Topic: Algebra including Logs/Indices in Book 1 (Topics 1 – 14, 20 – 25, 43/44)

Q1. Simplify $(a^2 - b^2)^{\frac{1}{2}}(a+b)^{-\frac{1}{2}}(a-b)^{\frac{3}{2}}$.
Ans: $(a-b)^2$

Q2. Solve $2x^3 - 3x^2 - 8x - 3 = 0$ and hence sketch the function $f(x) = 2x^3 - 3x^2 - 8x - 3$.
Ans: x = -1, 3, -0.5

Q3. Solve $\sqrt{3x+1} = \sqrt{x-1} + 2$. **Ans:** x = 1, 5

Q4. Simplify $\frac{2-\sqrt{3}}{2+\sqrt{3}}$. **Ans:** $7 - 4\sqrt{3}$

Q5. Solve the equation $4|x+1| = 3|x+1|$.
Ans: x = -1

Q6. Solve $|2x-1| < 3$. **Ans:** $-1 < x < 2$

Q7. Solve $3e^x - 7 + 2e^{-x} = 0$. **Ans:** $\ln 2, -\ln 3$

Q8. Show that $\frac{a+b}{2} \le \sqrt{\frac{a^2+b^2}{2}}$, where a and b are real.

Q9. Solve the inequality $\frac{x+3}{2x-1} \le 4$, for $x \in R, x \ne \frac{1}{2}$ **Ans:** $x < \frac{1}{2}$ or $x \ge 1$

Q10. If the equation $x^2 + kx + (k+3) = 0$, has two equal roots, find two possible values of k.
Ans: k = -2, 6

Q11. Given that $\frac{3}{5}$ is a root of $5x^3 + 7x^2 - kx + 3 = 0$, find the value of k. Find the other 2 roots in the form $a \pm \sqrt{b}$. **Ans:** k = 11, $-1 \pm \sqrt{2}$

Q12. Simplify $2^{\frac{1}{4}} + 2^{\frac{1}{4}} + 2^{\frac{1}{4}} + 2^{\frac{1}{4}}$. **Ans:** $2^{\frac{9}{4}}$

Q13. Solve: $6\log_x 2 + \log_2 x = 5$. **Ans:** x = 4, 8

Q14. Determine the values of $m \in R$ for which the quadratic equation
$10x^2 + 4x + 1 = 2mx(2-x)$ has real roots.
Ans: $m \le -\frac{1}{2}$ or $m \ge 3$

Q15. If $2^x + 2^{x+1} + 2^{x+2} = k.2^x$, find the value of k. **Ans:** k = 7

Q16. Solve the equation
$\log_2(7x+2) - \log_2(x+2) = 2$. **Ans:** x = 2

Q17. If $y = a^2$ and $a^3 b = 1$, express y in the form b^n. **Ans:** $b^{-\frac{2}{3}}$

Q18. Solve the equation $2^{2y+1} - 5(2^y) + 2 = 0$.
Ans: $y = \pm 1$

Q19. Solve the equations $2x - 3y = 1$ and $x^2 + xy - 4y^2 = 2$. **Ans:** (2, 1) and (11, 7)

Q20. Solve $32^{x-1} = 28$ for x, correct to two places of decimals. **Ans:** x = 1.96

Q21. Form the quadratic equation whose roots are $-\frac{2}{3}$ and 4. **Ans:** $3x^2 - 10x - 8 = 0$

Q22. Simplify $\frac{x^{\frac{3}{2}} - x^{-\frac{1}{2}}}{x^{\frac{1}{2}} - x^{-\frac{1}{2}}}$. **Ans:** x + 1

Q23. The population of a city grows according to the law $P = 40000(1.03)^n$, where n is the time in years and P is the population size.
(i) Estimate the size of the population in 12 years' time (ii) Determine when the population with have doubled (to the nearest half-year).
Ans: (i) 57,030 (ii) 23.5yrs

Q24. Sketch a rough graph of the polynomial $f(x) = 2(x+2)^3x^2(x-2)^2$.

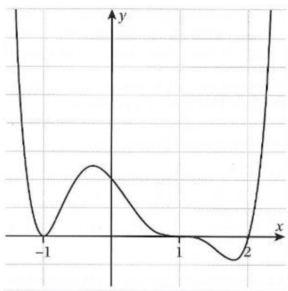
Q25. If $(x+a)^2$ is a factor of $x^3 + 6px + k$, show that (i) $k + 2a^3 = 0$ (ii) $k^2 + 32p^3 = 0$

Q26. Solve the equations A: $\frac{2x}{5} + \frac{y}{8} + z = \frac{5}{2}$
B: $\frac{x+1}{3} - \frac{y}{2} - 4z = 0$ and C: $\frac{x+y+z}{2} = 1$.
Ans: x = 5, y = -4, z = 1

Q27. (i) Construct a quadratic equation with roots k and 2k. (ii) Hence, show that if one root of the equation $x^2 + ax + b = 0$ is twice the other, then $9b = 2a^2$. **Ans:** (i) $x^2 - 3kx + 2k^2 = 0$

Q28. A dose of radioactive medicine decays at the rate of 15% per hour. If the original dose is 80mg, and f(x) represents the amount left in her body x hours after receiving the dose,
(i) calculate the amount left after 4 hours
(ii) find the value of x when f(x) = 20,
(iii) the amount by which f(x) decreases in the third hour. **Ans:** (i) 41.76 (ii) 8.53 (iii) 8.67

Q29. The graph of the polynomial y = f(x) of degree 6 is shown below.



(i) Find an expression for the polynomial f(x).
(ii) If the curve contains the point (0, 2), find the value of $c \in R$ if (3, c) lies on the curve.
Ans: (i) $f(x) = (x-1)^3(x+1)^2(x-2)$ (ii) 128