# Trigonometry

**Q1**



0.4m

$\Rightarrow$ Circum $= 2\pi(0.2)$

$= 1.2566$

$\Rightarrow$ No. of rotations $= \dfrac{100}{1.2566}$

$= 79.57$

$= \boxed{80}$

**Q2**

$1 - \dfrac{\sin^2\theta}{1-\cos\theta} = -\cos\theta$

$= \dfrac{1-\cos\theta - \sin^2\theta}{1-\cos\theta}$

$= \dfrac{1-\cos\theta - (1-\cos^2\theta)}{1-\cos\theta}$

$= \dfrac{1-\cos\theta - 1 + \cos^2\theta}{1-\cos\theta}$

$= \dfrac{\cos^2\theta - \cos\theta}{1-\cos\theta}$

$= \dfrac{\cos\theta(\cos\theta - 1)}{-1(\cos\theta - 1)}$

$= -\cos\theta = R.H.S.$

**Q3**

$\sin 2A + \tan 2A = \dfrac{\sin 2A \tan 2A}{\tan A}$

L.H.S

$\dfrac{2\tan A}{1+\tan^2 A} + \dfrac{2\tan A}{1-\tan^2 A}$

$= \dfrac{2\tan A - 2\tan^3 A + 2\tan A + 2\tan^3 A}{(1+\tan^2 A)(1-\tan^2 A)}$

$= \dfrac{4\tan A}{1-\tan^4 A}$
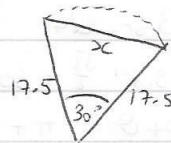
R.H.S

$\dfrac{\left(\dfrac{2\tan A}{1+\tan^2 A}\right)\left(\dfrac{2\tan A}{1-\tan^2 A}\right)}{\tan A}$

$= \dfrac{\dfrac{4\tan^2 A}{1-\tan^4 A}}{\tan A}$

$= \dfrac{4\tan^2 A}{1-\tan^4 A} \times \dfrac{1}{\tan A}$

$= \dfrac{4\tan A}{1-\tan^4 A} = L.H.S.$

**Q4**

$\tan^2 30 + \sin^2 60$

$= \left(\dfrac{1}{\sqrt{3}}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2$

$= \dfrac{1}{3} + \dfrac{3}{4}$

$= \boxed{\dfrac{13}{12}}$

**Q5.**



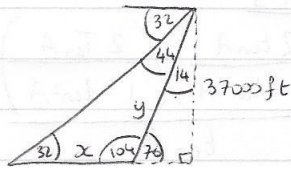17.5  30°  17.5

Cosine Rule

$x^2 = (17.5)^2 + (17.5)^2 - 2(17.5)(17.5)\cos 30$

$= 612.5 - 530.44$

$= 82.06$

$\Rightarrow x = \sqrt{82.06} = \boxed{9.06\text{ cm}}$

**Q6**



$$\cos 14 = \frac{A}{H} = \frac{37000}{y}$$

$$\Rightarrow y = \frac{37000}{\cos 14}$$

$$= 38132.7$$

Sine Rule

$$\frac{x}{\sin 44} = \frac{38132.7}{\sin 32}$$

$$x = \frac{38132.7 (\sin 44)}{\sin 32}$$

$$= \boxed{49987 \ ft}$$

**Q7**  $\sin 2x = -\frac{\sqrt{3}}{2}$



$\ominus \Rightarrow$ Q3 or Q4

Ref Angle = $\sin^{-1} \frac{\sqrt{3}}{2} = 60° = \frac{\pi}{3}$

$\Rightarrow$ Q3: $180 + \theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

Q4: $360 - \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$$\Rightarrow 2x = \frac{4\pi}{3} + 2n\pi, \quad \frac{5\pi}{3} + 2n\pi$$

$$\Rightarrow \boxed{x = \frac{2\pi}{3} + n\pi, \quad \frac{5\pi}{6} + n\pi}$$
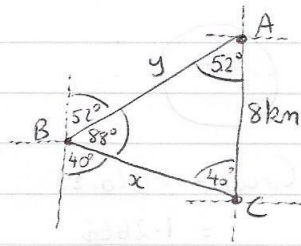
**Q8.**  $\cos 2A = 0.3420$

$\oplus \Rightarrow$ Q1 or Q4

R.A = $\cos^{-1} 0.342 = 70°$

$\Rightarrow$ Q1: $70°$  Q4: $360 - 70 = 290°$

$\Rightarrow 2A = 70 + n360, \quad 290 + n360$

$$\boxed{A = 35 + 180n, \quad 145 + 180n}$$

**Q9.**



Sine Rule

$$\frac{x}{\sin 52} = \frac{8}{\sin 88}$$
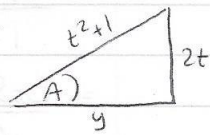
$$x = \frac{8(\sin 52)}{\sin 88} = 6.3 \ km$$

$$\frac{y}{\sin 40} = \frac{8}{\sin 88}$$

$$y = \frac{8(\sin 40)}{\sin 88} = 5.15 \ km$$

$$\Rightarrow \text{Tot Dist} = 6.3 + 5.15 + 8 = \boxed{19.45k}$$

**Q10.**  $\sin A = \frac{2t}{t^2 + 1}$



Pyt Thm

$$(t^2 + 1)^2 = (2t)^2 + y^2$$

$$t^4 + 2t^2 + 1 = 4t^2 + y^2$$
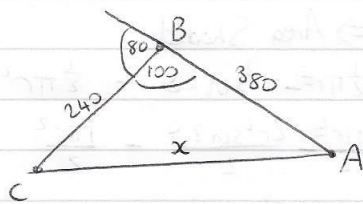
$$t^4 - 2t^2 + 1 = y^2$$

$$(t^2 - 1)(t^2 - 1) = y^2$$

$$(t^2 - 1)^2 = y^2$$

$$\Rightarrow y = t^2 - 1$$

$$\Rightarrow \cos A = \frac{y}{t^2 + 1} = \boxed{\frac{t^2 - 1}{t^2 + 1}}$$
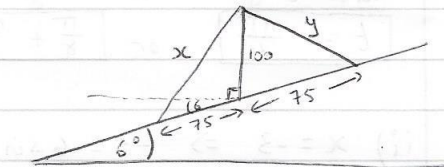
## Q11.



Cosine Rule

$$x^2 = (240)^2 + (380)^2 - 2(240)(380)\cos 100$$
$$= 202000 - (-31673.42761)$$
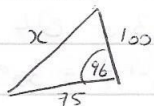$$= 233673.4276$$
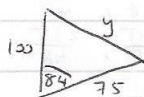$$\Rightarrow x = \boxed{483.4m}$$

## Q12.



To find $x$



Cosine Rule

$$x^2 = 75^2 + 100^2 - 2(75)(100)\cos 96$$
$$= 15625 - (-1567.92)$$
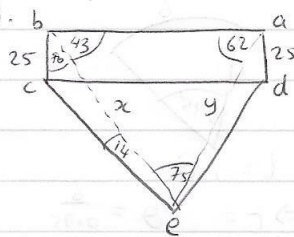$$= 17192.93$$
$$\Rightarrow x = \boxed{131.12 ft}$$



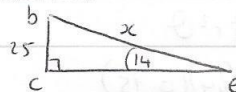$$180 - 96 = 84$$

$$y^2 = 100^2 + 75^2 - 2(100)(75)\cos 84$$
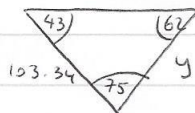$$= 14057.07$$
$$\Rightarrow y = \boxed{118.56 ft}$$

## Q13.



$\triangle$ bce



$$\sin 14 = \frac{25}{x}$$
$$\Rightarrow x = \frac{25}{\sin 14} = 103.34$$
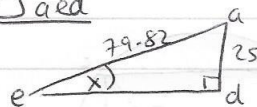
$\triangle$ bae



$$\frac{y}{\sin 43} = \frac{103.34}{\sin 62}$$
$$y = 79.82$$

$\triangle$ aed



$$\sin x = \frac{25}{79.82}$$
$$\Rightarrow x = \sin^{-1}\left(\frac{25}{79.82}\right)$$
$$= 18.25°$$
$$= \boxed{18°}$$

## Q14.

Hr hand speed $= \frac{360}{12} = 30°$ in 1hr

$\Rightarrow$ Speed $= \frac{30}{60} = \frac{1}{2}°$ every min

Speed of min hand $= 360°/hr = 6°/min$

$\Rightarrow$ Min hand gains $\boxed{5.5°}$ every minute

Let $x =$ no. of mins past 1:00 and 30° between

12 and 1 $\Rightarrow$ Min hand moves $30° + 6x$
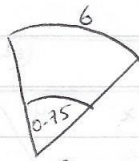
while hr hand moves $0.5x$

$\Rightarrow 0.5x = 6x - 30$

$\Rightarrow 5.5x = 30 \Rightarrow x = 5.45 = 5$ mins 27 secs

$\Rightarrow \boxed{1:05:27}$
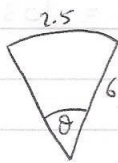
**Q15.**



$$L = r\theta$$
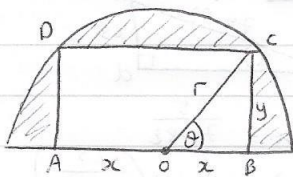$$\Rightarrow r = \frac{L}{\theta} = \frac{6}{0.75}$$
$$= 8 \text{cm}$$
$$\Rightarrow A = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2}(64)(0.75)$$
$$= \boxed{24 \text{cm}^2}$$

**Q16.**



$$L = r\theta$$
$$2.5 = 6\theta$$
$$\Rightarrow \theta = \frac{2.5}{6} = \boxed{\frac{5}{12}}$$

**Q17**



i) $\cos\theta = \frac{x}{r}$  $\quad \sin\theta = \frac{y}{r}$
$$\Rightarrow x = r\cos\theta \quad \Rightarrow y = r\sin\theta$$
$$\Rightarrow \text{Perimeter } ABCD$$
$$= 2x + y + 2x + y$$
$$= 4r\cos\theta + 2r\sin\theta$$
$$= 2r(\sin\theta + 2\cos\theta) \quad \text{Q.E.D.}$$

ii) Area ⌒ $= \frac{1}{2}\pi r^2$
Area ▭ $= 2x \cdot y$
$$= 2r\cos\theta \cdot r\sin\theta$$
$$= 2r^2\cos\theta \sin\theta \quad \text{(Pg14)}$$
$$= r^2 \sin 2\theta$$

$\Rightarrow$ Area Shaded
$$= \frac{1}{2}\pi r^2 - r^2 \sin 2\theta$$
$$= \frac{\pi r^2}{2} - \frac{2r^2 \sin 2\theta}{2}$$
$$= \frac{r^2}{2}(\pi - 2\sin 2\theta) \quad \text{Q.E.D}$$

**Q18.**  $x = 6\sin 8t$
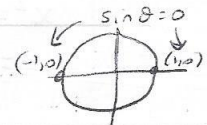


i) $6\sin 8t = 0$
$$\sin 8t = 0 \quad \Rightarrow 8t = 0° \text{ or } \pi$$
$$8t = 0 + 2n\pi \quad \text{or} \quad \pi + 2n\pi$$
$$t = \frac{2n\pi}{8} \quad \text{or} \quad \frac{\pi + 2n\pi}{8}$$
$$\boxed{t = \frac{n\pi}{4}} \quad \text{or} \quad \boxed{\frac{\pi}{8} + \frac{n\pi}{4}}$$

ii) $x = -3 \Rightarrow -3 = 6\sin 8t$
$$\Rightarrow \sin 8t = -\frac{1}{2}$$
$$\ominus \Rightarrow Q3 \text{ or } Q4$$
$$R.A = \sin^{-1}\frac{1}{2} = 30° = \frac{\pi}{6}$$
$$\Rightarrow Q3: \quad 180 + \theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$
$$Q4: \quad 360 - \theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$$\Rightarrow 8t = \frac{7\pi}{6} + 2n\pi \text{ or } \frac{11\pi}{6} + 2n\pi$$
$$\Rightarrow t = \frac{7\pi}{48} + \frac{n\pi}{4} \text{ or } \frac{11\pi}{48} + \frac{n\pi}{4}$$

$\underline{n=0}$  $\quad t = \frac{7\pi}{48}, \frac{11\pi}{48}$
$\underline{n=1}$  $\quad t = \frac{19\pi}{48}, \frac{23\pi}{48}$
$\underline{n=2}$  $\quad t = \frac{23\pi}{48}$

**Q19.** Max $Hgt = 12m$

Low $Hgt = 7m$

$\Rightarrow$ Amplitude $= \dfrac{12-7}{2} = 2.5$

Max to min $= 6hrs$

$\Rightarrow$ 1 full cycle $= 12hrs$

$\Rightarrow$ Period $= 12hrs$
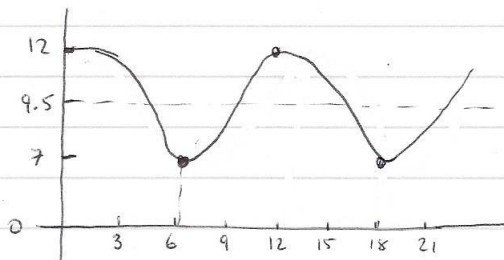
Starts @ max $\Rightarrow$ Cosine wave

$\cos t$ : period $= 360°$

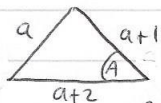$\cos 2t$ : period $= 180°$

$\cos 30t$ : period $= 12$

Sketch:



Normal $\cos$ wave centred about 0

$\Rightarrow$ Shift Up by $9.5$

$\Rightarrow$ Model : $9.5 + 2.5 \cos 30t$

$\Rightarrow$ $\boxed{x = \dfrac{19}{2} + \dfrac{5}{2} \cos 30t}$

**Q20**



$\cos A = \dfrac{(a+2)^2 + (a+1)^2 - a^2}{2(a+2)(a+1)}$

$= \dfrac{a^2 + 4a + 4 + a^2 + 2a + 1 - a^2}{2(a+2)(a+1)}$

$= \dfrac{a^2 + 6a + 5}{2(a+2)(a+1)} = \dfrac{(a+1)(a+5)}{2(a+2)(a+1)}$

$= \dfrac{a+5}{2(a+2)} = \boxed{\dfrac{a+5}{2a+4}}$ Q.E.D