Revision Sheet 4 Worked Solutions

Q1. $(1,1) \xrightarrow[-1]{-1} (0,0)$

$(8,-5) \xrightarrow[-1]{-1} (7,-6) = (x_1, y_1)$

$(5,-2) \xrightarrow[-1]{-1} (4,-3) = (x_2, y_2)$

$$\text{Area} = \frac{1}{2}|x_1 y_2 - x_2 y_1|$$
$$= \frac{1}{2}|(7)(-3) - (-6)(4)|$$
$$= \frac{1}{2}|-21 + 24|$$
$$= \frac{1}{2}|3|$$
$$= \boxed{1.5 \text{ units}^2}$$

Q2. i) $m: 7x - y + 7 = 0$

Point on line

$$\Rightarrow 7(k-2) - (7k-7) + 7 \stackrel{?}{=} 0$$
$$7k - 14 - 7k + 7 + 7 \stackrel{?}{=} 0$$
$$14 - 14 = 0$$
$$0 = 0$$

$\Rightarrow$ ON line Q.E.D

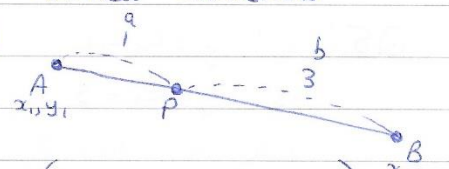
ii) Can't be done

Q3. $A = (5,-9)$ $B(-3,3)$

i) $$\text{mp} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{5-3}{2}, \frac{-9+3}{2}\right)$$
$$= \boxed{(1,-3)}$$

ii) $$\text{mp} = \left(\frac{5+1}{2}, \frac{-9-3}{2}\right)$$
$$= \boxed{(3,-6)}$$

iii)

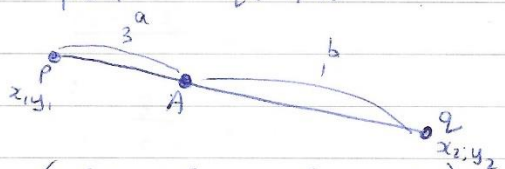


$$P = \left(\frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a}\right)$$

$$P = \left(\frac{3(5) + 1(-3)}{1+3}, \frac{3(-9) + 1(3)}{1+3}\right)$$
$$= \left(\frac{15-3}{4}, \frac{-27+3}{4}\right)$$
$$= \boxed{(3,-6)}$$

iv) As N is the midpoint of $[AM]$ it is the point that is $\frac{1}{4}$ of the way along the line, which is P.

Q4 $p(4,-3)$ $q(-4,9)$



$$A = \left(\frac{1(4) + 3(-4)}{3+1}, \frac{1(-3) + 3(9)}{3+1}\right)$$
$$= \left(\frac{4-12}{4}, \frac{-3+27}{4}\right)$$
$$= \boxed{(-2,6)}$$

Q5. $L_1: y = 4x - 1$

$L_2: y = 2x + 3$

$\Rightarrow m_1 = 4 \quad m_2 = 2$

$$\tan\theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \pm \frac{4 - 2}{1 + (4)(2)}$$

$$= \pm \frac{2}{9}$$

Acute angle $\Rightarrow \tan\theta = 2/9$

$\Rightarrow \theta = \tan^{-1} 2/9$

$= 12.53$

$= \boxed{13^\circ}$

Q6. To find slope of line joining $(3, -6)$ and $(-7, 12)$

$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{12 - (-6)}{-7 - 3}$$

$$= \frac{18}{-10}$$

$$= -9/5$$

Line 2 has slope of $-\frac{x \text{ no.}}{y \text{ no.}}$

$= -5/9$

$= 5/9$

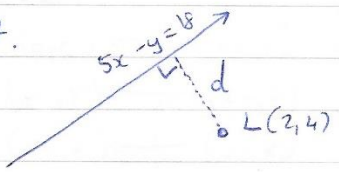
If $\perp$ $m_1 \times m_2 = -1$

$$-\frac{9}{5} \times \frac{5}{9} = -1$$

$-1 = -1$

$\Rightarrow \perp$ Q.E.D.

Q7.



$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|5(2) + 1(4) - 18|}{\sqrt{(5)^2 + (-1)^2}}$$

$$= \frac{|10 - 4 - 18|}{\sqrt{26}}$$

$$= \frac{|-12|}{\sqrt{26}} = \frac{12}{\sqrt{26}} = \boxed{2.4}$$

Q8. To find pt. of intersection:

A: $2x - 3y = -1$

B: $x + 2y = 3$

A×2: $4x - 6y = -2$

B×3: $3x + 6y = 9$

$7x = 7$

Sub x into B

$\boxed{x = 1} \Rightarrow \boxed{y = 1}$

i) Eqn of L ← PT. ON LINE $(2, -1)$, SLOPE ?

To find slope

$(x_1, y_1) = (2, -1)$

$(x_2, y_2) = (1, 1)$

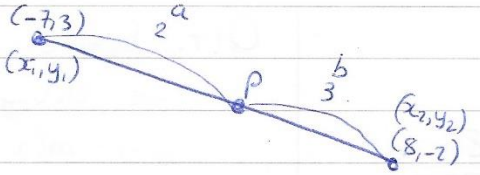
$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{1 - 2} = -2$$

$\Rightarrow$ Eqn: $y - (-1) = -2(x - 2)$

$y + 1 = -2x + 4$

$\boxed{2x + y - 3 = 0}$

Q12.



$$P = \left(\frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a} \right)$$

$$= \left(\frac{3(-7) + 2(8)}{2+3}, \frac{3(3) + 2(-2)}{3+2} \right)$$

$$= \left(\frac{-21+16}{5}, \frac{9-4}{5} \right)$$

$$= \boxed{(-1, 1)}$$

Q13. i) $x + 2y = -4$

$2y = -x - 4$

$y = -\frac{1}{2}x - 2 \Rightarrow$ L

ii) $2x - y = -4$

$y = 2x + 4 \Rightarrow$ m

iii) $x + 2y = 8$

$2y = -x + 8$

$y = -\frac{1}{2}x + 4 \Rightarrow$ j

iv) $2x - y = 2$

$y = 2x - 2 \Rightarrow$ n

Q14. Slope of $L = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3 - (-1)}{0 - 5}$

$= -4/5$

Slope of $K = \frac{-x \text{ no.}}{y \text{ no.}} = -25/k$

$\Rightarrow -25/k = 5/4$

$\Rightarrow 5k = -100$

$\Rightarrow \boxed{k = -20}$

Q15. Equations of any line through (4,3) is:

$y - 3 = m(x - 4)$

$y - 3 = mx - 4m$

$mx - y + 3 - 4m = 0$ *

Slope of $6x + y - 5 = 0$ given by: $\frac{-x \text{ no}}{y \text{ no}} = -6/1 = -6$

Angle between lines = 45°

$\Rightarrow \left| \frac{m_1 - (-6)}{1 + (m_1 \times -6)} \right| = \text{Tan } 45$

$\Rightarrow 1 = \left| \frac{m_1 + 6}{1 - 6m_1} \right|$

$\Rightarrow 1(1 - 6m_1) = m_1 + 6$

$1 - 6m_1 = m_1 + 6$

$-5 = 7m_1$

$\Rightarrow m_1 = -5/7$

OR

$-1(1 - 6m_1) = m_1 + 6$

$-1 + 6m_1 = m_1 + 6$

$5m_1 = 7$

$m_1 = 7/5$

Put two m values into *

$\frac{-5}{7}x - y + 3 - 4(-5/7) = 0$

$-5x - 7y + 21 + 20 = 0$

$\boxed{5x + 7y - 41 = 0}$

$\frac{7}{5}x - y + 3 - 4(\frac{7}{5}) = 0$

$7x - 5y + 15 - 28 = 0$

$\boxed{7x - 5y - 13 = 0}$

Q16. Distance from $(-2,a)$ to both lines is the same

$$\left|\frac{4(-2)+3(a)-3}{\sqrt{(4)^2+(3)^2}}\right| = \left|\frac{12(-2)+5(a)-13}{\sqrt{(12)^2+(5)^2}}\right|$$

$$\left|\frac{-11+3a}{5}\right| = \left|\frac{5a-37}{13}\right|$$

Square both sides:

$$\left(\frac{-11+3a}{5}\right)^2 = \left(\frac{5a-37}{13}\right)^2$$

$$\frac{9a^2-66a+121}{25} = \frac{25a^2-370a+1369}{169}$$

$$1521a^2-11154a+20449 = 625a^2-9250a+34225$$

$$896a^2-1904a-13776=0$$

$$224a^2-476a-3444=0$$

$$32a^2-68a-492=0$$

$$8a^2-17a-123=0$$

$$a = \frac{17\pm\sqrt{(-17)^2-4(8)(-123)}}{2(8)}$$

$$= \frac{17\pm\sqrt{289+3936}}{16}$$

$$= \frac{17\pm 65}{16}$$

$$= \boxed{-3 \text{ or } 5.125}$$

Q17. Equation of any line through $(4,1)$ is:

$$y-1 = m(x-4)$$

$$y-1 = mx-4m$$

$$mx-y+1-4m=0 \quad *$$

Distance to $(1,2) = 2\sqrt{2}$

$$\frac{|m(1)-(2)+1-4m|}{\sqrt{(m)^2+(-1)^2}} = 2\sqrt{2}$$

$$\frac{|-3m-1|}{\sqrt{m^2+1}} = 2\sqrt{2}$$

$$|-3m-1| = 2\sqrt{2}\sqrt{m^2+1}$$

$$|-3m-1| = 2\sqrt{2m^2+2}$$

Square both sides:

$$(-3m-1)^2 = \left(2\sqrt{2m^2+2}\right)^2$$

$$9m^2+6m+1 = 4(2m^2+2)$$

$$9m^2+6m+1 = 8m^2+8$$

$$m^2+6m-7=0$$

$$(m+7)(m-1)=0$$

$$m=-7 \quad m=1$$

Put m values into *:

$m=-7$

$$-7x-y+1-4(-7)=0$$

$$-7x-y+29=0$$

$$\boxed{7x+y-29=0}$$

$m=1$

$$1x-y+1-4(1)=0$$

$$\boxed{x-y-3=0}$$

Q18. $3x + 2y = c$

Cuts x-axis ($y=0$)

$3x + 2(0) = c$

$3x = c$

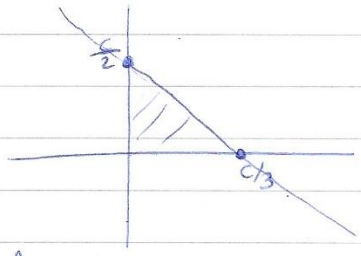
$x = c/3$

Cuts y-axis ($x=0$)

$3(0) + 2y = c$

$2y = c$

$y = c/2$



Area = 24

$\Rightarrow \frac{1}{2}bh = 24$

$\frac{1}{2}\left(\frac{c}{3}\right)\left(\frac{c}{2}\right) = 24$

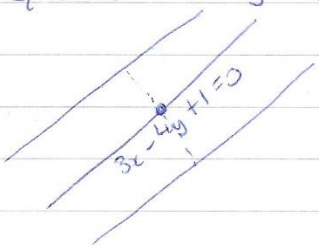
$\frac{c^2}{12} = 24$

$c^2 = 288$

$c = \sqrt{288}$

$\Rightarrow c = \boxed{12\sqrt{2}}$

Q19. Lines parallel to $3x - 4y + 1 = 0$ have equation $3x - 4y + d = 0$ *



To find a point on $3x - 4y + 1 = 0$

Let $x = 1$

$\Rightarrow 3(1) - 4y + 1 = 0$

$-4y = -4$

$y = 1$

$\Rightarrow (1,1)$ is on $3x - 4y + 1 = 0$

Dist from $(1,1)$ to parallel lines is 2

$\Rightarrow \frac{|3(1) - 4(1) + d|}{\sqrt{(3)^2 + (4)^2}} = 2$

$\frac{|d - 1|}{5} = 2$

$|d - 1| = 10$

$\Rightarrow d - 1 = 10$ or $d - 1 = -10$

$d = 11$ or $d = -9$

Put d values into *

$\boxed{3x - 4y + 11 = 0}$

or

$\boxed{3x - 4y - 9 = 0}$