Past Exam Questions: The Line

Week 20 revision

(a) $[AB]$ is a line segment.
The point $C\ (6, 11)$ divides the line segment $[AB]$ internally in the ratio $1:3$.

A is the point $(1, 13)$.
Find the co-ordinates of the point B.

(b) Find the perpendicular distance from the point $(5, -2)$ to the line:

$$y = \frac{4}{3}x - 11$$

(a) Find the area of the triangle with vertices $(4, 6)$, $(-3, -1)$, and $(0, 11)$.

(a) The points $A\ (8, -4)$ and $B\ (-1, 3)$ are the endpoints of the line segment $[AB]$.
Find the coordinates of the point C, which divides $[AB]$ internally in the ratio $4:1$.

(b) $A(-1, k)$ and $B(5, l)$ are two points, where $k, l \in \mathbb{Q}$.

(i) Show that the midpoint of $[AB]$ is $\left(2, \frac{k+l}{2}\right)$.

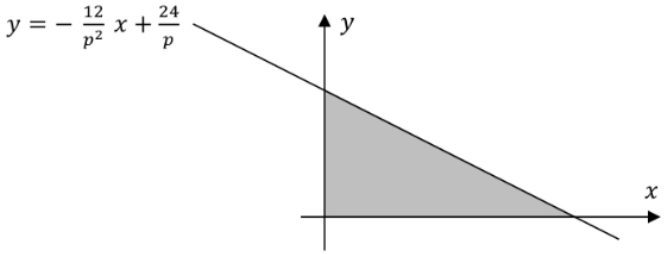
(ii) The perpendicular bisector of $[AB]$ is:

$$3x + 2y - 14 = 0$$

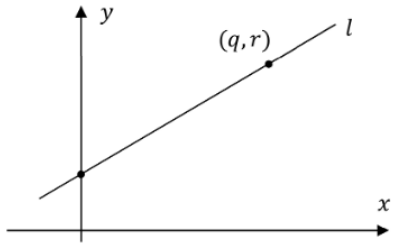
Find the value of l and the value of k.

(ii) The **area of the triangle** formed by the x-axis, the y-axis, and the tangent $y = -\frac{12}{p^2}x + \frac{24}{p}$ is always k square units, where $k \in \mathbb{N}$ is a constant.

Work out the value of k.



(b) The line l has a slope of m and contains the point (q, r), where $m, q, r \in \mathbb{R}$ are all positive.
Find the co-ordinates of the point where l cuts the y-axis, in terms of m, q, and r.



(a) The line $3x - 6y + 2 = 0$ contains the point $\left(k, \frac{2k+2}{3}\right)$, where $k \in \mathbb{R}$.
Find the value of k.

(c) The line k has a slope of -2.
The line j makes an angle of $30°$ with k.

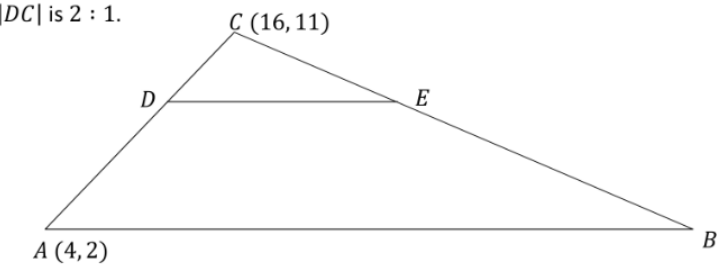
Find **one** possible value of the slope of the line j.
Give your answer in the form $d + e\sqrt{f}$, where $d, e, f \in \mathbb{Z}$.

(a) The coordinates of three points are $A(2, -6)$, $B(6, -12)$, and $C(-4, 3)$.
Find the perpendicular distance from A to BC.

Based on your answer, what can you conclude about the relationship between the points A, B, and C?

(b) The point $P(s,t)$ is on the line $x - 2y - 8 = 0$.
The point P is also a distance of 1 unit from the line $4x + 3y + 6 = 0$.
Find a value of s and the corresponding value of t.

(c) The points $A(4,2)$ and $C(16,11)$ are vertices of the triangle ABC shown below.
D and E are points on $[CA]$ and $[CB]$ respectively.
The ratio $|AD| : |DC|$ is $2 : 1$.

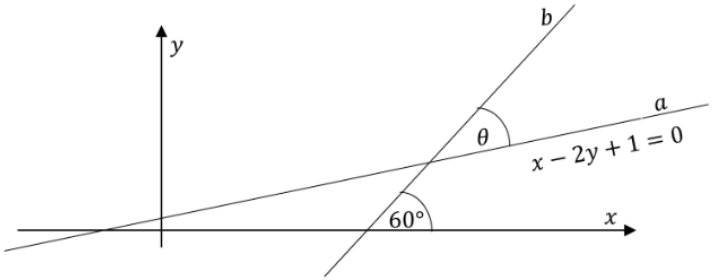


(i) Find $|AD|$.

(ii) $[AB]$ and $[DE]$ are **horizontal** line segments.
$|AB| = 33$ units.
Find the coordinates of B and of E.

$B($, $)$ $E($, $)$

(b) The diagram below shows two lines a and b. The equation of a is $x - 2y + 1 = 0$.
The acute angle between a and b is θ. Line b makes an angle of $60°$ with the positive sense of the x-axis, as shown in the diagram.
Find the value of θ, in degrees, correct to 3 decimal places.



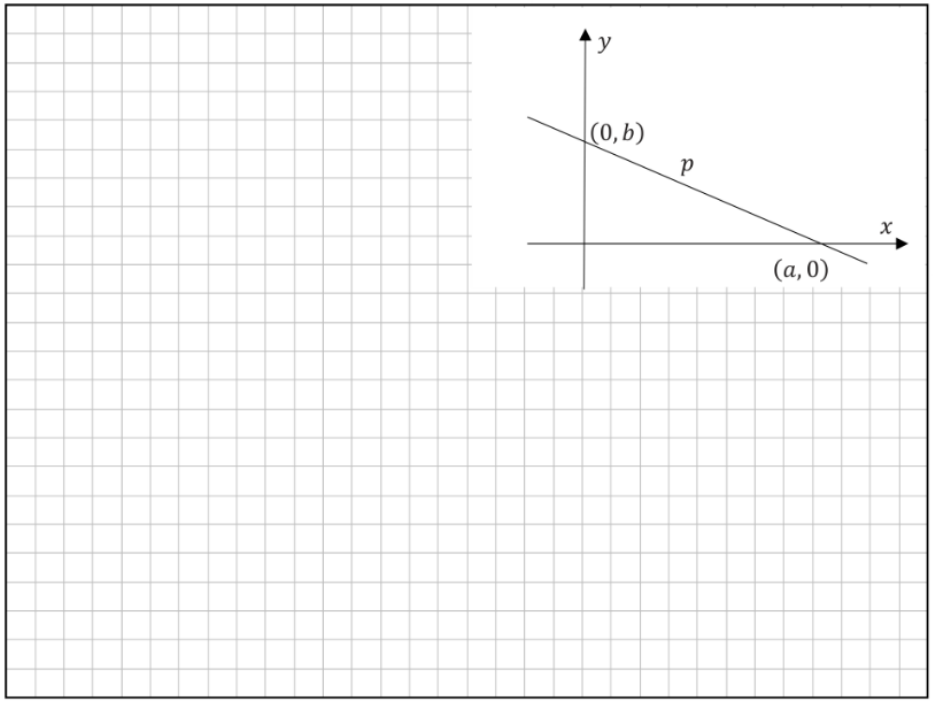
(b) The line l has a slope m, and contains the point $A(6, 0)$.

(i) Write the equation of the line l in terms of m.

(ii) The line l cuts the line k: $4x + 3y = 25$ at P.
Find the co-ordinates of P in terms of m.
Give each co-ordinate as a fraction in its simplest form.

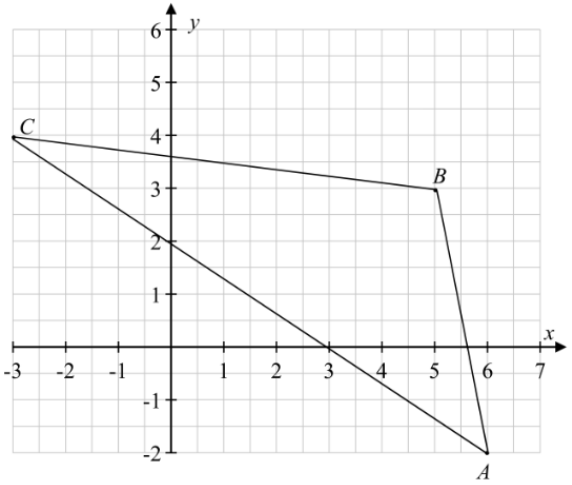
(a) The line p makes an intercept on the x-axis at $(a, 0)$ and on the y-axis at $(0, b)$, where $a, b \neq 0$.

Show that the equation of p can be written as $\frac{x}{a} + \frac{y}{b} = 1$.



The points $A(6, -2)$, $B(5, 3)$ and $C(-3, 4)$ are shown on the diagram.

(a) Find the equation of the line through B which is perpendicular to AC.

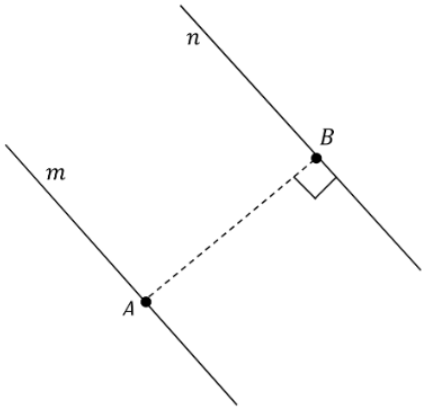


(b) Use your answer to part **(a)** above to find the co-ordinates of the orthocentre of the triangle ABC.

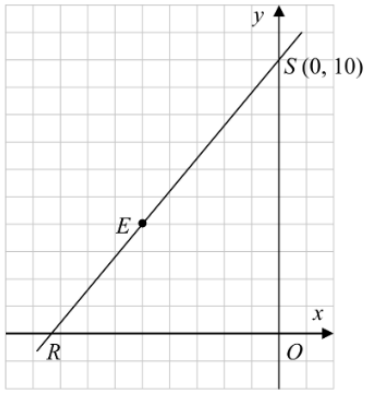
The line $m: 2x + 3y + 1 = 0$ is parallel to the line $n: 2x + 3y - 51 = 0$.

(a) Verify that $A(-2, 1)$ is on m.

(b) Find the coordinates of B, the point on the line n closest to A, as shown below.



The line RS cuts the x-axis at the point R and the y-axis at the point $S(0, 10)$, as shown. The area of the triangle ROS, where O is the origin, is $\frac{125}{3}$.



(a) Find the co-ordinates of R.

(b) Show that the point $E(-5, 4)$ is on the line RS.

(c) A second line $y = mx + c$, where m and c are positive constants, passes through the point E and again makes a triangle of area $\frac{125}{3}$ with the axes. Find the value of m and the value of c.

(a) The co-ordinates of two points are $A(4, -1)$ and $B(7, t)$.

The line $l_1 : 3x - 4y - 12 = 0$ is perpendicular to AB. Find the value of t.

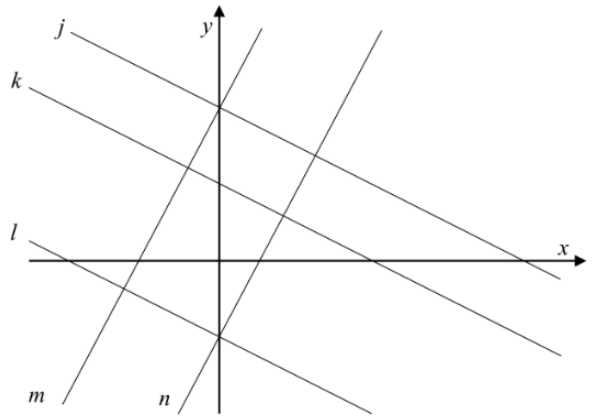
(b) Find, in terms of k, the distance between the point $P(10, k)$ and l_1.

(c) $P(10, k)$ is on a bisector of the angles between the lines l_1 and $l_2 : 5x + 12y - 20 = 0$.

(i) Find the possible values of k.

(ii) If $k > 0$, find the distance from P to l_1.

In the co-ordinate diagram shown, the lines j, k, and l are parallel, and so are the lines m and n.
The equations of four of the five lines are given in the table below.



Equation	Line
$x + 2y = -4$	
$2x - y = -4$	
$x + 2y = 8$	
$2x - y = 2$	

(a) Complete the table, by matching four of the lines to their equations.

(b) Hence, insert scales on the x-axis and y-axis.

(c) Hence, find the equation of the remaining line, given that its x-intercept and y-intercept are both integers.

The equations of six lines are given:

Line	Equation
h	$x = 3 - y$
i	$2x - 4y = 3$
k	$y = -\frac{1}{4}(2x - 7)$
l	$4x - 2y - 5 = 0$
m	$x + \sqrt{3}y - 10 = 0$
n	$\sqrt{3}x + y - 10 = 0$

(a) Complete the table below by matching each description given to one or more of the lines.

Description	**Line(s)**
A line with a slope of 2.	
A line which intersects the y-axis at $(0, -2\frac{1}{2})$.	
A line which makes equal intercepts on the axes.	
A line which makes an angle of $150°$ with the positive sense of the x-axis.	
Two lines which are perpendicular to each other.	

(b) Find the acute angle between the lines m and n.