> Went through all this page on the board getting them to work with me at the back of their copies.

- Chapter 8: Difference Equations
- Topic 37: Recap on JC Patterns

- Linear Patterns from JC:

- A linear sequence of numbers is a list of numbers where there is a common difference between each term.
  - E.g. 3, 8, 13, 18, 23........which has a common difference of +5.
- In Junior Cycle, we learned how to find the General Term (T_n) of a linear sequence.

Step 1: Multiply the common difference by 'n'............in the sequence above: 5n Step 2: See what needs to be added or subtracted to 5n to get each of the terms in the sequence.........in the sequence above: -2 Step 3: Write the General Term $T_n = 5n - 2$.

- Once we had the general term, there were two useful things that we were able to do:

Finding any term in the sequence	Finding the term number of a value
For example, to find the 50th term: $T_n = 5n - 2$ $\Rightarrow T_n = 5(50) - 2$ $= 250 - 2$ $= 248$	For example, what term is 568? $5n - 2 = 568$ $\Rightarrow 5n = 570$ $\Rightarrow n = 114$ => the 114th term is 568

- Quadratic Patterns from JC:

- We also learned in Junior Cycle about Quadratic Sequences.
- A quadratic sequence is a list of numbers where the second difference between each term is the same every time.
  - E.g. 4, 7, 12, 19, 28, 39...........which has first differences of +3, +5, +7, +9......and a second difference of +2.
- To find the General Term of a quadratic sequence we:

Step 1: Let the General Term $T_n = an^2 + bn + c$. Step 2: The second difference represents 2a, so halving the second difference gave us a value for a......in the sequence above, the second difference is +2, so 'a' would be 1. Step 3: Use two of the terms in the sequence to make two simultaneous equations, which we solve to find 'b' and 'c'...... $T_n = an^2 + bn + c$	
$T_2 = (2)^2 + b(2) + c = 7$ $\Rightarrow 4 + 2b + c = 7$ $\Rightarrow 2b + c = 3$.....Eqn 1	$T_3 = (3)^2 + b(3) + c = 12$ $\Rightarrow 9 + 3b + c = 12$ $\Rightarrow 3b + c = 3$......Eqn 2
Solving Equations 1 and 2 gives b = 0 and c = 3 $\Rightarrow T_n = n^2 + (0)n + 3$ $\Rightarrow T_n = n^2 + 3$	

- Note: An alternative method to find Tn of a Quadratic Sequence is use three terms and form three equations in a, b and c and solve for those values then.

- If a sequence is such that the third difference between its terms is the same every time, then that is known as a cubic sequence.
  E.g. 11, 31, 69, 131, 223……

- Topic 38: Arithmetic Sequences/Series
  - In your LC Maths course, the term we use to describe linear sequences is Arithmetic Sequences.
  - We use a formula at senior cycle to help us find the General Term of this type of sequence quickly:

$$T_n = a + (n - 1)d$$

See pg22 Tables Book

where 'a' is the 1st term and 'd' is the common difference

  - If we add together the terms of an arithmetic sequence, we get an Arithmetic Series.
  - It can be useful to be able to find the sum of the terms of an arithmetic series.
  - We use a formula to help us find the sum of the first n terms:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

See pg22 Tables Book

where 'a' is the 1st term and 'd' is the common difference

- **Example:** A sequence is 3, 9, 15, 21, 27……
  i) Find the 60th term. ii) Find the sum of the first 60 terms. iii) What term is the first term to be bigger than 10000?

Solution:

i) Firstly, we will find the General Term:
- As a = 3 and d = 6, then

$T_n = a + (n - 1)d$
$\Rightarrow T_n = 3 + (n - 1)(6)$
$\Rightarrow T_n = 3 + 6n - 6$
$\Rightarrow T_n = 6n - 3$

- So, now we can find the 60th term:

$T_{60} = 6(60) - 3$
$= 360 - 3$
$= 357$

ii) This time we need to use the S_n formula:

$S_n = \frac{n}{2}\{2a + (n-1)d\}$
$= \frac{60}{2}\{2(1) + (60-1)(3)\}$
$= 30\{2 + 177\}$
$= 5370$

iii) To find this term, we need to solve $T_n > 10000$

$6n - 3 > 10000$
$\Rightarrow 6n > 10003$
$\Rightarrow n > 1667.17$
=> the 1668th term will be the first term to be bigger than 10000

Day 1: Classwork Questions: Sheet: Exercise 1 Qs 4/5/9 and Exercise 2 Qs 1(ii)(vi)/4 and then give Pg 146 Ex 8A Qs 2/3/5 for HW to finish also

Topic 39: Geometric Sequences:

- A Geometric sequence is a set of numbers where each term is found by multiplying the previous term by the same number, known as the common ratio.
  E.g. 10, 30, 90, 270………
- The common ratio is denoted 'r'.
- We also use a formula to help us find the General Term of this type of sequence:

$$T_n = ar^{n-1}$$

See pg22 Tables Book

where 'a' is the 1st term and 'r' is the common ratio.

- Similarly, a Geometric Series is a series where the terms of a Geometric Sequence are added together.
- It can be easily shown that the sum of the first n terms of a Geometric Series can be found using the formula:

$$S_n = \frac{a(1-r^n)}{1-r} \text{ if } r < 1 \text{ or } S_n = \frac{a(r^n - 1)}{r-1} \text{ if } r > 1$$

See pg22 Tables Book

where 'a' is the 1st term and 'r' is the common ratio.

- In the specific case of an infinite Geometric series, the formulae above simplify to:

$$S_\infty = \frac{a}{1-r}$$

Classwork Questions: Questions from Sheet on Geo S Ex 4 Qs 1(ii)(iv)(v)(vi)/2(ii)(iv)/3 and Ex 5 Qs 1(ii)/2(ii)

Topic 40: Recurrence Relations

- A recurrence relation is a sequence which is defined differently to how previous sequences we have come across are defined.
- Up to now, the General Term described any term in terms of 'n'.
- In a recurrence relation a sequence is defined showing how any term is connected to the previous term.
- Examples of recurrence relations would be:

Note the use of 'u' instead of 'T'

i) $T_n = 3T_{n-1}$ — Any term = 3 times the previous term

ii) $T_{n+1} = T_n - 4$ — Any term = the previous term less 4

iii) $u_{n+1} = \frac{1}{2}u_n - 3$ — Any term = half the previous term less 3

- Example: Pg 149 Ex 8B Q5

A sequence $T_1, T_2, T_3, T_4, T_5, T_6$ .... is defined by the recurrence relation $T_{n+1} = \frac{1}{4}T_n$, for $n \in N$ where $T_1 = 144$.

i) Write down the first four terms.

ii) Find S_∞ the sum of the entire series.

Solution:

i) Firstly, the key information we need is the recurrence relation itself $T_{n+1} = \frac{1}{4}T_n$, which tells us that any term in this sequence is a quarter of the previous term.

- We were also given $T_1 = 144$, so the next term will be $\frac{1}{4}(144) = 36$.
- The third term will be $\frac{1}{4}(36) = 9$ and the fourth term will be $\frac{1}{4}(9) = \frac{9}{4}$

ii)

- Let's start by writing out the sequence and identifying what type it is:

144, 36, 9, $^9/_4$, ……

- This is a Geometric Sequence with a = 144 and r = $\frac{1}{4}$.
- To find S_∞, we use the formula above for the sum of an infinite Geometric sequence:

$$S_\infty = \frac{a}{1-r}$$

$$\Rightarrow S_\infty = \frac{144}{1-\frac{1}{4}} = \frac{144}{\frac{3}{4}} = 192$$

Not enough in this lesson, so went through Example 1 on next page at the end of this lesson and did both examples then in the next lesson.

Classwork Questions: Pg 149 Ex 8B Qs 2/3/7/8/10

Topic 41: First Order Difference Equations

- We saw in the previous topic that when we are given a recurrence relation, we have to go through the process of working out the previous term if we want to know ANY term in the sequence.
- This can be a fairly time-consuming process, if for example, we wanted to know the 5000th term of a sequence, as we'd have to work out the previous 4999 terms first.
- For this reason, it is very useful if we can work out the General Term from a recurrence relation.
- In order to do this, we have to learn how to solve a difference equation.

- First Order Difference Equations:
  - These difference equations are ones where each term is defined in terms of one previous term. e.g. $T_{n+1} = \frac{1}{4}T_n - 3$

- Example 1: Pg 151 Ex 8C Q3

i) Solve the difference equation $u_n = 3 + 2u_{n-1}$ given that $u_0 = 1$.

ii) As $n \to \infty$, which of these is true? A: u_n gets smaller and smaller. B: u_n gets bigger and bigger. C: u_n tends to a finite limit k.

Solution:

- The strategy to solve this type is to sub in increasing values for n starting at 1, and see if we can spot a pattern to link u_n and the term we've been given (in this example u_0):

n = 1	n = 2	n = 3
$u_n = 3 + 2u_{n-1}$ => $u_1 = 3 + 2u_0$ => $u_1 = 3 + 2(1)$	$u_n = 3 + 2u_{n-1}$ => $u_2 = 3 + 2u_1$ => $u_2 = 3 + 2(3 + 2(1))$ => $u_2 = 3 + 2(3) + 2^2(1)$	$u_n = 3 + 2u_{n-1}$ => $u_3 = 3 + 2u_2$ => $u_3 = 3 + 2(3 + 2(3) + 2^2(1))$ => $u_3 = 3 + 2(3) + 2^2(3) + 2^3(1)$

- Hopefully, you can now spot from looking at u_1, u_2 and u_3 above that
  a) the terms in red form a pattern and are of the form $2^n(1)$
  b) the terms in blue form a geometric series with $a = 3$ and $r = 2$, so we can find the sum of that series using our formula from the first topic:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_n = \frac{3((2)^n - 1)}{2 - 1}$$

$$\Rightarrow S_n = \frac{3((2)^n - 1)}{1}$$

$$\Rightarrow S_n = 3(2)^n - 3$$

> Show them that the solution works for the original equation i.e.
> $U_{n-1} = 3(2)^{n-1} - 3 = \frac{3}{2}(2)^n - 3$
> => $2U_{n-1} = 2\left[\frac{3}{2}(2)^n - 3\right] = 3(2)^n - 6$
> => $3 + 2U_{n-1} = 3(2)^n - 3 = U_n$

- So, combining the two gives us: $1(2)^n + 3(2)^n - 3$.
- And finally, adding together the like terms gives us our final solution: $4(2)^n - 3$

ii)

- As we now know the General Term, we can write out the first few terms of the sequence:

$u_1 = 4(2^1) - 3 = 5,\ u_2 = 4(2^2) - 3 = 13,\ u_3 = 4(2^3) - 3 = 29,\ u_4 = 4(2^4) - 3 = 61$

=> Sequence is: 5, 13, 29, 61…….

- We can see from the sequence above that the distance between the terms is increasing as we add on more terms, so u_n is just going to get bigger and bigger as $n \to \infty$.

- Example 2: Pg 151 Ex 8C Q4

Savani has €500 in her savings account. She decides that, now that she has a new job, she will put €1000 on January 1st every year into her savings account. The bank offers 1% compound interest per annum.

i) Show that the amount in her savings account after n years (A_n) is determined by the difference equation $A_n = 1.01A_{n-1} + 1000$.

ii) Given that $A_0 = 500$, solve this difference equation.

iii) How much will she have in her account in 20 years' time?

Solution :

i) If she has 'x' euro in her account at the start of ANY year, then she will have 1.01x at the end of the year as the bank adds 1% compound interest.

- She then adds in €1000 at the start of the next year.
- So, at the end of ANY year she will have 1.01 of what she had the previous year plus an additional €1000 => $A_n = 1.01A_{n-1} + 1000$ Q.E.D

ii) As before,

n = 1	n = 2	n = 3
$A_n = 1.01A_{n-1} + 1000$ => $A_1 = 1.01A_0 + 1000$ $A_1 = 1.01(500) + 1000$	$A_n = 1.01A_{n-1} + 1000$ => $A_2 = 1.01A_1 + 1000$ => $A_2 = 1.01(1.01(500) + 1000) + 1000$ $A_2 = (1.01^2)(500) + 1.01(1000) + 1000$	$A_n = 1.01A_{n-1} + 1000$ => $A_3 = 1.01A_2 + 1000$ $= 1.01((1.01^2)(500) + 1.01(1000) + 1000) + 1000$ $= (1.01^3)(500) + (1.01^2)1000 + (1.01)1000 + 1000$

- So hopefully, we see the pattern again…….
    - The red terms being $(1.01^n)(500)$ and the blue terms being a Geometric Series with a = 1000 and r = 1.01.
- We can find the sum of this series using our formula:

> Let them try this first and then follow in behind on the board with the solution.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_n = \frac{1000((1.01)^n - 1)}{1.01 - 1}$$

$$\Rightarrow S_n = \frac{1000((1.01)^n - 1)}{0.01}$$

$$\Rightarrow S_n = 100{,}000((1.01)^n - 1)$$

- So, our solution will be: $(1.01^n)(500) + 100{,}000((1.01)^n - 1)$.

$$= 500(1.01^n) + 100{,}000(1.01)^n - 100{,}000$$

$$\Rightarrow A_n = 100{,}500(1.01)^n - 100{,}000$$

iii) To calculate how much she will have after 20 years, we just have to sub in n = 20:

$$A_n = 100{,}500(1.01)^n - 100{,}000$$

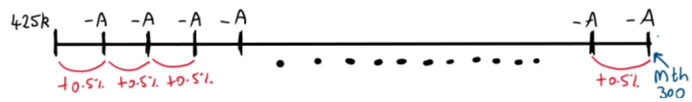
$$A_{20} = 100{,}500(1.01)^{20} - 100{,}000$$

$$\Rightarrow A_{20} = €22{,}629.10$$

Day 1: Classwork Questions: Pg 151 Ex 8C Qs 1/2/5(a)

Day 2: Classwork Questions: Pg 152 Ex 8C Qs 5(b)/6/8/9

- <u>Interest Repayments:</u>



- **Example:** Pg 154 Ex 8D Q3

  A couple borrows €425,000 to buy a house. They will repay the same amount (A) each month for 25 years. The Building Society charges a monthly interest rate of 0.5%.

  i) How many monthly repayments will there be ?

  ii) If D_n is the amount of debt owing after n months write down a difference equation in D_n.

  iii) Solve the difference equation.

  iv) Find A to the nearest euro.

<u>Solution:</u>

i) The number of monthly repayments over 25 years will be 25 x 12 = 300.

ii) In ANY month, the couple will owe 1.005 of what they owe the previous month D_{n-1} (as the interest rate is 0.5%) and then they will make a repayment of A

=> D_n will be: $1.005(D_{n-1}) - A$

iii) We now solve as before:

n = 1	n = 2	n = 3
$D_n = 1.005D_{n-1} - A$ => $D_1 = 1.005D_0 - A$	$D_n = 1.005D_{n-1} - A$ => $D_2 = 1.005D_1 - A$ => $D_2 = 1.005(1.005D_0 - A) - A$ => $D_2 = 1.005^2D_0 - 1.005A - A$	$D_n = 1.005D_{n-1} - A$ => $D_3 = 1.005D_2 - A$ => $D_3 = 1.005(1.005^2D_0 - 1.005A - A) - A$ => $D_3 = 1.005^3D_0 - 1.005^2A - 1.005A - A$ => $D_3 = 1.005^3D_0 - 1(1.005^2A + 1.005A + A)$

- So the pattern this time is…….
  - The first term being $(1.005^n)D_0$ and the terms in the brackets being a Geometric Series with a = A and r = 1.005.
- Again, we can find the sum of this series using our formula:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_n = \frac{A((1.005)^n - 1)}{1.005 - 1}$$

$$\Rightarrow S_n = \frac{A((1.005)^n - 1)}{0.005}$$

$$\Rightarrow S_n = 200A((1.005)^n - 1)$$

- So, our solution will be: $D_n = (1.005^n)D_0 - 200A((1.005)^n - 1)$.

iv)

- We know $D_0 = €425{,}000$ and n = 300 from part (i) and we also know that the debt will be repaid when $D_n = 0$ =>

=> $(1.005^{300})(425{,}000) - 200A((1.005)^{300} - 1) = 0$

=> $1897612.17 - 200A(3.464969812) = 0$

=> $1897612.17 - 692.9939624A = 0$

=> $1897612.17 = 692.9939624A$

=> $A = \frac{1897612.17}{692.9939624} = €2738.28 = €2738$

Put this in memory M on Calculator

Classwork Questions: Pg Ex 8D Qs 1/2/5

Notes for board to introduce Topic 42:
e.g.s $T_{n+1} = 3T_n - 2T_{n-1}$ or
$u_{n+2} - 2u_{n+1} + 3u_n = 0$
2 types:
(i) Homogeneous (ii) inhomogeneous
(i) Homogeneous:
- 2 theorems we need:
For characteristic equation $px^2 + qx + r = 0$
Different roots α and β => $u_n = l\alpha^n + m\beta^n$
Same roots α and α => $u_n = l\alpha^n + mn\alpha^n$

Topic 42: Second Order Difference Equations

- **Second Order Homogeneous Equations:**
  - These difference equations are ones where each term is defined in terms of two previous terms. e.g. $T_{n+1} = 3T_n - \frac{3}{4}T_{n-1}$ or $u_{n+2} - 2u_{n+1} + 3u_n = 0$
  - Second order difference equations are called homogeneous, if they only contain T_n, T_{n+k} or T_{n-k}.
  - If they contain other terms other than those, they are known as inhomogeneous equations or non-homogeneous ones. E.g. $u_{n+2} - 2u_{n+1} + 3u_n - 4 = 0$
  - The example above is non-homogeneous as it contains an extra constant of 4.
  - When solving homogeneous equations, we first solve what is known as the characteristic quadratic equation.
  - If the difference equation is: $u_{n+2} - 2u_{n+1} + 3u_n = 0$ then the corresponding characteristic equation will be: $x^2 - 2x + 3 = 0$. (Note that the coefficients have to match in both equations).
  - There are two theorems we need then to solve the difference equation:

Difference Equation Theorem 1	Difference Equation Theorem 2
If α and β are the two roots of the characteristic equation $px^2 + qx + r = 0$, then the solution will be in the form: $u_n = l\alpha^n + m\beta^n$	If α and α are the two equal roots of the characteristic equation $px^2 + qx + r = 0$, then the solution will be in the form: $u_n = l\alpha^n + mn\alpha^n$

- **Example 1:** Pg 156 Ex 8E Q4 Solve $2u_{n+2} - 11u_{n+1} + 5u_n = 0$ given $u_0 = 2$ and $u_1 = -8$.

  Solution:
  - First we form the characteristic equation and solve it:
    $2u_{n+2} - 11u_{n+1} + 5u_n = 0$ will have characteristic equation
    $$2x^2 - 11x + 5 = 0$$
    $$\Rightarrow (2x-1)(x-5) = 0$$
    $$\Rightarrow x = \frac{1}{2} \quad or \quad x = 5$$
  - Using Theorem 1 above with our two roots a = $\frac{1}{2}$ and b = 5, our solution will be in the form:
    $$u_n = la^n + mb^n$$
    $$\Rightarrow u_n = l(\tfrac{1}{2})^n + m(5)^n$$
  - Now we use the other information we were given in the question:

If $u_0 = 2$: $\Rightarrow u_0 = l(\frac{1}{2})^0 + m(5)^0 = 2$ $\Rightarrow l + m = 2$ (as $(\frac{1}{2})^0$ and $(5)^0$ = 1)	If $u_1 = -8$: $\Rightarrow u_1 = l(\frac{1}{2})^1 + m(5)^1 = -8$ $\Rightarrow \frac{1}{2}l + 5m = -8$ $\Rightarrow l + 10m = -16$ (multiplying by 2)

  - We now solve the two simultaneous equations above giving $l = 4$ and $m = -2$.
  - So, the solution to our difference equation is: $u_n = 4(\frac{1}{2})^n - 2(5)^n$.

- **Example 2:** Pg 156 Ex 8E Q9 Solve $u_{n+1} - 4u_n + 4u_{n-1} = 0$ given $u_0 = -1$ and $u_1 = 8$.

  Solution:
  - Again, we start by forming the characteristic equation and solving it:
    $u_{n+1} - 4u_n + 4u_{n-1} = 0$ will have characteristic equation
    $$x^2 - 4x + 4 = 0$$
    $$\Rightarrow (x-2)(x-2) = 0$$
    $$\Rightarrow x = 2 \quad or \quad x = 2$$
  - Using Theorem 2 above with our two roots a = 2 and a = 2, our solution will be in the form:
    $$u_n = la^n + mna^n$$
    $$\Rightarrow u_n = l(2)^n + mn(2)^n$$
  - Using the other information given:

If $u_0 = -1$: $\Rightarrow u_0 = l(2)^0 + m(0)(2)^0 = -1$ $\Rightarrow l = -1$	If $u_1 = 8$: $\Rightarrow u_1 = l(2)^1 + m(1)(2)^1 = 8$ $\Rightarrow 2l + 2m = 8$ $\Rightarrow 2(-1) + 2m = 8$ $\Rightarrow m = 5$

  - So, the solution to our difference equation is: $u_n = -1(2)^n + 5n(2)^n$, which can be written in a tidier way if we factorise out the 2^n: $u_n = 2^n(5n - 1)$.

Day 1: Classwork Questions: Pg 156 Ex 8E Qs 1/2/6/8/11/12

Day 2: Classwork Questions: Pg 156/157 Ex 8E Qs 14/16/19/21

- Second Order Inhomogeneous Equations:
  - Firstly, a reminder that inhomogeneous equations are ones of the form ones $u_{n+2} + u_{n+1} + u_n = \pm A$ where A is some other function not related to u_n.
  - To solve inhomogeneous equations there are two parts to the solution: the particular solution and the complementary solution.
  - For the particular solution, we use the table below, depending on what form the right-hand side of the equation is.

Type	Particular Solution
Constant e.g. 7	$u_n = a$
Linear e.g. $3n - 5$	$u_n = an + b$
Quadratic e.g. $3n^2 + 5n - 2$	$u_n = an^2 + bn + c$
Exponential: e.g. 3^n	$u_n = a(3^n)$

- Example 1: Pg 160 Ex 8F Q4

  Solve the difference equation $T_{n+2} - 4T_{n+1} + 4T_n = 7n - 14$ given $T_0 = 1$ and $T_1 = 15$.

  Solution:
  - We begin by noticing that the right-hand side of the equation above is $7n - 14$, which is a linear expression, so our particular solution from the table above will be of the form

$$T_n = an + b$$

  - We now need to get expressions for T_{n+1} and T_{n+2}:

$$T_{n+1} = a(n+1) + b = an + a + b$$
$$T_{n+2} = a(n+2) + b = an + 2a + b$$

  - Subbing into our initial equation gives:

$$T_{n+2} - 4T_{n+1} + 4T_n = 7n - 14$$
$$\Rightarrow an + 2a + b - 4(an + a + b) + 4(an + b) = 7n - 14$$
$$\Rightarrow an - 2a + b = 7n - 14$$

  - If we now compare the left-hand side to the right side, it can be seen that $a = 7$ and $-2a + b = -14$.
  - Subbing in our value of a gives: $-2(7) + b = -14 \Rightarrow b = 0$
  - So, our particular solution in this case is: $T_n = an + b = 7n$.
  - We now find the complementary solution by solving the homogeneous equation $T_{n+2} - 4T_{n+1} + 4T_n = 0$, which we learned how to do in the last section:

    Characteristic Equation to solve: $x^2 - 4x + 4 = 0$

$$\Rightarrow (x-2)(x-2) = 0$$
$$\Rightarrow x - 2 = 0 \quad or \quad x - 2 = 0$$
$$\Rightarrow x = 2 \quad\quad or \quad x = 2$$

  - Using Theorem 2 above with our two roots a = 2 and a = 2, our solution will be in the form:

$$T_n = la^n + mna^n \Rightarrow T_n = l(2)^n + mn(2)^n$$

  - To get the final solution, we add the particular solution and the complementary solution: $T_n = l(2)^n + mn(2)^n + 7n$

- We finally evaluate the values of l and m as before:

If $T_0 = 1$: => $T_0 = l(2)^0 + m(0)(2)^0 + 7(0) = 1$ => $l = 1$	If $T_1 = 15$: => $T_1 = l(2)^1 + m(1)(2)^1 + 7(1) = 15$ => $2l + 2m + 7 = 15$ => $2(1) + 2m + 7 = 15$ => $2m = 6$ => $m = 3$

- So, the final solution to our difference equation is: $T_n = 1(2)^n + 3n(2)^n + 7n$, which can be written in a tidier way if we factorise out the 2^n: $u_n = 2^n(3n + 1) + 7n$.

Classwork Questions: Pg 160 Ex 8F Qs 2/7/1 and then try Q6

- Example 2:

  Solve the difference equation $2u_{n+2} - u_{n+1} - 3u_n = 3^n$ given $u_0 = 3$ and $u_1 = -5$.

  Solution:

- This time the right-hand side is an exponential expression, so our solution will be of the form i.e. $u_n = a.3^n$.
- Again, we begin by getting expressions for u_{n+1} and u_{n+2}:

$$u_{n+1} = a.3^{(n+1)} = a.3^n.3^1 = 3a.3^n$$
$$u_{n+2} = k.3^{(n+2)} = a.3^n.3^2 = 9a.3^n$$

- Subbing these expressions into the equation we were asked to solve initially gives:

$$2u_{n+2} - u_{n+1} - 3u_n = 3^n$$
$$=> 2(9a.3^n) - (3a.3^n) - 3(a.3^n) = 3^n$$

- If we factorise out the 3^n on the left-hand side:

$$3^n(18a - 3a - 3a) = 3^n$$
$$=> 3^n(12a) = 3^n(1)$$

- For this to be true 12a must be equal to 1

$$=> 12a = 1$$
$$=> a = \frac{1}{12}$$

- So, our particular solution is: $u_n = \frac{1}{12}.3^n$.
- As in example 1, we now must find the complementary solution by solving the associated characteristic equation:

$$2x^2 - x + 3 = 0$$
$$=> (2x - 3)(x + 1) = 0$$
$$=> x = \frac{3}{2} \quad or \quad x = -1$$

- As we have two roots our complementary solution will be in the form $u_n = la^n + mb^n$ where a = $\frac{3}{2}$ and b = -1: $=> u_n = l(\frac{3}{2})^n + m(-1)^n$
- We now combine the particular and complementary solutions again to get the full solution:

$$u_n = l(\frac{3}{2})^n + m(-1)^n + \frac{1}{12}.3^n$$

- Using the other information given in the question to find l and m:

If $u_0 = 3$: => $u_0 = l(\frac{3}{2})^0 + m(-1)^0 + \frac{1}{12}.3^0 = 3$ => $l + m = 3 - \frac{1}{12}$ => $12l + 12m = 35$	If $u_1 = -5$: => $u_1 = l(\frac{3}{2})^1 + m(-1)^1 + \frac{1}{12}.3^1 = -5$ => $\frac{3}{2}l - m + \frac{1}{4} = -5$ => $6l - 4m + 1 = -20$ => $6l - 4m = -21$

- Solving these two simultaneous equations gives: $l = -\frac{14}{15}$ and $m = \frac{77}{20}$, so our final solution will be: $u_n = (-\frac{14}{15})\left(\frac{3}{2}\right)^n + (\frac{77}{20})(-1)^n + \frac{1}{12}.3^n$

Day 1: Classwork Questions: Solve: $u_{n+2} - u_{n+1} - 6u_n = 2^n$ given $u_0 = 2$ and $u_1 = 3$ and then try Qs 3/8 on Pg 160

Day 2: Classwork Questions: Pg 160 Ex 8F Qs 14/15/16

Revision Questions and Test

Solution:
$u_n = \frac{8}{5}(3)^n + \frac{13}{20}(-2)^n - \frac{1}{4}(2)^n$