

Worksheet: Induction

L.C. Higher Level

Q1. Prove that $2^{3n} - 1$ is divisible by 7, for all $n \in \mathbb{N}$

Q2. Prove that $3^n > 2n$ for all $n \in \mathbb{N}$

Q3. Prove that $1 + 5 + 9 + \dots + (4n-3) = \frac{n}{2}(4n-2)$ for all positive integers n .

Q4. Prove that $n^2 > 4n + 3$ for $n \in \mathbb{N}$ and $n \geq 5$.

Q5. Prove that $5^{2n} + 12^{n-1}$ is divisible by 13 for all $n \in \mathbb{N}$.

Q6. Prove by induction that $\sum_{r=1}^n 5r = \frac{5n}{2}(n+1)$

Q7. Prove that 3 is a factor of $4^n - 1$, $n \in \mathbb{N}$.

Q8. Prove that $(1+3x)^n \geq 1+3nx$ for $x > 0$ and $n \in \mathbb{N}$.

Q9. Prove that $\sum_{r=1}^n 2^{r-1} = 2^n - 1$ for all $n \in \mathbb{N}$

Q10. Prove that $\sum_{r=2}^n \frac{1}{r^2-1} = \frac{3}{4} - \frac{2n+1}{2n(n+1)}$ for $n \geq 2$, $n \in \mathbb{N}$

Q11. Prove that $n^2 + 15n + 4$ is an even number for $n \in \mathbb{N}$.

Q12. Prove De Moivre's Theorem using induction.