Probability

Q1. i) $7! = \boxed{5040}$

ii) 3 vowels together = 1 block of $\boxed{IEA}$

+ 4 other objects to rearrange

$\Rightarrow$ 5 objects in total

$\Rightarrow 5! \times 3! = \boxed{720}$

possible arrangements of I, E & A.

Q2. $\underline{D}\ \_\ \_\ \_\ \_\ \_$

$1 \times 5 \times 4 \times 3 \times 2 \times 1$

$\Rightarrow 5! = \boxed{120}$

Q3. $\binom{10}{4} = \boxed{210}$

Q4. i) $\_\ \_\ \_\ \_\ \_$

$5 \times 4 \times 3 \times 2 \times 1 = \boxed{120}$

ii) $\_\ \_\ \_\ \_\ \_$ (can be 4 or 6)

$4 \times 3 \times 2 \times 1 \times 2 = \boxed{48}$

iii) (5, 6 or 7) $\_\ \_\ \_\ \_\ \_ = \boxed{72}$

$3 \times 4 \times 3 \times 2 \times 1$

Q5. Total arrangements $= 7! = 5040$

Arrangements with AE together $= 6! \times 2! = 1440$

$\Rightarrow$ AE not together $= \boxed{3600}$

Q6. $\binom{6}{2} = \boxed{15}$

Q7. $\binom{7}{3} = \boxed{35}$

Q8. i) P(Both red) = P(1st red) × P(2nd red)

$= \frac{6}{10} \times \frac{5}{9}$

$= \frac{30}{90} = \boxed{\frac{1}{3}}$

ii) P(Both Same) = P(Both red) OR P(Both Black)

$= \frac{1}{3} + \left(\frac{4}{10} \times \frac{3}{9}\right)$

$= \frac{30}{90} + \frac{12}{90}$

$= \frac{42}{90}$

$= \boxed{\frac{7}{15}}$

iii) P(Different Col) = 1 − P(Both Same)

$= 1 - \frac{7}{15}$

$= \boxed{\frac{8}{15}}$

Q9. i) 3 Men AND 2 Women

$= \binom{8}{3} \times \binom{7}{2}$

$= \boxed{1176}$

ii) (3M AND 2W) or (4M AND 1W) or (5M AND 0W)

$= 1176 + \left[\binom{8}{4} \times \binom{7}{1}\right] + \left[\binom{8}{5} \times \binom{7}{0}\right]$

$= 1176 + 490 + 56$

$= \boxed{1722}$

Q10. i) If one person has birthday on a particular day

$\Rightarrow$ all the other 6 must have birthday on the same day

$\Rightarrow 1 \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30}$

$= \boxed{\frac{1}{729{,}000{,}000}}$

ii) $1 \times \frac{29}{30} \times \frac{28}{30} \times \frac{27}{30} \times \frac{26}{30} \times \frac{25}{30} \times \frac{24}{30}$

$= \boxed{0.47}$

iii) P(At Least 2) = 1 − P(1) − P(0)

$= 1 - \left(1 \times \frac{1}{30} \times \frac{29}{30} \times \frac{28}{30} \times \frac{27}{30} \times \frac{26}{30}\right) - 0.47$

$= 1 - 0.023 - 0.47 = \boxed{0.51} > 0.5$

Q11. Total No. of Coins = $x + 6$

i) P(Both Copper) = P(1st Cpp) AND P(2nd Cp)

$$= \frac{x}{x+6} \times \frac{x-1}{x+5}$$

$$= \frac{x(x-1)}{(x+6)(x+5)}$$

$$= \boxed{\frac{x^2-x}{x^2+11x+30}}$$

ii) $$\frac{x^2-x}{x^2+11x+30} = \frac{4}{13}$$

$$13x^2 - 13x = 4(x^2+11x+30)$$

$$13x^2 - 13x = 4x^2 + 44x + 120$$

$$9x^2 - 57x - 120 = 0$$

$$3x^2 - 19x - 40 = 0$$

$$(3x + 5)(x - 8) = 0$$

$$3x+5 = 0 \quad \text{or} \quad x - 8 = 0$$

$$\cancel{x = -5/3} \qquad \boxed{x = 8}$$

iii) P(One of Coins is Copper)

= P(1st is Cp AND 2nd Not) or Vice Versa

$$= \left(\frac{8}{14} \times \frac{6}{13}\right) + \left(\frac{6}{14} \times \frac{8}{13}\right)$$

$$= \frac{24}{91} + \frac{24}{91}$$

$$= \boxed{\frac{48}{91}}$$

Q12 i) $$\frac{5}{16} \times \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13} = \boxed{\frac{1}{364}}$$

ii) P(Same) = All Red or All Blue

$$= \left(\frac{6}{16} \times \frac{5}{15} \times \frac{4}{14} \times \frac{3}{13}\right) + \left(\frac{1}{364}\right)$$

$$= \frac{3}{364} + \frac{1}{364} = \boxed{\frac{1}{91}}$$

iii) P(1st Blue, 2nd Red, 3rd Y, 4th Green) × 4!

$$= \left(\frac{5}{16} \times \frac{6}{15} \times \frac{2}{14} \times \frac{3}{13}\right) \times 4!$$

$$= \frac{3}{728} \times 24 = \boxed{\frac{9}{91}}$$

iv) P(2 are blue AND 2 are not blue)

$$= \frac{5}{16} \times \frac{4}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{55}{1092}$$

Block of 2 Blues + 2 other colours

⇒ 3 Objects to rearrange = 3!

$$\Rightarrow \frac{55}{1092} \times 3! = \boxed{\frac{55}{182}}$$

Q13. i) $8! = \boxed{40{,}320}$

ii) Choose 5 lanes from 8 = $\binom{8}{5}$

and then arrange 5 runners = 5!

$$\Rightarrow \text{Total} = \binom{8}{5} \times 5! = \boxed{6720}$$

Q14. One person serves on both committee & subcommittee

⇒ choose 4 remaining places from 9 others on committee

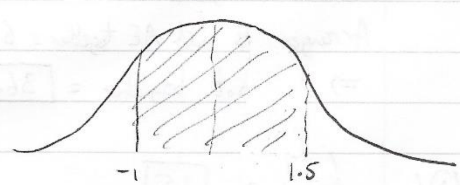
$$\Rightarrow \binom{9}{4} = \boxed{126}$$

Q15. $$z = \frac{x - \mu}{\sigma}$$

For 53: $$z = \frac{53-65}{12} = -1$$

For 83: $$z = \frac{83-65}{12} = 1.5$$

$$\Rightarrow P(-1 \le z \le 1.5)$$



$$\Rightarrow = P(z \le 1.5) - P(z \le -1)$$

$$= P(z \le 1.5) - (1 - P(z \le 1))$$

$$= 0.9332 - 1 + 0.8413$$

$$= \boxed{0.7745}$$

Q16 i) _ _ _ _ _ WINNING NUMBERS

Match 4 = 4 of winning nos + 1 not

$= \binom{5}{4} \times \binom{35}{1} = \boxed{150}$

ii) Match 3 = 3 of winning nos + 2 not

$= \binom{5}{3} \times \binom{30}{2} = \boxed{4350}$

iii) At least 3 = Match 3 or 4 or 5

Match 5 = $\binom{5}{5}$ = 1

Match 3 = 4350

Match 4 = 150

$\Rightarrow$ Total = 4501

$\Rightarrow P(\text{at least } 3) = \dfrac{4501}{\binom{35}{5}}$

$= \dfrac{4501}{324,632}$

$= 0.0138$

$= \boxed{0.014}$

Q17. Front Row

i) Total for front row $= \binom{15}{3} = 455$

3 Girls in front row $= \binom{7}{3} = 35$

$\Rightarrow P(3 \text{ Girls}) = \dfrac{35}{455} = \boxed{\dfrac{1}{13}}$

ii) 3 Boys or 2 Boys + 1 Girl

$\dfrac{\binom{8}{3} + \left[\binom{8}{2} \times \binom{7}{1}\right]}{455} = \boxed{\dfrac{36}{65}}$

iii) GGB or BGG but not GBG

$= \Rightarrow \dfrac{\frac{2}{3} \text{ of } \left[\binom{7}{2} \times \binom{8}{1}\right]}{455} = \boxed{\dfrac{16}{65}}$

Q18.

i) P(WLWLWW)

$p = 3/5$

$\Rightarrow q = 2/5$

$\Rightarrow P(\;) = \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5}$

or $\left(\frac{3}{5}\right)^4 \times \left(\frac{2}{5}\right)^2 = \boxed{0.021}$

ii) $P(4 \text{ Wins}) = \binom{6}{4}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^2$

$= \boxed{0.311}$

iii) Lose @ least 4 = Lose 4, 5 or 6

$= \binom{6}{4}\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^2 + \binom{6}{5}\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right) + \binom{6}{6}\left(\frac{2}{5}\right)^6$

$= 0.13824 + 0.036864 + 0.004096$

$= \frac{112}{625} = \boxed{0.1792}$

Q19. $p = 2/3$

$\Rightarrow q = 1/3$

i) $P(\text{None pass}) = \left(\frac{1}{3}\right)^6 = \boxed{\dfrac{1}{729}}$

ii) P(Half Pass) = P(3 Pass AND 3 Fail)

$= \binom{6}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3$

$= \boxed{\dfrac{160}{729}}$

Q20.

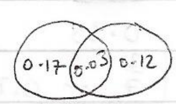
i) Independent $\Rightarrow P(A \cap B) = P(A) \times P(B)$

$= 0.2 \times 0.15$

$= \boxed{0.03}$

ii) $P(A|B) = P(A)$ if independent $= \boxed{0.2}$

iii)



$\Rightarrow P(A \cup B) = \boxed{0.32}$

OR

$(A \cap B) \neq \emptyset \Rightarrow$ Not mutually excl

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.2 + 0.15 - 0.03$

$= \boxed{0.32}$

Q21.

i) $P(Z \leq 1.32) = \boxed{0.9066}$

ii) $P(Z > -0.4) = P(Z < 0.4)$

$= \boxed{0.6554}$

iii) $P(1.3 \leq Z \leq 2)$

$= P(Z \leq 2) - P(Z \leq 1.3)$

$= 0.9772 - 0.9032$

$= \boxed{0.074}$

iv) $P(-0.67 \leq Z \leq 1.5)$

$= P(Z \leq 1.5) - P(Z \leq -0.67)$

$= P(Z \leq 1.5) - (1 - P(Z \leq 0.67))$

$= 0.9332 - 1 + 0.7486$

$= \boxed{0.6818}$

Q22. $P(A \cup B) \neq P(A) + P(B)$

$\Rightarrow$ not mutually exclusive

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.7 = 0.45 + 0.35 - x$

$\Rightarrow x = 0.1$

i) $P(A \cap B) = \boxed{0.1}$

ii) $P(A \cap B) \stackrel{?}{=} P(A) \times P(B)$

$0.1 \stackrel{?}{=} 0.45 \times 0.35$

$0.1 \neq 0.1575$

$\Rightarrow$ Not Independent

iii) $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{0.1}{0.35}$

$= \boxed{\frac{2}{7} \text{ or } 0.286}$

Q23. $P(E|F) = \dfrac{P(E \cap F)}{P(F)}$

$\Rightarrow \frac{1}{2} = \dfrac{\frac{1}{7}}{x}$

$\Rightarrow x = \dfrac{\frac{1}{7}}{\frac{1}{2}} = \frac{2}{7}$

$P(F|E) = \dfrac{P(E \cap F)}{P(E)}$

$\Rightarrow \frac{1}{3} = \dfrac{\frac{1}{7}}{y}$

$\Rightarrow y = \dfrac{\frac{1}{7}}{\frac{1}{3}} = \frac{3}{7}$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= \frac{3}{7} + \frac{2}{7} - \frac{1}{7}$

$= \boxed{\frac{4}{7}}$

Q24. i) $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$\Rightarrow 0.8 = \dfrac{x}{0.7}$

$\Rightarrow P(A \cap B) = 0.8 \times 0.7$

$= \boxed{0.56}$

ii) $P(A \cap B) = P(A) \times P(B)$

$0.56 = 0.8 \times 0.7$

$0.56 = 0.56$

$\Rightarrow$ Independent Q.E.D.

Q25. $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$\Rightarrow \frac{1}{5} = \dfrac{P(A \cap B)}{\frac{1}{3}}$

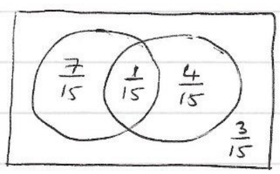
$\Rightarrow P(A \cap B) = \frac{1}{5} \times \frac{1}{3}$

$= \frac{1}{15}$

i) $P(A \cap B) \neq P(A) \times P(B)$

$\Rightarrow$ Not independent

$\Rightarrow P(A \cap B) = \boxed{\frac{1}{15}}$

ii) 

$P(\text{Only One}) = \frac{7}{15} \overset{\text{OR}}{+} \frac{4}{15}$

$= \boxed{\frac{11}{15}}$

iii) $P(\text{Neither}) = \boxed{\frac{3}{15}}$

Q26.

$P(x)$	x Winnings	$E(x) = x.P(x)$
$^{12}/_{52}$	€20	€4.62
$^{4}/_{52}$	€50	€3.85
$^{36}/_{52}$	€0	€0
		€8.47

$\Rightarrow E(x) =$ €8.47 $-$ €5

$=$ €3.47

$\Rightarrow$ Good bet as you can expect to win €3.47 on average when you play.