Revision Sheet 3 – Algebra 3 (Chap 3) **Review: Wed 7/11/18**

Q1. Write $\frac{4+\sqrt{3}}{2+\sqrt{3}}$ in the form $a+b\sqrt{3}$.	**Q2.** Determine the values of $k \in R$ for which the quadratic equation $x^2-(3k+1)x+(2k^2+k+4)=0$ has real roots.
Q3. Solve the equation $\|2x-1\|=7$ for $x \in R$.	**Q4.** Prove that if $a>0$ and $b>0$ then: (i) $\frac{1}{a}+\frac{1}{b} \geq \frac{2}{a+b}$ (ii) $(a+b)(\frac{1}{a}+\frac{1}{b}) \geq 4$ (iii) $a+b \geq 2\sqrt{ab}$
Q5. Show that for any real values of a, b and h, the quadratic equation $(x-a)(x-b)-h^2=0$ has real roots.	**Q6.** Write $\frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}$ in the form $a+b\sqrt{c}$.
Q7. Show that the equation $4ax^2-4ax+a+c^2=0$ has no real roots for $a \in N, c \in R, c \neq 0$.	**Q8.** Solve the inequality $\frac{x+3}{2x-1} \leq 4$, for $x \in R, x \neq \frac{1}{2}$.
Q9. Prove that for any real numbers p and q: (i) $p^2+4q^2 \geq 4pq$ (ii) $(p+q)^2 \leq 2(p^2+q^2)$	**Q10.** Solve the inequality $\frac{x}{x-1}-\frac{3}{2} \geq \frac{1}{2(x-1)}$, for $x \in R, x \neq 1$.
Q11. Express $\frac{1-\sqrt{3}}{1+\sqrt{3}}$ in the form $a\sqrt{3}-b$ where $a, b \in N$.	**Q12.** Solve the equation $\|3x-1\|=\|5x-7\|$ for $x \in R$.
Q13. Determine the values of $k \in R$ for which the quadratic equation $kx(1-x)=1$ has real roots.	**Q14.** Prove that $a^2-6a+9+b^2 \geq 0$ for all real values of a and b.
Q15. Solve the equation $\sqrt{2x+5}-x=1$.	**Q16.** Solve the equation $\sqrt{6x+4}-1=\sqrt{3x+1}$.

Answers:

Q1. $5-2\sqrt{3}$	Q2. $k \leq -5 \; and \; k \geq 3$
Q3. x = -3 or x = 4	Q6. $(2x+1)-2\sqrt{x^2+x}$
Q8. $x \leq 0.5 \; and \; x \geq 1$	Q10. $1 \leq x \leq 2$
Q11. $\sqrt{3}-2$	Q12. x = 1 or x = 3
Q13. $k \leq 0 \; and \; k \geq 4$	Q15. x = 2
Q16. x = 0	