

Worked Solutions

Q1. LCM

$$(0.2)(200) + (2)(0) = (0.2)(120) + (2)(v)$$

$$40 = 24 + 2v$$

$$2v = 16$$

$$v = \boxed{8 \text{ m/s}}$$



Q2.

i)

Before	Mass	After
11 $\hat{e}$	4	$p \hat{e}$
7 $\hat{e}$	6	10 $\hat{e}$

NLR

$$\frac{p-10}{11-7} = -e$$

$$p-10 = -4e \quad (*)$$

LCM

$$11(4) + 7(6) = p(4) + 10(6)$$

$$4p + 60 = 86$$

$$4p = 26$$

$$p = 13/2$$

$$\Rightarrow \text{Speed : } \boxed{13/2 \text{ m/s}} \text{ or } \boxed{6.5 \text{ m/s}}$$

ii) Using (*)

$$p-10 = -4e$$

$$13/2 - 10 = -4e$$

$$-4e = -7/2$$

$$\Rightarrow e = \boxed{7/8}$$

Q3.

Before	Mass	After
$16\vec{i}$	1	$p\vec{i}$
$-9\vec{i}$	2	$q\vec{i}$

$$e = \frac{5}{7}$$

NLR

$$\frac{p - q}{16 + 9} = \frac{-5}{7}$$

$$7p - 7q = -125 : I$$

LCM

$$16(1) + (-9)(2) = p(1) + q(2)$$

$$p + 2q = -2 : II$$

Solving I & II:

$$7p - 7q = -125$$

$$(-) \quad 7p + 14q = +14$$

$$-21q = -111$$

$$q = \frac{37}{7}$$

$$\Rightarrow II: p + 2\left(\frac{37}{7}\right) = -2$$

$$p + \frac{74}{7} = -2$$

$$p = -2 - \frac{74}{7}$$

$$p = -\frac{88}{7}$$

$$KE_{\text{loss}} = KE_{\text{bef}} - KE_{\text{aft}}$$

$$= \left[\frac{1}{2}(1)(16)^2 + \frac{1}{2}(2)(9)^2 \right] - \left[\frac{1}{2}(1)\left(\frac{88}{7}\right)^2 + \frac{1}{2}(2)\left(\frac{37}{7}\right)^2 \right]$$

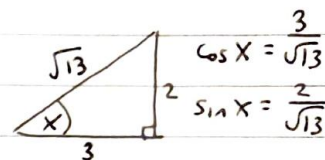
$$= 209$$

$$- 106.959$$

$$= \boxed{102.041}$$

Q4.

Before	Mass	After
$3\vec{i} + 2\vec{j}$	M	$p\vec{i} + 2\vec{j}$
$0\vec{i} + 0\vec{j}$	$2M$	$q\vec{i} + 0\vec{j}$

i) NLR

$$\frac{p - q}{3} = -\frac{1}{2}$$

$$2p - 2q = -3 \quad \text{I}$$

LCM

$$3(m) + 0 = p(m) + q(2m)$$

$$p + 2q = 3 \quad \text{II}$$

Solving I & II:

$$\text{I: } 2p - 2q = -3$$

$$\text{II: } p + 2q = 3$$

$$3p = 0$$

$$p = 0 \Rightarrow q = \frac{3}{2}$$

$$\Rightarrow \text{Ans: } \boxed{0\vec{i} + 2\vec{j}, \frac{3}{2}\vec{i} + 0\vec{j}}$$

$$\text{ii) } KE_{\text{Bef}} = \frac{1}{2}(m)(3)^2 = \frac{9m}{2}$$

$$KE_{\text{Aft}} = \frac{1}{2}(m)(0)^2 + \frac{1}{2}(2m)\left(\frac{3}{2}\right)^2 = \frac{9m}{4}$$

$$\Rightarrow \text{Loss} = \frac{18m}{4} - \frac{9m}{4}$$

$$= \boxed{\frac{9m}{4} \text{ J}}$$

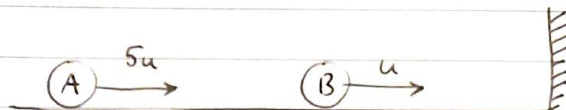
$$\text{iii) } \vec{I} = m\vec{v} - m\vec{u} \text{ (for } M)$$

$$= M(0\vec{i}) - M(3\vec{i})$$

$$= -3M\vec{i}$$

$$\Rightarrow \text{Ans: } \boxed{3M \text{ N}_s}$$

Q5.



i) A → B

$e = \frac{3}{4}$

Before	Mass	After
$5u \vec{i}$	m	$p \vec{i}$
$u \vec{i}$	$6m$	$q \vec{i}$

NLR

$$\frac{p - q}{4u} = -\frac{3}{4}$$

$$4p - 4q = -12u$$

$$p - q = -3u : I$$

LCM

$$5u(m) + u(6m) = p(m) + q(6m)$$

$$p + 6q = 11u : II$$

Solving I & II:

$$p - q = -3u$$

$$(-) p + 6q = 11u$$

$$-7q = -14u$$

$$q = 2u \Rightarrow I: p - 2u = -3u$$

$$p = -u$$

$$\Rightarrow \text{Ans: } \boxed{-u \vec{i}, 2u \vec{i}}$$

ii) B → Wall $e = \frac{1}{4}$

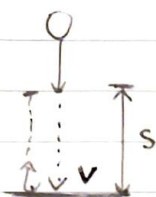
$$\text{Vel before hits wall} = 2u \vec{i}$$

$$\Rightarrow \text{Vel after hits wall} = -\frac{1}{4}(2u) \vec{i} = -\frac{u}{2} \vec{i}$$

As $p < -u$ and $-\frac{u}{2} < 0$, both spheres are now moving to the left and speed of B is half of speed of A $\Rightarrow$ no more collisions occur.

Q6.

i)



$$u = 0 \quad a = g \quad s = 3.5 \quad v = ?$$

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2(g)(3.5)$$

$$v^2 = 68.6$$

$$v = \sqrt{68.6} = \boxed{8.29 \text{ m/s}}$$

$$\text{ii) Vel before impact} = -8.29 \text{ j}$$

$$\Rightarrow \text{Vel after impact} = +0.6(8.29)$$

$$= 4.97$$

$$u = 4.97 \quad v = 0 \quad a = -g \quad s = s$$

$$v^2 = u^2 + 2as$$

$$0 = (4.97)^2 + 2(-g)(s)$$

$$2gs = 24.7$$

$$s = \frac{24.7}{2g} = \boxed{1.26 \text{ m}}$$

Q7.

Before	Mass	After
$4\vec{e}$	2	$p\vec{e}$
$-2\vec{e}$	5	$0\vec{e}$

i) NLR

$$\frac{p - 0}{4 + 2} = -e$$

$$p = -6e$$

LCM

$$4(2) + (-2)(5) = p(2) + 0$$

$$2p = -2$$

$$\boxed{p = -1}$$

$$\text{Ans : } \boxed{1 \text{ m/s}}$$

$$\text{ii) } p = -6e$$

$$\Rightarrow -1 = -6e$$

$$\Rightarrow e = \boxed{1/6}$$

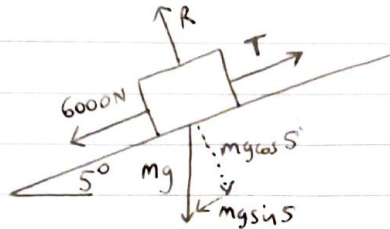
$$\begin{aligned} \text{iii) Loss in KE} &= KE_{\text{Before}} - KE_{\text{After}} \\ &= \left[\frac{1}{2}(2)(4)^2 + \frac{1}{2}(5)(2)^2 \right] - \left[\frac{1}{2}(2)(1)^2 + 0 \right] \\ &= 26 - 1 \\ &= 25 \text{ J} \end{aligned}$$

$$\Rightarrow \% \text{ Loss} = \frac{\text{Loss}}{\text{Before}} \times \frac{100}{1}$$

$$= \frac{25}{26} \times \frac{100}{1}$$

$$= \boxed{96\%}$$

Q8.
i)



$$P = T v$$

$$400000 = T v$$

$$\Rightarrow v = \frac{400000}{T}$$

Forces up hill (Max Speed $\Rightarrow a=0$)

$$T = 6000 + mg \sin 5$$

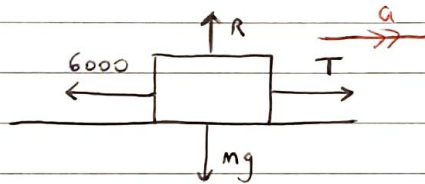
$$T = 6000 + 100000 g \sin 5$$

$$T = 91412.63$$

$$T = 91413 \text{ N}$$

$$\Rightarrow \text{Max Speed} = \frac{400000}{91413} = \boxed{4.38 \text{ m/s}}$$

ii)



$$T - 6000 = 100000 a$$

$$91413 - 6000 = 100000 a$$

$$100000 a = 85413$$

$$a = \frac{85413}{100000} = \boxed{0.85 \text{ m/s}^2}$$

Q9.

i)

$$\text{Power} = Tv$$

$$21000 = T(15)$$

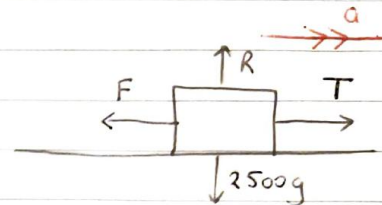
$$\Rightarrow T = \frac{21000}{15} = 1400 \text{ N}$$

$$F = ma$$

$$1400 - 400 = 2500a$$

$$1000 = 2500a$$

$$\Rightarrow a = \frac{1000}{2500} = \boxed{\frac{2}{5} \text{ m/s}^2} \text{ or } \boxed{0.4 \text{ m/s}^2}$$



ii) Max Speed $\Rightarrow a = 0$

$$\text{Power} = Tv$$

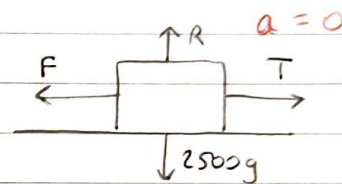
$$21000 = Tv$$

$$\Rightarrow T = \frac{21000}{v}$$

$$a = 0 \Rightarrow \frac{21000}{v} = F = 400$$

$$\Rightarrow 400v = 21000$$

$$\Rightarrow v = \frac{21000}{400} = \boxed{52.5 \text{ m/s}}$$



Q10.	Before	Mass	After	
	$u \cos 30 \vec{i} + u \sin 30 \vec{j}$	1	$p \vec{i} + u \sin 30 \vec{j}$	$u \cos 30 = \frac{u\sqrt{3}}{2}$
	$0 \vec{i} + 0 \vec{j}$	2	$q \vec{i} + 0 \vec{j}$	$u \sin 30 = \frac{u}{2}$

Perfectly elastic $\Rightarrow e = 1$

NLR

$$\frac{p - q}{\frac{u\sqrt{3}}{2}} = -1$$

$$2p - 2q = -u\sqrt{3} \quad \text{I}$$

LCM

$$\frac{u\sqrt{3}}{2}(1) + 0 = p(1) + q(2)$$

$$p + 2q = \frac{u\sqrt{3}}{2}$$

$$2p + 4q = u\sqrt{3} \quad \text{II}$$

Solving I & II:

$$\text{I} \times 2: 4p - 4q = -2u\sqrt{3}$$

$$\text{II}: 2p + 4q = u\sqrt{3}$$

$$6p = -u\sqrt{3}$$

$$p = -u\sqrt{3}/6$$

Solving I & II:

$$\text{I}: 2p - 2q = -u\sqrt{3}$$

$$-\text{II}: 2p - 4q = -u\sqrt{3}$$

$$-6q = -2u\sqrt{3}$$

$$q = \frac{u\sqrt{3}}{3}$$

$$\text{Vel 1 kg After} = \left| -\frac{u\sqrt{3}}{6} \vec{i} + \frac{u}{2} \vec{j} \right|$$

$$= \sqrt{\left(-\frac{u\sqrt{3}}{6}\right)^2 + \left(\frac{u}{2}\right)^2}$$

$$= \sqrt{\frac{3u^2}{36} + \frac{u^2}{4}}$$

$$= \sqrt{\frac{3u^2}{36} + \frac{9u^2}{36}}$$

$$= \sqrt{\frac{12u^2}{36}}$$

$$= \boxed{\frac{u\sqrt{3}}{3} = u}$$

Q.E.D.

Q11.

Before	Mass	After
$2u\vec{i}$	m	$p\vec{i}$
$u\vec{i}$	$2m$	$q\vec{i}$

i) NLR

$$\frac{p-q}{u} = -e$$

$$p-q = -eu : I$$

LCM

$$2u(m) + u(2m) = p(m) + q(2m)$$

$$p + 2q = 4u : II$$

Solving I & II:

$$p - q = -eu$$

$$(-) p + 2q = (-) 4u$$

$$-3q = -eu - 4u$$

$$3q = eu + 4u$$

$$q = \frac{u(e+4)}{3}$$

Solving I & II:

$$I \times 2: 2p - 2q = -2eu$$

$$II: p + 2q = 4u$$

$$3p = 4u - 2eu$$

$$p = \frac{2u(2-e)}{3} (*)$$

As $0 \leq e \leq 1$, $\frac{e+4}{3}$ will be $> 1 \Rightarrow$ velocity of 2nd sphere increases.

ii) $p = u$

$$\Rightarrow \frac{u}{1} = \frac{2u(2-e)}{3} \text{ (Using } (*) \text{)}$$

$$\Rightarrow 3 = 2(2-e)$$

$$3 = 4 - 2e$$

$$2e = 1$$

$$e = \boxed{\frac{1}{2}}$$

Q12. Before	Mass	After
$u \cos \alpha \vec{i} + u \sin \alpha \vec{j}$	m	$p \vec{i} + u \sin \alpha \vec{j}$
$0 \vec{i} + 0 \vec{j}$	m	$q \vec{i} + 0 \vec{j}$

NLR

$$\frac{p - q}{u \cos \alpha} = -e$$

$$p - q = -e u \cos \alpha$$

$$p - q = -e u \cos \alpha : I$$

LCM

$$u \cos \alpha (m) + 0 = p(m) + q(m)$$

$$p + q = u \cos \alpha : II$$

Solving I & II:

$$p - q = -e u \cos \alpha$$

$$p + q = u \cos \alpha$$

$$2p = u \cos \alpha (1 - e)$$

$$p = \frac{u \cos \alpha (1 - e)}{2}$$

$$\text{Slope before} = \frac{\vec{j}_{\text{comp}}}{\vec{i}_{\text{comp}}} = \frac{u \sin \alpha}{u \cos \alpha} = \tan \alpha$$

$$\text{Slope after} = \frac{u \sin \alpha}{\frac{u \cos \alpha (1 - e)}{2}} = \frac{2 \tan \alpha}{1 - e}$$

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{\tan \alpha - \frac{2 \tan \alpha}{1 - e}}{1 + \tan \alpha \left(\frac{2 \tan \alpha}{1 - e} \right)} \right| \times \frac{1 - e}{1 - e}$$

$$= \left| \frac{(1 - e) \tan \alpha - 2 \tan \alpha}{(1 - e) + 2 \tan^2 \alpha} \right|$$

$$= \left| \frac{-\tan \alpha (1 + e)}{1 - e + 2 \tan^2 \alpha} \right|$$

$$= \boxed{\frac{\tan \alpha (1 + e)}{(1 - e) + 2 \tan^2 \alpha}}$$

Q.E.D.

Q13.

Before	Mass	After
$u\vec{c}$	km	$0\vec{c}$
$ku\vec{c}$	m	$p\vec{c}$

i)

NLR

$$0 - p = -e$$

$$u - ku$$

$$-p = -eu + eku$$

$$p = eu - eku$$

$$p = eu(1-k)$$

$$4 \quad 6 \quad 8$$

LCM

$$u(km) + ku(m) = 0 + p(m) \quad 2m$$

$$p \geq 2uk$$

$$\text{Ans } 4p = \boxed{9\sqrt{3}u m! 8u} : II$$

ii)

$$2uk = e(1-k)$$

$$\Rightarrow e = \frac{2k}{8-k}$$

$$0 < \frac{6}{6\sqrt{3}+8} \leq 1 \Rightarrow \frac{2k}{6\sqrt{3}+8} \leq 1$$

$$6 \frac{2k}{1-k} \leq \frac{(1-k)^2 + 1}{(1-k)^2}$$

$$2k(1-k) \leq k^2 - 2k + 1$$

$$2k - 2k^2 - k^2 + 2k - 1 \leq 0$$

$$-3k^2 + 4k - 1 \leq 0$$

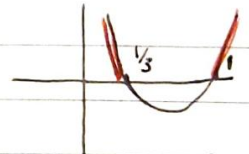
$$3k^2 - 4k + 1 \geq 0$$

$$(3k-1)(k-1) = 0$$

$$\Rightarrow k \leq \frac{1}{3} \text{ or } k > 1 \text{ (If } k=1 \text{ then } e = \frac{2}{0})$$

This makes $e < 0$ $\Rightarrow$ exclude

$$\Rightarrow \boxed{k \leq \frac{1}{3}}$$



Q14.

Before	Mass	After	
$3u \cos 30 \hat{i} + 3u \sin 30 \hat{j}$	$3m$	$0 \hat{i} + 3u \sin 30 \hat{j}$	$\cos 30 = \sqrt{3}/2$
$-2u \hat{i} + 0 \hat{j}$	$2m$	$p \hat{i} + 0 \hat{j}$	$\sin 30 = 1/2$

NLR

$$\frac{0 - p}{3u \cos 30 + 2u} = -e$$

$$3u \cos 30 + 2u$$

$$\frac{-p}{3u(\sqrt{3}/2) + 2u} = -e$$

$$3u(\sqrt{3}/2) + 2u$$

$$-p = -3eu(\frac{\sqrt{3}}{2}) - 2eu$$

$$4p = 6\sqrt{3}eu + 8eu : I$$

LCM

$$3u \cos 30 (3m) + (-2u)(2m) = 0 + p(2m)$$

$$2p = 9u(\sqrt{3}/2) - 4u$$

$$2p = \frac{9}{2}\sqrt{3}u - 4u$$

$$4p = 9\sqrt{3}u - 8u : II$$

Equating I & II:

$$6\sqrt{3}eu + 8eu = 9\sqrt{3}u - 8u$$

$$e(6\sqrt{3} + 8) = 9\sqrt{3} - 8$$

$$6\sqrt{3} + 8$$

$$6\sqrt{3} + 8$$

$$e = \frac{9\sqrt{3} - 8}{6\sqrt{3} + 8} = \boxed{0.41}$$

Q15.

Before	Mass	After
$u\vec{i}$	$2m$	$p\vec{i}$
$-2u\vec{i}$	m	$q\vec{i}$

NLR

$$\frac{p-q}{u+2u} = -e$$

$$p-q = -3eu \quad \text{I}$$

LCM

$$u(2m) + (-2u)(m) = p(2m) + q(m)$$

$$2p + q = 0$$

$$\Rightarrow q = -2p \quad \text{II}$$

Put II into I:

$$p - (-2p) = -3eu$$

$$3p = -3eu$$

$$p = -eu$$

$$\Rightarrow q = -2(-eu)$$

$$= 2eu$$

$$\begin{aligned} KE_{\text{bef}} &= \frac{1}{2}(2m)(u)^2 + \frac{1}{2}(m)(-2u)^2 \\ &= mu^2 + 2mu^2 \\ &= 3mu^2 = E \end{aligned}$$

$$\begin{aligned} KE_{\text{aft}} &= \frac{1}{2}(2m)(-eu)^2 + \frac{1}{2}(m)(2eu)^2 \\ &= me^2u^2 + 2me^2u^2 \\ &= 3me^2u^2 = F \end{aligned}$$

$$\sqrt{\frac{F}{E}} = \sqrt{\frac{3me^2u^2}{3mu^2}} = \sqrt{\frac{e^2}{1}} = \boxed{e}$$

Q.E.D.

Q16.

Before	Mass	After
$u\vec{c}$	$2m$	$p\vec{c}$
$o\vec{c}$	m	$q\vec{c}$

i)

NLR

$$\frac{p-q}{u} = -e$$

$$p - q = -eu \quad \text{I}$$

LCM

$$u(2m) + 0 = p(2m) + q(m)$$

$$2p + q = 2u \quad \text{II}$$

Solving I & II:

$$p - q = -eu$$

$$2p + q = 2u$$

$$3p = 2u - eu$$

$$p = \frac{u(2-e)}{3} = \boxed{\frac{1}{3}(2-e)u} \quad \text{Q.E.D.}$$

ii) Solving I & II:

$$\text{I} \times 2: 2p - 2q = -2eu$$

$$\text{II} : (-) 2p + q = (-) 2u$$

$$-3q = -2eu - 2u$$

$$3q = 2eu + 2u$$

$$q = \frac{2u(e+1)}{3}$$

$$KE_{\text{bef}} = \frac{1}{2}(2m)(u)^2 = mu^2$$

$$KE_{\text{aft}} = \frac{1}{2}(2m)\left(\frac{u(2-e)}{3}\right)^2 + \frac{1}{2}(m)\left(\frac{2u(1+e)}{3}\right)^2$$

$$= mu^2 \left(\frac{4+e^2-4e}{9} \right) + 2mu^2 \left(\frac{1+2e+e^2}{9} \right)$$

$$= \frac{4mu^2 + mu^2e^2 - 4emu^2 + 2mu^2 + 4emu^2 + 2mu^2e^2}{9}$$

$$= \frac{2mu^2 + mu^2e^2}{3}$$

$$Loss = \frac{3mu^2}{3} - \left(\frac{2mu^2 + mu^2e^2}{3} \right) = \frac{mu^2 - mu^2e^2}{3}$$

$$= \boxed{\frac{mu^2(1-e^2)}{3}}$$

Q.E.D.

Q17.

Before	Mass	After	
$\frac{\sqrt{3}}{2} \vec{i} + \frac{v}{2} \vec{j}$	M	$p \vec{i} + \frac{v}{2} \vec{j}$	$v \cos 30 = \frac{v\sqrt{3}}{2}$
$0 \vec{i} + 0 \vec{j}$	$2M$	$q \vec{i} + 0 \vec{j}$	$v \sin 30 = \frac{v}{2}$

NLR

$$\frac{p - q}{v\sqrt{3}/2} = -e$$

$$2p - 2q = -ev\sqrt{3} : I$$

LCM

$$\frac{v\sqrt{3}}{2}(m) + 0 = p(m) + q(2m)$$

$$p + 2q = \frac{v\sqrt{3}}{2}$$

$$2p + 4q = v\sqrt{3} : II$$

Solving I & II:

$$I \times 2: 4p - 4q = -2ev\sqrt{3}$$

$$II: 2p + 4q = v\sqrt{3}$$

$$6p = v\sqrt{3}(1 - 2e)$$

$$p = \frac{v\sqrt{3}(1 - 2e)}{6}$$

$$\text{Slope of } P \text{ before} = \frac{\vec{j}_{\text{comp}}}{\vec{i}_{\text{comp}}} = \frac{\frac{v}{2}}{\frac{v\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\text{Slope of } P \text{ after} = \frac{\vec{j}_{\text{comp}}}{\vec{i}_{\text{comp}}} = \frac{v/2}{\frac{v\sqrt{3}(1 - 2e)}{6}} = \frac{3}{\sqrt{3}(1 - 2e)}$$

$$\text{These are } \perp \Rightarrow m_1 \times m_2 = -1$$

$$\Rightarrow \frac{1}{\sqrt{3}} \left(\frac{3}{\sqrt{3}(1 - 2e)} \right) = -1$$

$$\Rightarrow \frac{1}{1 - 2e} = -\frac{1}{1}$$

$$\Rightarrow 2e - 1 = 1$$

$$2e = 2$$

$$\boxed{e = 1}$$

2012 Q5
Q5 a) $A \rightarrow B$

Before	Mass	After
$5\vec{v}$	$3m$	$p\vec{v}$
$0\vec{v}$	$2m$	$q\vec{v}$

NLR

$$\frac{p-q}{5} = -e$$

$$p-q = -5e \quad \text{I}$$

LCM

$$15m+0 = 3mp+2mq$$

$$3p+2q = 15 \quad \text{II}$$

Solving I & II:

$$\text{I} \times 2: 2p-2q = -10e$$

$$\text{II}: 3p+2q = 15$$

$$5p = 15-10e$$

$$p = 3-2e$$

Using I:

$$3-2e-q = -5e$$

$$q = 3+3e$$

No more collisions between

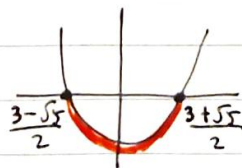
A and B as long as $p < r$

$$3-2e < -e^2+e+2$$

$$e^2-3e+1 < 0$$

$$a=1 \quad b=-3 \quad c=1$$

$$e = \frac{3 \pm \sqrt{5}}{2}$$



$$\Rightarrow \frac{3-\sqrt{5}}{2} < e < \frac{3+\sqrt{5}}{2}$$

$$\Rightarrow \boxed{e > \frac{3-\sqrt{5}}{2}}$$

Q.E.D.

$B \rightarrow C$

Before	Mass	After
$(3+3e)\vec{v}$	$2m$	$r\vec{v}$
$0\vec{v}$	m	$s\vec{v}$

NLR

$$\frac{r-s}{3+3e} = -e$$

$$r-s = -3e-3e^2$$

LCM

$$(6+6e)m = 2mr+sm$$

$$2r+s = 6+6e$$

Solving:

$$r-s = -3e-3e^2$$

$$2r+s = 6+6e$$

$$3r = -3e^2+3e+6$$

$$r = \boxed{-e^2+e+2}$$

Solving same eqns for s gives:

$$s = \boxed{2e^2+4e+2}$$

$s > r \Rightarrow C$ will move away from B $\Rightarrow$ no more collisions between them

b)	Before	Mass	After
	$u \cos \alpha \hat{i} + u \sin \alpha \hat{j}$	m	$p \hat{i} + u \sin \alpha \hat{j}$
	$0 \hat{i} + 0 \hat{j}$	m	$q \hat{i} + 0 \hat{j}$

NLR

$$\frac{p - q}{u \cos \alpha} = \frac{-1}{3}$$

$$3p - 3q = -u \cos \alpha \quad \text{I}$$

LCM

$$u \cos \alpha (m) + 0 = p(m) + q(m)$$

$$p + q = u \cos \alpha \quad \text{II}$$

Solving I & II:

$$\text{I: } 3p - 3q = -u \cos \alpha$$

$$\text{II} \times 3: 3p + 3q = 3u \cos \alpha$$

$$6p = 2u \cos \alpha$$

$$p = \frac{u \cos \alpha}{3}$$

$$\text{Slope before} = \frac{\cancel{3} \cancel{u} \sin \alpha}{\cancel{3} \cancel{u} \cos \alpha} = \frac{\sin \alpha}{\cos \alpha} = \tan \alpha = m_1$$

$$\text{Slope after} = \frac{u \sin \alpha}{\frac{u \cos \alpha}{3}} = 3 \tan \alpha = m_2$$

$$\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{\tan \alpha - 3 \tan \alpha}{1 + \tan \alpha (3 \tan \alpha)}$$

$$= \left| \frac{-2 \tan \alpha}{1 + 3 \tan^2 \alpha} \right|$$

$$= \boxed{\frac{2 \tan \alpha}{1 + 3 \tan^2 \alpha}}$$

Q.E.D.

2014 Q5

Q5 a)

i)

Before	Mass	After
$u\vec{i}$	$2m$	$p\vec{i}$
$0\vec{i}$	$7m$	$q\vec{i}$

NLR

$$\frac{p-q}{u} = -\frac{1}{2}$$

$$2p - 2q = -u \quad \text{I}$$

LCM

$$u(2m) + 0 = p(2m) + q(7m)$$

$$2p + 7q = 2u \quad \text{II}$$

Solving I & II:

$$\text{I: } 2p - 2q = -u$$

$$\text{II: } 2p + 7q = 2u$$

$$-9q = -3u$$

$$q = \frac{u}{3}$$

Using I:

$$2p - 2\left(\frac{u}{3}\right) = -u$$

$$6p = -u$$

$$p = -\frac{u}{6}$$

As A rebounds in opposite direction and even after B hits the wall its speed coming back will only match A's $\Rightarrow$ no further collision.

$$\text{ii) } KE_{\text{before}} = \frac{1}{2}(2m)(u)^2 + 0 = mu^2$$

$$KE_{\text{After}} = \frac{1}{2}(2m)(p)^2 + \frac{1}{2}(7m)(q)^2 = \frac{mu^2}{36} + \frac{14mu^2}{36} = \frac{15mu^2}{36} = \frac{5mu^2}{12}$$

$$\Rightarrow KE_{\text{loss}} = mu^2 - \frac{5mu^2}{12} = \frac{7mu^2}{12}$$

Collision of B with wall:

$$KE_{\text{before}} = \frac{1}{2}(7m)\left(\frac{u}{3}\right)^2 = \frac{7mu^2}{18}$$

$$KE_{\text{After}} = \frac{1}{2}(7m)\left(-\frac{u}{6}\right)^2 = \frac{7mu^2}{72}$$

$$\Rightarrow \text{Loss} = \frac{28mu^2}{72} - \frac{7mu^2}{72} = \frac{7mu^2}{24}$$

$$\Rightarrow \text{Total Loss} = \frac{7mu^2}{12} + \frac{7mu^2}{24} = \boxed{\frac{7mu^2}{8}}$$

b)	Before	Mass	After
i)	$3u\vec{i} + 4u\vec{j}$	$2m$	$p\vec{i} + 4u\vec{j}$
	$-4u\vec{i} + 3u\vec{j}$	m	$q\vec{i} + 3u\vec{j}$

NLR

$$\frac{p-q}{7u} = -e$$

$$p-q = -7eu : I$$

LCM

$$3u(2m) + (-4u)(m) = (p)(2m) + (q)(m)$$

$$2p + q = 2u : II$$

Solving I & II:

$$I: p - q = -7eu$$

$$II: 2p + q = 2u$$

$$3p = 2u - 7eu$$

$$p = \frac{2u - 7eu}{3}$$

Solving I & II:

$$I \times 2: 2p - 2q = -14eu$$

$$II: 2p + q = 2u$$

$$-3q = -14eu - 2u$$

$$q = \frac{2u + 14eu}{3}$$

$$KE_{\text{before}} = \frac{1}{2}(2m)(3u)^2 + \frac{1}{2}(m)(-4u)^2 = 9mu^2 + 8mu^2 = 17mu^2$$

$$KE_{\text{after}} = \frac{1}{2}(2m)\left(\frac{u(2-7e)}{3}\right)^2 + \frac{1}{2}(m)\left(\frac{2u(1+7e)}{3}\right)^2$$

$$= mu^2\left(\frac{49e^2 - 28e + 4}{9}\right) + 2mu^2\left(\frac{49e^2 + 14e + 1}{9}\right)$$

$$= \frac{mu^2}{9}(6 + 147e^2)$$

$$\Rightarrow KE_{\text{loss}} = 17mu^2 - \frac{mu^2}{9}(6 + 147e^2)$$

$$= \frac{153mu^2 - 6mu^2 - 147mu^2e^2}{9}$$

$$= \frac{147mu^2 - 147mu^2e^2}{9} = \frac{25mu^2}{2}$$

$$\Rightarrow 294 - 294e^2 = 225$$

$$294e^2 = 69$$

$$e^2 = \frac{69}{294}$$

$$\Rightarrow e = \boxed{0.484}$$

$$ii) \Rightarrow p = \frac{2u - 7(0.484)u}{3} = -0.463u$$

$$I = m\vec{v} - m\vec{u} = 2m(-0.463u) - 2m(3u) = -6.93mu$$

$$\Rightarrow k = \boxed{6.93}$$

2017 Q5

Q5 a)
i)

Before	Mass	After
$6\vec{e}$	1.5	$-v\vec{e}$
$0\vec{e}$	m	$2v\vec{e}$

This couldn't be the $\ominus$ one



NLR

$$\frac{-v - 2v}{6} = -e$$

$$-3v = -6e$$

$$v = 2e \quad \text{I}$$

LCM

$$6(1.5) + 0 = 1.5(-v) + m(2v)$$

$$2mv - 1.5v = 9 \quad \text{II}$$

$$KE_A \text{ Before} = \frac{1}{2}(1.5)^2(6)^2 = 27$$

$$KE_A \text{ After} = \frac{1}{2}(1.5)(v)^2 = 0.75v^2$$

$$\Rightarrow \text{Loss in KE} = 27 - 0.75v^2$$

$$80\% \text{ of KE Loss} = 21.6 - 0.6v^2$$

$$KE_B \text{ Before} = \frac{1}{2}(m)(0)^2 = 0$$

$$KE_B \text{ After} = \frac{1}{2}(m)(2v)^2 = 2v^2m$$

$$\Rightarrow KE \text{ Gain} = 2v^2m = 21.6 - 0.6v^2$$

$$\Rightarrow m = \frac{21.6 - 0.6v^2}{2v^2}$$

Using II:

$$2\sqrt{\frac{21.6 - 0.6v^2}{2v^2}} - 1.5v = 9$$

$$21.6 - 0.6v^2 - 1.5v^2 = 9v$$

$$-2.1v^2 - 9v + 21.6 = 0$$

$$2.1v^2 + 9v - 21.6 = 0$$

$$a = 2.1 \quad b = 9 \quad c = -21.6$$

$$\Rightarrow v = \frac{9 \pm \sqrt{(9)^2 - 4(2.1)(-21.6)}}{2(2.1)} = \boxed{\frac{12}{7} \text{ m/s}} \text{ or } -6 \text{ m/s}$$

ii)

Using I: $e = \frac{v}{2} = \frac{12/7}{2} = \boxed{\frac{6}{7}}$

	Before	Mass	After	
i)	$u \cos \alpha \vec{i} + u \sin \alpha \vec{j}$ $0\vec{i} + 0\vec{j}$	$3m$ $7m$	$p\vec{i} + u \sin \alpha \vec{j}$ $v\vec{i} + 0\vec{j}$	$e = \frac{2}{7}$

NLR

$$\frac{p - v}{u \cos \alpha} = \frac{-2}{7}$$

$$7p - 7v = -2u \cos \alpha \quad \text{I}$$

LCM

$$u \cos \alpha (3m) + 0 = p(3m) + v(7m)$$

$$3p + 7v = 3u \cos \alpha \quad \text{II}$$

Solving I & II:

$$\text{I: } 7p - 7v = -2u \cos \alpha$$

$$\text{II: } 3p + 7v = 3u \cos \alpha$$

$$10p = u \cos \alpha$$

$$p = \frac{u \cos \alpha}{10}$$

Using I:

$$7\left(\frac{u \cos \alpha}{10}\right) - 7v = -2u \cos \alpha$$

$$7u \cos \alpha - 70v = -20u \cos \alpha$$

$$27u \cos \alpha = 70v$$

$$\Rightarrow u = \boxed{\frac{70v}{27 \cos \alpha}}$$

$$\text{ii) Slope before} = \frac{\vec{j}_{\text{comp}}}{\vec{i}_{\text{comp}}} = \frac{u \sin \alpha}{u \cos \alpha} = \tan \alpha = \tan 30^\circ = \frac{1}{\sqrt{3}} = m_1$$

$$\text{Slope after} = \frac{u \sin \alpha}{p} = \frac{u \sin \alpha}{\frac{u \cos \alpha}{10}} = 10 \tan \alpha = 10 \tan 30^\circ = \frac{10}{\sqrt{3}} = m_2$$

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{\frac{1}{\sqrt{3}} - \frac{10}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)\left(\frac{10}{\sqrt{3}}\right)} \right|$$

$$= \left| \frac{-9/\sqrt{3}}{13/3} \right|$$

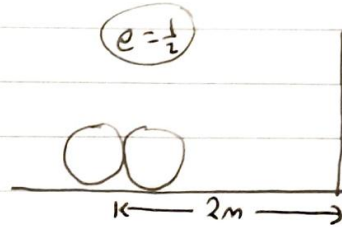
$$\tan \theta = \frac{9\sqrt{3}}{13}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{9\sqrt{3}}{13}\right) = \boxed{50.2^\circ}$$

2019 Q5

Q5 a)

	Before	Mass	After
i) A:	$u\vec{i}$	$3m$	$p\vec{i}$
B:	$-u\vec{i}$	m	$q\vec{i}$



NLR

$$\frac{p-q}{2u} = -\frac{1}{2}$$

$$2p-2q = -2u$$

$$p-q = -u : I$$

LCM

$$u(3m) + (-u)(m) = p(3m) + q(m)$$

$$3p+q = 2u : II$$

Solving I & II:

$$I: p - q = -u$$

$$II: 3p + q = 2u$$

$$4p = u$$

$$p = \frac{u}{4}$$

Using I:

$$\frac{u}{4} - q = -u$$

$$q = \frac{u}{4} + u$$

$$q = \frac{5u}{4}$$

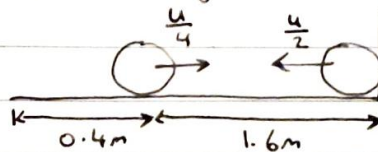
ii) Vel B before hitting wall = $\frac{5u}{4} \vec{i}$

$$\Rightarrow \text{Vel after hitting wall} = -\frac{2}{5}\left(\frac{5u}{4}\right) = -\frac{u}{2} \vec{i}$$

$$\text{Dist to wall} = 2m$$

$$\Rightarrow \text{time to hit wall for B} = \frac{2}{5u/4} = \frac{8}{5u} \text{ secs}$$

$$\text{Dist travelled by A in that time} = \frac{u}{4}\left(\frac{8}{5u}\right) = \frac{2}{5} = 0.4m$$



Let T = time to collide

$$\text{Dist travelled by A} = \frac{uT}{4} \quad \text{Dist travelled by B} = \frac{uT}{2}$$

$$\Rightarrow \frac{uT}{4} + \frac{uT}{2} = 1.6$$

$$uT = \frac{32}{15}$$

$$\Rightarrow T = \frac{32}{15u}$$

$$\Rightarrow \frac{8}{5u} + \frac{32}{15u} = 4$$

$$\Rightarrow \frac{56}{15u} = 4$$

$$\Rightarrow 60u = 56$$

$$\Rightarrow u = \boxed{\frac{14}{15} \text{ m/s}} \quad \text{or} \quad \boxed{0.93 \text{ m/s}}$$

	Before	Mass	After
i)	$3u\vec{i} + 4u\vec{j}$ $-4u\vec{i} + 3u\vec{j}$	$2m$ m	$p\vec{i} + 4u\vec{j}$ $q\vec{i} + 3u\vec{j}$

NLR

$$\frac{p - q}{7u} = \frac{-5}{7}$$

$$7p - 7q = -35u$$

$$p - q = -5u : I$$

LCM

$$3u(2m) + (-4u)(m) = p(2m) + q(m)$$

$$2p + q = 2u : II$$

Solving I & II:

$$p - q = -5u$$

$$2p + q = 2u$$

$$3p = -3u$$

$$p = -u$$

Using I:

$$-u - q = -5u$$

$$q = 4u$$

$$\Rightarrow \text{Speed P: } \sqrt{(-u)^2 + (4u)^2} = \boxed{u\sqrt{17} \text{ m/s}}$$

$$\text{Speed Q: } \sqrt{(4u)^2 + (3u)^2} = \boxed{5u \text{ m/s}}$$

$$ii) \text{ Slope P after} = \frac{j_{\text{comp}}}{i_{\text{comp}}} = \frac{4u}{-u} = -4$$

$$\text{Slope Q after} = \frac{3u}{4u} = \frac{3}{4}$$

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{-4 - \frac{3}{4}}{1 + (-4)(\frac{3}{4})} \right|$$

$$= \left| \frac{-19/4}{-2} \right|$$

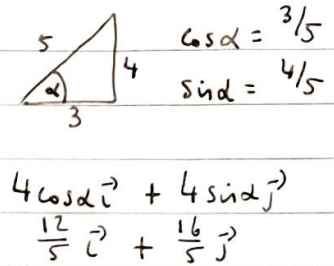
$$= |2.375|$$

$$\Rightarrow \theta = \tan^{-1}(2.375) = \boxed{67.17^\circ}$$

2023 Q2(b)

Q2 (b)

Before	Mass	After
$\frac{12}{5}\vec{i} + \frac{16}{5}\vec{j}$	m	$p\vec{i} + \frac{16}{5}\vec{j}$
$0\vec{i} + 3.2\vec{j}$	$2m$	$q\vec{i} + 3.2\vec{j}$



NLR

$$\frac{p-q}{\frac{12}{5}} = -e$$

$$5p - 5q = -12e \quad \text{I}$$

LCM

$$\frac{12}{5}(m) + 0 = p(m) + q(2m)$$

$$p + 2q = \frac{12}{5}$$

$$5p + 10q = 12 \quad \text{II}$$

Solving I & II:

$$\text{I: } 5p - 5q = -12e$$

$$\text{II: } 5p + 10q = 12$$

$$-15q = -12e - 12$$

$$15q = 12e + 12$$

$$5q = 4e + 4$$

$$q = \frac{4(e+1)}{5}$$

$$\Rightarrow \text{Vel: } \boxed{\frac{4e+4}{5} \vec{i} + 3.2\vec{j}}$$

Solving I & II:

$$\text{I} \times 2: 10p - 10q = -24e$$

$$\text{II: } 5p + 10q = 12$$

$$15p = 12 - 24e$$

$$5p = 4 - 8e$$

$$p = \frac{4-8e}{5}$$

$$\Rightarrow \text{Vel} = \boxed{\frac{4-8e}{5} \vec{i} + \frac{16}{5} \vec{j}}$$