The Line

Q1. EQN $\nearrow$ PT. ON LINE, $\searrow$ SLOPE

To find where $L \cap M$

$L: 3x - \cancel{2y} = -7$

$M \times 2: 10x + \cancel{2y} = -6$

$13x = -13$

$x = -1$

$\Rightarrow y = 2$

Slope $M = \frac{-x}{y} = \frac{-5}{1}$

$\Rightarrow \perp$ Slope $= \frac{1}{5}$

$y - 2 = \frac{1}{5}(x - (-1))$

$y - 2 = \frac{1}{5}(x + 1)$

$5(y - 2) = x + 1$

$5y - 10 = x + 1$

$\boxed{x - 5y + 11 = 0}$

Q2. Eqn of any line through $(0,0)$:

$y - 0 = m(x - 0)$

$y = mx$

$mx - y = 0$

Slope of $2x + 3y - 4 = 0$: $\frac{-2}{3} = m_2$

$\Rightarrow \text{Tan } 45 = \left| \frac{m_1 - (-\frac{2}{3})}{1 + m_1(-\frac{2}{3})} \right|$

$\Rightarrow 1 = \frac{m_1 + \frac{2}{3}}{1 - \frac{2m_1}{3}} \cdot \frac{3}{3}$

$\Rightarrow 1 = \left| \frac{3m_1 + 2}{3 - 2m_1} \right|$

Squaring both sides gives:

$1 = \frac{9m_1^2 + 12m_1 + 4}{9 + 4m_1^2 - 12m_1}$

$\Rightarrow 9m_1^2 + 12m_1 + 4 = 4m_1^2 - 12m_1 + 9$

$5m_1^2 + 24m_1 - 5 = 0$

$(5m_1 - 1)(m_1 + 5) = 0$

$m_1 = \frac{1}{5}$ or $m_1 = -5$

$\Rightarrow$ Eqns: $y = \frac{1}{5}x$ or $y = -5x$

$5y = x$ or $y = -5x$

$\boxed{x - 5y = 0}$ $\boxed{5x + y = 0}$

Q3. $\frac{|3(3) - 4(k) + 7|}{\sqrt{(3)^2 + (-4)^2}} = 6$

$|16 - 4k| = 30$

$16 - 4k = 30$ or $16 - 4k = -30$

$-4k = 14$ $\quad -4k = -46$

$k = \frac{-14}{4} = \frac{-7}{2}$ $\quad k = \frac{23}{2}$

$k < 0 \Rightarrow \boxed{k = \frac{-7}{2}}$

Q4. Any line parallel to $12x - 5y + 3 = 0$ has equation $12x - 5y + d = 0$

$\frac{|12(1) - 5(2) + d|}{\sqrt{(12)^2 + (-5)^2}} = 2$

$|2 + d| = 26$

$2 + d = 26$ or $2 + d = -26$

$d = 24$ or $d = -28$

$\Rightarrow \boxed{12x - 5y + 24 = 0 \text{ or } 12x - 5y - 28 = 0}$

Q5. Slope of line $A = \frac{-a}{6}$

$\Rightarrow \perp$ slope $= \frac{6}{a}$

Slope of $B = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{10 - (-2)}{-4 - 6}$

$= \frac{12}{-10}$

$= -\frac{6}{5} = \frac{6}{a}$

$\Rightarrow -6a = 30$

$\Rightarrow \boxed{a = -5}$

Q6. Equidistant

$\Rightarrow \frac{|3(-1) - 4(-5) - 2|}{\sqrt{(3)^2 + (-4)^2}} = \frac{|3(-1) - 4(-5) + k|}{\sqrt{(3)^2 + (-4)^2}}$

$\Rightarrow \frac{|15|}{5} = \frac{|17 + k|}{5}$

$\Rightarrow k + 17 = 15$ or $k + 17 = -15$

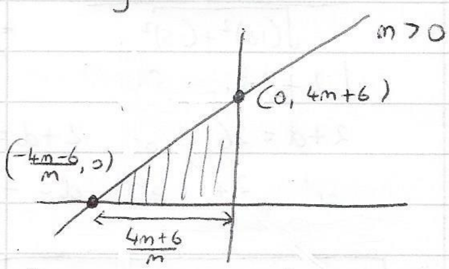
$k = -2$ or $k = -32$

$k \neq -2 \Rightarrow \boxed{k = -32}$

Q7. $y - 6 = m(x + 4)$

$y - 6 = mx + 4m$

$mx - y + 4m + 6 = 0$



Cuts x-axis

$mx = -4m - 6$

$x = \frac{-4m - 6}{m}$

Cuts y-axis

$y = 4m + 6$

Area $\Delta = \frac{1}{2}\left(\frac{4m+6}{m}\right)(4m+6) = 54$

$\Rightarrow \frac{(4m+6)^2}{m} = 108$

$16m^2 + 48m + 36 = 108m$

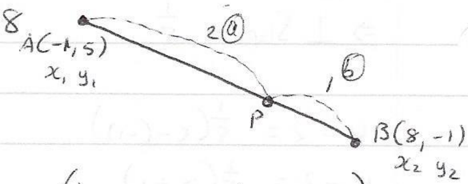
$16m^2 - 60m + 36 = 0$

$4m^2 - 15m + 9 = 0$

$(4m - 3)(m - 3) = 0$

$\boxed{m = 3/4}$ or $\boxed{m = 3}$

Q8



$P = \left(\frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a}\right)$

$= \left(\frac{1(-1) + 2(8)}{2 + 1}, \frac{1(5) + 2(-1)}{2 + 1}\right)$

$= \left(\frac{15}{3}, \frac{3}{3}\right)$

$= \boxed{(5, 1)}$

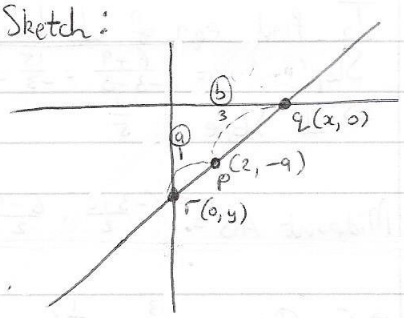
Q9. $m_1 = \frac{-3}{-1} = 3 \quad m_2 = \frac{-1}{2}$

$\tan\theta = \left|\frac{3 + \frac{1}{2}}{1 + (3)(-\frac{1}{2})}\right|$

$\tan\theta = |-7|$

$\Rightarrow$ Acute Angle $= \tan^{-1}(7) = \boxed{82^\circ}$

Q10. Sketch:



$$(2,-9) = \left(\frac{3(0)+1(x)}{3+1}, \frac{3(y)+1(0)}{3+1}\right)$$

$$\Rightarrow (2,-9) = \left(\frac{x}{4}, \frac{3y}{4}\right)$$

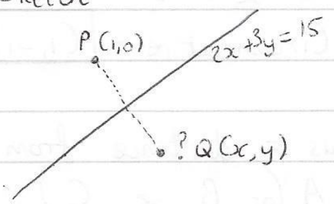
$$\Rightarrow \frac{x}{4} = 2 \quad \text{and} \quad \frac{3y}{4} = -9$$

$$x = 8 \qquad 3y = -36$$

$$y = -12$$

$$\Rightarrow \boxed{q = (8,0) \quad r = (0,-12)}$$

Q11. Sketch:



$\perp$ dist from P to mirror must be equal to $\perp$ dist from Q.

$$\frac{|2(1)+3(0)-15|}{\sqrt{(2)^2+(3)^2}} = \frac{|2x+3y-15|}{\sqrt{(2)^2+(3)^2}}$$

$$\Rightarrow |-13| = |2x+3y-15|$$

$$\Rightarrow 2x+3y-15 = 13$$

$$2x+3y = 28$$

$$\Rightarrow 2x = 28-3y$$

$$\Rightarrow \boxed{x = \frac{28-3y}{2}} \quad *$$

Also:

$$\text{Slope mirror} = -\frac{2}{3}$$

$$\Rightarrow \text{Slope} \perp \text{ line } PQ = \frac{3}{2}$$

$\Rightarrow$ Eqn of PQ:

$$y - 0 = \frac{3}{2}(x-1)$$

$$2(y) = 3(x-1)$$

$$2y = 3x - 3$$

$$3x - 2y = 3$$

Sub $*$ into eqn above:

$$3\left(\frac{28-3y}{2}\right) - 2y = 3$$

$$84 - 9y - 4y = 6$$

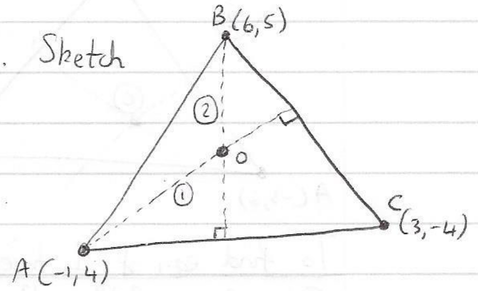
$$-13y = -78$$

$$y = 6$$

$$\Rightarrow x = \frac{28-3(6)}{2} = 5 \quad (\text{Using } *)$$

$$\Rightarrow \boxed{(5,6)}$$

Q12. Sketch



<u>To find eqn of altitude 1:</u>

$$\text{Slope } BC = \frac{-4-5}{3-6} = \frac{-9}{-3} = 3$$

$$\Rightarrow \perp \text{ Slope} = \frac{-1}{3} \qquad \text{Pt on Line} = (-1,4)$$

$$y - 4 = \frac{-1}{3}(x+1)$$

$$3y - 12 = -x - 1$$

$$x + 3y = 11 \quad \text{I}$$

To find eqn of altitude 2:

Slope $AC = \frac{-4-4}{3+1} = \frac{-8}{4} = -2$

$\Rightarrow \perp$ Slope $= \frac{1}{2}$ Pt. on line $= (6,5)$

$y - 5 = \frac{1}{2}(x-6)$

$2y - 10 = x - 6$

$x - 2y = -4$ II

Solving I + II

$\cancel{x} + 3y = 11$

$(-)\cancel{x} \overset{(+)}{-} 2y = \overset{(+)}{-}4$

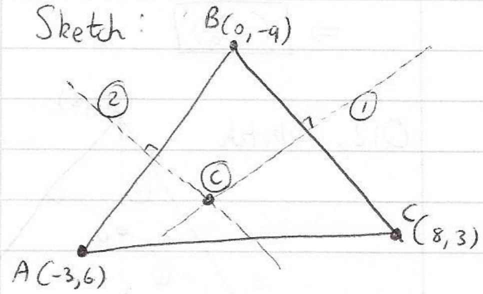
$5y = 15$

$y = 3$

$\Rightarrow x = 2$

$\Rightarrow$ Orthocentre $= \boxed{(2,3)}$

Q13. Sketch:



To find eqn of $\perp$ bisector 1:

Slope $BC = \frac{3+9}{8-0} = \frac{12}{8} = \frac{3}{2}$

$\Rightarrow \perp$ slope $= -\frac{2}{3}$

Midpoint $BC = \left(\frac{0+8}{2}, \frac{-9+3}{2}\right) = (4,-3)$

$\Rightarrow$ Eqn 1: $y + 3 = -\frac{2}{3}(x-4)$

$3y + 9 = -2x + 8$

$2x + 3y = -1$ I

To find eqn of $\perp$ bisector 2:

Slope $AB = \frac{6+9}{-3-0} = \frac{15}{-3} = -5$

$\Rightarrow \perp$ Slope $= \frac{1}{5}$

Midpoint $AB = \left(\frac{-3+0}{2}, \frac{6-9}{2}\right) = \left(\frac{-3}{2}, \frac{-3}{2}\right)$

$\Rightarrow$ Eqn 2: $y + \frac{3}{2} = \frac{1}{5}\left(x + \frac{3}{2}\right)$

$5y + \frac{15}{2} = x + \frac{3}{2}$

$10y + 15 = 2x + 3$

$2x - 10y = 12$ II

Solving I & II:

$\cancel{2x} + 3y = -1$

$(-)\cancel{2x} \overset{(+)}{-} 10y \overset{(-)}{=} 12$

$13y = -13$

$y = -1$

$\Rightarrow x = 1$

$\Rightarrow$ Circumcentre $= \boxed{(1,-1)}$

Radius = distance from $(1,-1)$ to A (or B or C)

$= \sqrt{(1+3)^2 + (-1-6)^2}$

$= \sqrt{16 + 49}$

$= \boxed{\sqrt{65}}$

Q14. $y = k^2x + 12$ $\qquad 2ky = 4x+5$

i) $\text{Slope} = k^2$ $\qquad y = \frac{4}{2k}x + \frac{5}{2k}$

$\text{Slope} = \frac{4}{2k}$

$\perp \Rightarrow$ Slopes multiply to give -1

$\Rightarrow k^2\left(\frac{4}{2k}\right) = -1$

$2k = -1$

$\boxed{k = \frac{-1}{2}}$

ii) $y = \left(-\frac{1}{2}\right)^2 x + 12$ $\quad\Big|\quad 2\left(-\frac{1}{2}\right)y = 4x+5$

$y = \frac{1}{4}x + 12$ $\quad\Big|\quad -y = 4x+5$

$\Rightarrow \frac{1}{4}x + 12 = -(4x+5)$

$x + 48 = -16x - 20$

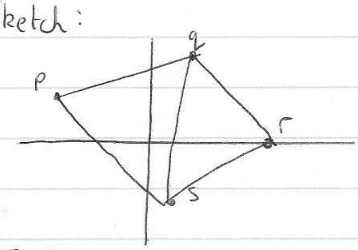
$17x = -68$

$x = -4$

$\Rightarrow y = 11$

$\Rightarrow \boxed{(-4, 11)}$

Q15. Sketch:



To find Area Δqrs

$r(3,0) \xrightarrow[0]{-3} (0,0)$

$q(1,3) \xrightarrow[0]{-3} (-2,3)$

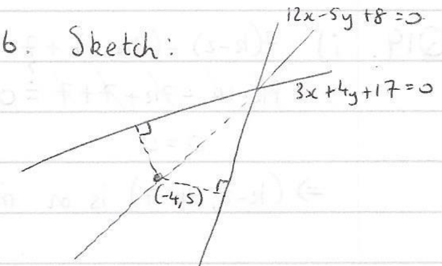
$s(-1,-2) \xrightarrow[0]{-3} (-4,-2)$

$\text{Area} = \frac{1}{2}|(-2)(-2) - (3)(-4)|$

$= \frac{1}{2}|16| = 8$

$\Rightarrow$ Area Parallelogram $= 8 \times 2 = \boxed{16}$

Q16. Sketch:



If $(-4,5)$ is on bisector

$\Rightarrow \perp$ dist to both lines must be the same i.e.

$$\frac{|12(-4)-5(5)+8|}{\sqrt{(12)^2+(-5)^2}} = \frac{|3(-4)+4(5)+17|}{\sqrt{(3)^2+(4)^2}}$$

$$\frac{|-65|}{13} = \frac{|25|}{5}$$

$5 = 5 \Rightarrow (-4,5)$ on bisector

Q17. $\text{Centroid} = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$\Rightarrow \left(\frac{4-1+h}{3}, \frac{2+7+k}{3}\right) = (2,4)$

$\Rightarrow \frac{h+3}{3} = 2$ and $\frac{k+9}{3} = 4$

$h+3 = 6$ $\qquad k+9 = 12$

$\boxed{h=3}$ $\qquad \boxed{k=3}$

Q18. If collinear then area of triangle pqr must be 0.

$p(-6,13) \xrightarrow[-13]{+6} (0,0)$

$q(-1,3) \xrightarrow[-13]{+6} (5,-10)$

$r(3,-5) \xrightarrow[-13]{+6} (9,-18)$

$\text{Area} = \frac{1}{2}|(5)(-18) - (9)(-10)|$

$= \frac{1}{2}|-90+90|$

$= 0 \Rightarrow$ Collinear

Q19. i) $7(k-2)-(7k-7)+7=0$

$$\cancel{7k}-14-\cancel{7k}+\cancel{7}+7 \overset{?}{=} 0$$

$$0=0$$

$\Rightarrow (k-2, 7k-7)$ is on m.

ii) $q(t+1, 3-t)$

As $+t$ and $-t$ in x and y we can simply add them to eliminate t:

$$t+1+3-t = 4$$

$$\Rightarrow x+y = 4$$

$$\Rightarrow \boxed{x-y-4=0}$$

Q20. $\sqrt{(k-5)^2+(2-6)^2} = 2\sqrt{5}$

$$\sqrt{k^2-10k+25+16} = 2\sqrt{5}$$

Squaring both sides:

$$k^2-10k+41 = 20$$

$$k^2-10k+21=0$$

$$(k-3)(k-7)=0$$

$$\boxed{k=3} \text{ or } \boxed{k=7}$$

Q21. Any line through $(4,1)$ has eqn:

$$y-1 = m(x-4)$$

$$y-1 = mx-4m$$

$$mx-y+1-4m=0 \quad *$$

$\perp$ dist to $(1,2) = 2\sqrt{2}$:

$$\frac{|m(1)-2+1-4m|}{\sqrt{m^2+(-1)^2}} = 2\sqrt{2}$$

$$\frac{|-3m-1|}{\sqrt{m^2+1}} = 2\sqrt{2}$$

$$|-3m-1| = 2\sqrt{2m^2+2}$$

Squaring both sides:

$$9m^2+6m+1 = 8m^2+8$$

$$\Rightarrow m^2+6m-7 = 0$$

$$(m+7)(m-1)=0$$

$$m=-7 \text{ or } m=1$$

Using $*$

$$-7x-y+1-4(-7)=0 \text{ or } x-y+1-4(1)=0$$

$$-7x-y+29=0 \text{ or } \boxed{x-y-3=0}$$

$$\boxed{7x+y-29=0}$$

Q22. $3x+2y = c$

Cuts x-axis	Cuts y-axis
$3x+2(0)=c$	$3(0)+2y=c$
$3x=c$	$2y=c$
$x = c/3$	$y=c/2$

$$\text{Area} = \tfrac{1}{2}\left(\tfrac{c}{3}\right)\left(\tfrac{c}{2}\right) = 24$$

$$\Rightarrow \frac{c^2}{12} = 24$$

$$\Rightarrow c^2 = 288$$

$$\Rightarrow c = \sqrt{288} = \boxed{12\sqrt{2}}$$