{d}$$

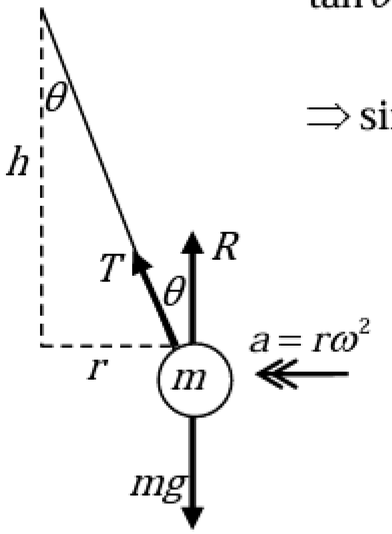
$$T_1 + mg = \frac{m}{\cancel{d}}(2g\cancel{d})$$

$$T_1 = mg$$

$$\Rightarrow k\cancel{mg} = 7\cancel{mg}$$

$$\Rightarrow \boxed{k = 7}$$

Solution: (i)



$$\tan\theta = \frac{r}{h},$$

$$\Rightarrow \sin\theta = \frac{r}{\sqrt{r^2+h^2}} \ \ \& \ \ \cos\theta = \frac{h}{\sqrt{r^2+h^2}}$$

When particle is about to lose contact

$\Rightarrow R = 0$

Resolve $\uparrow$: $T\cos\theta = mg$

$$\Rightarrow T = \frac{mg\sqrt{r^2+h^2}}{h}$$

$\vec{F} = m\vec{a} \ \leftarrow: \ T\sin\theta = mr\omega^2$ **A**

$$\Rightarrow \frac{mg\sqrt{r^2+h^2}}{h}\frac{r}{\sqrt{r^2+h^2}} = mr\omega^2$$

$$\Rightarrow \omega^2 = \frac{g}{h} \text{ when } R = 0$$

$$\Rightarrow \omega^2 \le \frac{g}{h} \text{ when } R \ge 0$$

(ii) $F = k\left(l - l_0\right)$, $\Rightarrow T = \dfrac{2mg}{h}\left(l - h\right)$

In **A**: $\dfrac{2mg}{h}\left(l-h\right)\dfrac{r}{l} = mr\left(\dfrac{2g}{5h}\right)$

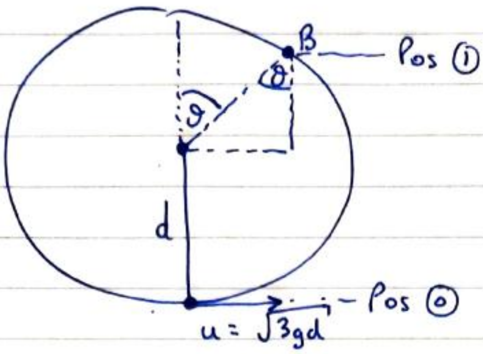
$$\Rightarrow \frac{2\left(l-h\right)}{l} = \frac{2}{5}, \qquad \Rightarrow l = \frac{5h}{4}$$

$$\Rightarrow \cos\theta = \frac{h}{l} = \frac{h}{5h/4} = \frac{4}{5}, \qquad \Rightarrow \theta = 36.9^\circ$$

2020 Q6

Q6 b)

i)



<u>LCE</u>

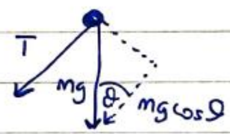
$$\cancel{m}g(0) + \tfrac{1}{2}\cancel{m}(\sqrt{3gd})^2 = \cancel{m}g(d + d\cos\theta) + \tfrac{1}{2}\cancel{m}v^2$$

$$3gd = 2gd + 2gd\cos\theta + v^2$$

$$gd - 2gd\cos\theta = v^2 \quad : \underline{\text{I}}$$

<u>Forces @ Pos ①</u>

$$T + mg\cos\theta = \frac{mv^2}{d}$$



$\underline{T=0} \Rightarrow v^2 = gd\cos\theta \quad \underline{\text{II}}$

$$\Rightarrow gd - 2gd\cos\theta = gd\cos\theta$$

$$\Rightarrow \boxed{\cos\theta = 1/3} \Rightarrow \sin\theta = \sqrt{8}/3$$

ii) When string goes slack, particle becomes a projectile: with initial velocity $v\cos\theta\,\vec{i} + v\sin\theta\,\vec{j}$

$$v^2 = gd\cos\theta$$

$$v^2 = \frac{gd}{3}$$

$$\Rightarrow v = \sqrt{\frac{gd}{3}}$$

$$\Rightarrow \vec{u} = \frac{1}{3}\sqrt{\frac{gd}{3}}\,\vec{i} + \frac{\sqrt{8}}{3}\sqrt{\frac{gd}{3}}\,\vec{j}$$

$$\Rightarrow \quad V_y = \frac{\sqrt{8}}{3}\sqrt{\frac{gd}{3}} - gt = 0$$

$$gt = \frac{\sqrt{8}}{3}\sqrt{\frac{gd}{3}}$$

$$t = \frac{\sqrt{8}}{3g}\sqrt{\frac{gd}{3}}$$

$$S_y = \frac{\sqrt{8}}{3}\sqrt{\frac{gd}{3}}\, t - \frac{1}{2} g t^2$$

$$= \frac{\sqrt{8}}{3}\sqrt{\frac{gd}{3}}\left(\frac{\sqrt{8}}{3g}\right)\left(\sqrt{\frac{gd}{3}}\right) - \frac{g}{2}\left(\frac{8}{9g^2}\right)\left(\frac{gd}{3}\right)$$

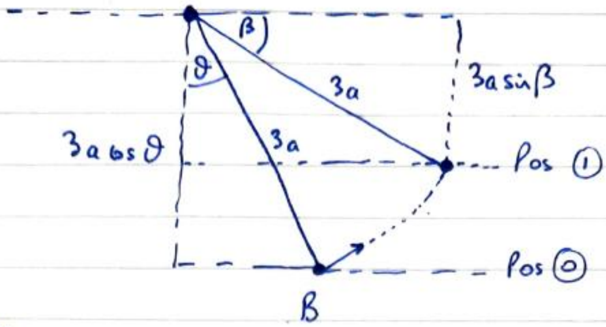
$$= \frac{8\cancel{g}d}{27\cancel{g}} - \frac{8\cancel{g^2}d}{54\cancel{g^2}}$$

$$= \frac{8d}{27} - \frac{8d}{54}$$

$$= \boxed{\frac{4d}{27}}$$

2017 Q6

Q6 b)



$\cos\theta = 2/3$
$\Rightarrow \sin\theta = \sqrt{5}/3$

LCE

$$\cancel{m}g(0) + \tfrac{1}{2}\cancel{m}(\sqrt{ag})^2 = \cancel{m}g\left(3a\left(\tfrac{2}{3}\right) - 3a\sin\beta\right) + \tfrac{1}{2}\cancel{m}v^2$$

$$\frac{ag}{2} = 2ag - 3ag\sin\beta + \frac{v^2}{2}$$

$$ag = 4ag - 6ag\sin\beta + v^2$$

$$6ag\sin\beta - 3ag = v^2$$

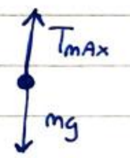
$$\Rightarrow v^2 = 3ag(2\sin\beta - 1) \quad \text{Q.E.D.}$$

Forces @ bottom for T_{max}

Bottom $\Rightarrow \beta = 90°$

$$\Rightarrow v^2 = 3ag(2(1) - 1)$$

$$v^2 = 3ag$$



$$T_{MAX} - mg = \frac{mv^2}{3a}$$

$$T_{MAX} = mg + \frac{m}{\cancel{3a}}(\cancel{3a}g)$$

$$= \boxed{2mg}$$

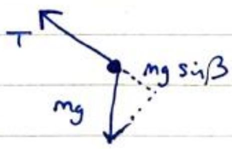
T_{min} will occur when $v = 0$

$$\Rightarrow 3ag(2\sin\beta - 1) = 0$$

$$\Rightarrow 2\sin\beta = 1$$

$$\sin\beta = 1/2 \Rightarrow \beta = 30°$$

Forces @ $\beta = 30°$



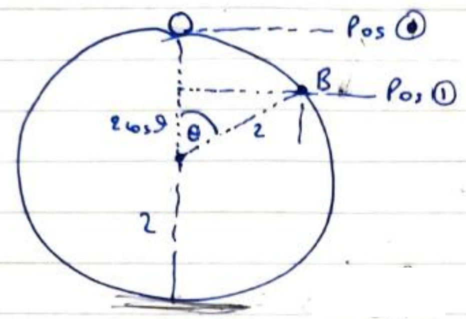
$$T - mg\sin\beta = \frac{mv^2}{3a}$$

$$T - mg/2 = 0$$

$$\Rightarrow T = \boxed{mg/2}$$

2010 Q6

Q6 a)

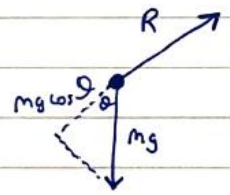


i) <u>LCE</u>

$$\cancel{m}g(0) + \tfrac{1}{2}\cancel{m}(0)^2 = \cancel{m}g(2\cos\theta - 2) + \tfrac{1}{2}\cancel{m}v^2$$

$$v^2 = 4g - 4g\cos\theta \quad \mathrm{I}$$

<u>Forces @ B</u>



$$\cancel{m}g\cos\theta - R = \frac{\cancel{m}v^2}{2}$$

<u>$R = 0$</u>

$$v^2 = 2g\cos\theta \quad \mathrm{II}$$

$$\Rightarrow \quad 2g\cos\theta = 4g - 4g\cos\theta$$

$$6\cancel{g}\cos\theta = 4\cancel{g}$$

$$\cos\theta = 2/3$$

$$\Rightarrow \quad v^2 = 4g - 4g(2/3)$$

$$v^2 = 196/15$$

$$v = \boxed{\frac{14\sqrt{15}}{15}} \text{ or } \boxed{3.61\ m/s}$$

ii)

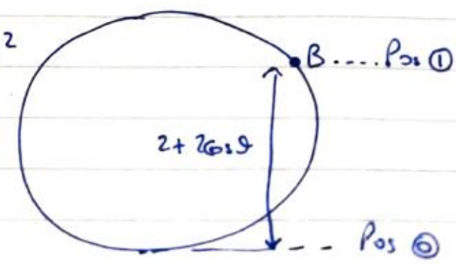
<u>Using LCE</u>

$$\cancel{m}g(2 + 2\cos\theta) + \tfrac{1}{2}\cancel{m}(3.61)^2 = \cancel{m}g(0) + \tfrac{1}{2}\cancel{m}v^2$$

$$2g + 2g(2/3) + 98/15 = v^2/2$$

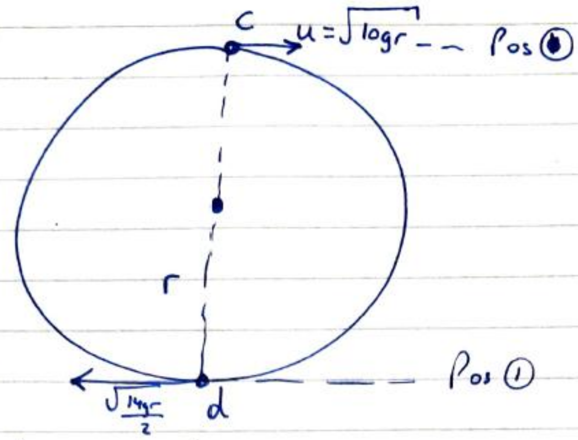
$$v^2 = \frac{392}{5}$$

$$v = \boxed{8.85\ m/s}$$



2007 Q6

Q6 b)



LCE

$$\cancel{m}g(2r) + \frac{1}{2}\cancel{m}(\sqrt{10gr})^2 = \cancel{m}g(0) + \frac{1}{2}\cancel{m}v^2$$

$$2gr + 5gr = \frac{1}{2}v^2$$

$$v^2 = 14gr$$

LCM

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$\cancel{m}(\sqrt{14gr}) + m(0) = (2\cancel{m})v$$

$$v = \frac{\sqrt{14gr}}{2}$$

LCE

$$2mg(0) + \frac{1}{\cancel{2}}(\cancel{2m})\left(\frac{14gr}{4}\right) = 2\cancel{m}g(2r) + \frac{1}{\cancel{2}}(\cancel{2m})v^2$$

$$\frac{7gr}{2} = 4gr + v^2$$

$$v^2 = \frac{7gr}{2} - \frac{8gr}{2}$$

$$v^2 = -\frac{gr}{2}$$

No soln $\Rightarrow$ doesn't make it to top