

Revision Sheet 1: Algebra (Chaps 1 → 3)

6TH YR H.L.

Q1. Simplify $x - (1-x)^2$ $\boxed{3x - x^2 - 1}$ Q2. Simplify $\frac{x^2 - 3x}{x^2 - 2x - 3}$ $\boxed{\frac{x}{x+1}}$

Q3. Simplify $\frac{1}{a-b} - \frac{1}{b-a}$ $\boxed{\frac{2}{a-b}}$ Q4. Simplify $\frac{(a^2-b^2)^4(a^4-b^4)}{(a^2+b^2)(a+b)^5}$ $\boxed{(a-b)^5}$

Q5. Simplify $(\sqrt{x} + \frac{1}{\sqrt{x}} + 1)(\sqrt{x} + \frac{1}{\sqrt{x}} - 1)$ $\boxed{x + \frac{1}{x} + 1}$

Q6. If $p/2 = \sqrt{\frac{1}{x^2-1}}$, express x^2 in terms of p . $\boxed{\frac{p^2+4}{p^2}}$

Q7. If $x = \sqrt{a} + \frac{1}{\sqrt{a}}$ and $y = \sqrt{a} - \frac{1}{\sqrt{a}}$, find the value of $\sqrt{x^2 - y^2}$. $\boxed{2}$

Q8. Solve $x^3 + 5x^2 - 4x - 20 = 0$ $\boxed{x = 2, -2, -5}$

Q9. Find the real numbers a and b such that $x^2 + 4x - 6 = (x+a)^2 + b$ $\boxed{a=2 \quad b=-10}$

Q10. If $f(x) = 3x^3 + mx^2 - 17x + n$ and $x-3$ and $x+2$ are factors of $f(x)$, find the value of m and n . $\boxed{m=-4 \quad n=6}$

Q11. Solve $2x^3 - 3x^2 - 8x - 3 = 0$ and sketch the curve. $\boxed{x = -1, 3, -\frac{1}{2}}$

Q12. If $x^2 - t$ is a factor of $x^3 - px^2 - qx + r$, show that
i) $pq = r$ and ii) express the roots of $x^3 - px^2 - qx + r = 0$ in terms of p and q . $\boxed{x = \pm\sqrt{q}, p}$

Q13. If $2x^2 + 5x + 6 = p(x+q)^2 + r$, find the values of p, q and r $\boxed{p=2 \quad q=5/4 \quad r=23/8}$

Q14. If $x^2 + bx + c$ is a factor of $x^3 - p$, show that i) $b^3 = p$
ii) $c^3 = p^2$.

Q15. $(x-1)^2$ is a factor of $ax^3 + bx^2 + 1 = 0$. Find a, b . $\boxed{a=2 \quad b=-3}$

Q16. Solve A: $3x - y + 3z = 1$ B: $x + 2y - 2z = -1$ and C: $4x - y + 5z = 4$
 $x = -1, y = 2, z = 2$

Q17. Solve the equations $2x - 3y = 1$ and $x^2 + xy - 4y^2 = 2$ $(2, 1)$
 $(11, 7)$

Q18. Form the quadratic equation whose roots are $-\frac{2}{3}$ and 4.
 $3x^2 - 10x - 8 = 0$

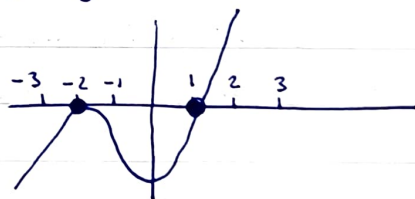
Q19. Given that $\frac{3}{5}$ is a root of $5x^3 + 7x^2 - kx + 3 = 0$, find the value of k . Find the other 2 roots in the form $a \pm \sqrt{b}$.
 $k = 11 \quad -1 \pm \sqrt{2}$

Q20. i) Factorise $x^3 + 1000$ ii) Prove $x^3 + 1000 = 0$ has only 1 real root.

Q21. Find the cubic equation with roots $-\frac{1}{2}, 2, 3$. $2x^3 - 9x^2 + 7x + 6 = 0$

Q22. The diagram shows the graph of $y = x^3 + ax^2 + bx + c$. Find the values of a, b and c .

$a = 3 \quad b = 0 \quad c = -4$



Q23. i) Solve $4x^2 - 23x + 15 = 0$ $\frac{3}{4}, 5$

ii) Hence solve $4(y^2 + y)^2 - 23(y^2 + y) + 15 = 0$ $\frac{1}{2}, -\frac{3}{2}, \frac{-1 \pm \sqrt{21}}{2}$

Q24. Simplify $\frac{8x^3 + 27}{4x^2 - 9}$ $\frac{4x^2 - 6x + 9}{2x - 3}$

Q25. Write $\frac{2 - \sqrt{3}}{2 + \sqrt{3}}$ in the form $a + b\sqrt{3}$. $7 - 4\sqrt{3}$

Q26. Solve $1 + \sqrt{2x - 1} = x - 1$ and check your answers. $x = 5$

Q27. Write the following as one fraction in the form $\frac{k}{x+3}$
 $\frac{4}{x-2} + \frac{5}{x+3} - \frac{20}{x^2 + x - 6}$ $\frac{9}{x+3}$

Revision Sheet 2: Algebra (Chaps 4 → 6)

6th YR H.L.

Q1. Show that $\frac{3}{1+x^p} + \frac{3}{1+x^{-p}}$ reduces to a constant.

Q2. Express $2^{1/4} + 2^{1/4} + 2^{1/4} + 2^{1/4}$ in the form $2^{p/4}$. $\boxed{2^{9/4}}$

Q3. Simplify $3^{\wedge} + 3^{\wedge} + 3^{\wedge}$. Q4. Solve $|2x-1| < 3$. $\boxed{-1 < x < 2}$

Q5. Solve $2x^2 + 5x - 3 > 0$ $\boxed{x < -3, x > 1/2}$

Q6. Solve for x $\frac{5-x}{x-2} < 1$ $\boxed{x < 2, x > 7/2}$

Q7. Solve for y $2^{2y+1} - 5(2^y) + 2 = 0$ $\boxed{y = \pm 1}$

Q8. Show for all real nos $a, b > 0$ that $a+b \geq 2\sqrt{ab}$

Q9. Solve $\log_2(7x+2) - \log_2(x+2) = 2$. $\boxed{x=2}$

Q10. Solve the simultaneous eqns $(2^x)(2^y) = 8$ and $\frac{3^{2x}}{3^y} = 1$
 $\boxed{x=1, y=2}$

Q11. Solve for x : $6\log_x 2 + \log_2 x = 5$. $\boxed{x=4, 8}$

Q12. If $3^x = 1000$, find the value of x . $\boxed{6.29}$

Q13. If $\log_6(x+1) + 2\log_6 y = \log_6(x-1)$, write y in terms of x and hence find the value of y if $x = 5/3$.
 $\boxed{y = \sqrt{\frac{x-1}{x+1}}}$
 $\boxed{y = 1/2}$

Q14. Solve for x : $2^x + 2^{-x} = 2$. $\boxed{x=0}$

Q15. Show that $\frac{a+b}{2} \leq \sqrt{\frac{a^2+b^2}{2}}$ where a and b are real.

Q16. If $U_n = (2n+1)3^n$, show that $U_{n+2} - 6U_{n+1} + 9U_n = 0$

Q17. If $U_n = n(4^n)$, show that $U_{n+2} - 8U_{n+1} + 16U_n = 0$

Q18. Solve $e^{2x} = 3$. $\boxed{\ln 3}$ Q19. Solve $e^{2x} - 3e^x - 4 = 0$ $\boxed{\ln 4}$

Q20. Solve $3e^x - 7 + 2e^{-x} = 0$. $\boxed{\ln 2, -\ln 3}$

Q21. i) If $2^x + 2^{x+1} + 2^{x+2} = k \cdot 2^x$, find the value of k . $\boxed{k=7}$
ii) Hence solve $2^x + 2^{x+1} + 2^{x+2} = 224$. $\boxed{x=5}$

Q22. A rectangle has length x cm and width $(25-x)$ cm. Given that the area of it must be at least 150 cm^2 , find the set of possible values of x . $\boxed{10 \leq x \leq 15}$

Algebra Worksheet:

L.C. Higher Level

Q1. Simplify $\frac{4+\sqrt{3}}{2+\sqrt{3}}$. $\boxed{5-2\sqrt{3}}$

Q2. $2x - t$ is a factor of $2x^3 + px + p$. Show that $p = \frac{-t^3}{2(t+2)}$

Q3. Show that $z^2 - 4z + 5$ is a factor of $z^3 + (i-4)z^2 + (5-4i)z + 5i$. Hence, find the three roots of $z^3 + (i-4)z^2 + (5-4i)z + 5i = 0$. $\boxed{z = 2 \pm i, -i}$

Q4. Solve for x : $\sqrt{2x+5} - x = 1$. $\boxed{x=2}$

Q5. i) Let $f(x) = 3tx^2 - 6x + (t-2)$, where $t \in \mathbb{R}$. Find the values of t for which $f(x) = 0$ has equal roots. $\boxed{t = -1, 3}$ Hence, for each value of t , find the roots of $f(x) = 0$. $\boxed{x = \frac{1}{3}}$ $\boxed{x = -1}$

ii) Find the range of values of x for which $\left| \frac{x-3}{x+4} \right| > 2$, where $x \in \mathbb{R}$ and $x \neq -4$. $\boxed{-11 < x < -\frac{5}{3}}$

Q6. Solve the simultaneous equations $x + 3z = -1$, $y - 3z = 6$, $x + y + z = 4$. $\boxed{\begin{matrix} x=2 \\ y=3 \\ z=-1 \end{matrix}}$

Q7. Let $f(x) = 2x^2 - ax + 5a$. Find the range of values of a for which $f(x) = 0$ has real roots. $\boxed{a \leq 0 \text{ or } a \geq 40}$

Q8. Solve for x and y . $\log(y-x) = 0$
 $2\log y = \log(21+x)$ $\boxed{(4, 5)}$

Q9. Solve the equation: $16 + 4^x = 5(2^{x+1})$ $\boxed{x=1, 3}$

Q10. Solve the equation $x = 2 - \frac{1}{x}$ $x = 1$

Q11. Solve for x and y : $(2^x)(2^y) = 8$
 $\frac{25^x}{5^y} = 5$ $x = \frac{4}{3}$ $y = \frac{5}{3}$

Q12. $xc^2 + bx + c$ is a factor of $x^3 + px^2 + qx + r$
Show that $q - c = b(p - b)$ and $br = c(q - c)$.

Q13. i) Solve for x : $|3x + 4| - |x + 2| \leq 0$ $-\frac{3}{2} \leq x \leq -1$

ii) Given that $f(x) = (2k)^x$ and $f(x+3) = 125f(x)$
find the value of k . $k = \frac{5}{2}$

Q14. Solve, without using a calculator, the following simultaneous equations: $2x + y = 3$ $3x^2 + 2xy = 9$
 $x = 3$ $y = -3$

Q15. $(x-1)$ is a factor of $f(x) = x^3 + ax^2 - (a+1)x + 12 = 0$
and $a > 0$. i) Find the value of a . $a = 3$
ii) Hence, find the two other factors of $f(x)$.
 $(x-2)$ and $(x+6)$

Q16. Solve $8x^2 - 2x - 15 = 0$ and hence, find the values of y for which $8y - 2\sqrt{y} - 15 = 0$.
 $x = \frac{3}{2}, -\frac{5}{4}$ $y = \frac{9}{4}, \frac{25}{16}$

5TH YR Higher Level

• Topics on Test:

- 1) Sim Eqns
 $\swarrow$ x, y, z
 $\searrow$ linear + quadratic
 $\swarrow$ normal ones with fractions
- 2) Logs - incl eqns with logs + word problems
- 3) Indices incl eqns with e
- 4) Recurrence Eqns - $U_{n+2} - 5U_{n+1}$ e.t.c.
- 5) Factorising - incl simplifying fractions using factors
- 6) Surds
- 7) Completing the Square
- 8) Undetermined Coefficients

Revision Questions:

Q1. If $U_n = 5^n - 14(2^n)$, show that $U_{n+2} - 7U_{n+1} + 10U_n = 0$

Q2. Solve the equation $2^{2x+1} - 15(2^x) - 8 = 0$ $x = 3$

Q3. Evaluate $2\log_2 2 + \log_2 12 - \log_2 6$ 3

Q4. Solve for x: $\log_2 x + 4\log_x 2 = 5$ $x = 2, 16$

Q5. Solve the Simultaneous Equations:

$$2\log x = \log(x+y) \quad ; \quad \log y = \log 2 + \log(x-1)$$
 $(1, 0) \text{ or } (2, 2)$

Q6. Solve for n: $3^{2n-1} = 4800$ $n = 4.358$

Q7. Solve for x: $3e^x - 7 + 2e^{-x} = 0$ $x = \ln 2, -\ln 3$

Q8. The amount A in an account after t years with initial investment P at a rate r is given by $A = P(e^{rt})$. How many yrs will it take an investment of €1000 to amount to €1700 at a rate of 4.5%? 12 yrs

Q9. Solve for x and y : $2x - y + 1 = 0$; $y^2 = 2x^2 + x$ $(-\frac{1}{2}, 0)$
 $(-1, -1)$

Q10. Solve for x, y and z

$$\begin{aligned} x + 2y + z &= 3 \\ 5x - 3y + 2z &= 19 \\ 3x + 2y - 3z &= -5 \end{aligned}$$

$x=2, y=-1, z=3$

Q11. Show that $\frac{-1+\sqrt{3}}{1+\sqrt{3}} = 2-\sqrt{3}$.

Q12. If $x = \sqrt{a} + \frac{1}{\sqrt{a}}$ and $y = \sqrt{a} - \frac{1}{\sqrt{a}}$ find the value of $\sqrt{x^2 - y^2}$. ANS: 2 * HARD ONE!

Q13. If $(2x+p)(x^2+qx-5) = 2x^3 + 5x^2 - 13x + 5$, find value of p and q . $p=-1, q=3$

Q14. If $(ax+k)(x^2-px+1) = ax^3+bx+c$ show that $c^2 = a(a-b)$

Q15. Solve the Sim Eqns

$$\frac{x+1}{2} - \frac{y+3}{3} = 4 \quad ; \quad x + \frac{y-3}{2} = \frac{1}{2}$$

$(5, -6)$

Q16. Simplify $\frac{4x^2-10x}{9x^2+6x} \div \frac{2x-5}{3x+2}$

$\frac{2}{3}$

Q17. Simplify $\frac{1 - \frac{3}{x}}{x - \frac{9}{x}}$

$\frac{1}{x+3}$

Q18. Simplify $\frac{3x-5}{x-2} + \frac{1}{2-x}$

3

Worksheet 1: Algebra Chaps 1 → 3

Q1. Simplify $\frac{8x^3+27}{4x^2-9}$ $\frac{4x^2-6x+9}{2x-3}$

Q2. Solve $\sqrt{x+6} = x-6$. $x=10$

Q3. Show that $\frac{3(x-4)}{x-3} - \frac{x}{3-x}$ reduces to a constant.

Q4. Simplify $(x^3 + \sqrt{8} + \frac{4}{x^3})(x^3 - \sqrt{8} + \frac{4}{x^3})$, giving your answer in the form $x^n + \frac{k}{x^n}$. $x^6 + \frac{16}{x^6}$

Q5. Express $\frac{1}{x+1} + \frac{3}{(x+1)(2x-1)}$ as a single algebraic fraction and simplify your answer $\frac{2}{2x-1}$

Q6. Simplify $x\sqrt{1 - \frac{8}{x} + \frac{16}{x^2}}$ $x-4$

Q7. If $p(x-q)^2 + r = 2x^2 - 12x + 5$, find p, q and r . $p=2$
 $q=3$
 $r=-13$

Q8. Solve the simultaneous equations $x-y+3=0$ and $3x^2+y^2=13$ $(-2, 1)$
 $(\frac{1}{2}, \frac{7}{2})$

Q9. Given that $x = -\frac{1}{2}$ and $x=1$ are both roots of the equation $ax^2+bx+1=0$, find the values of the constants a and b . $a=2$ $b=-1$

Q10. Solve the simultaneous equations $4x+y=17$ and $\frac{x-3}{4} + \frac{y}{2} = \frac{5}{2}$ $x=3$
 $y=5$ $k+1+\frac{1}{k}$

Q11. If $x = \sqrt{k} + \frac{1}{\sqrt{k}} + 1$, express $x^2 - 2x$ in terms of k .

Q12. Solve the simultaneous equations $3x+5y-z=-3$, $2x+y-3z=-9$, $x+3y+2z=7$ $x=2$ $y=-1$ $z=4$

Q13. Solve the equation $x^2 - 6x + 5 = 0$. Hence solve fully the equation $(t - \frac{6}{t})^2 - 6(t - \frac{6}{t}) + 5 = 0$ $t = -2, -1, 3, 6$

Q14. If $y = \frac{3x+2}{5-x}$, express x in terms of y . $x = \frac{5y-2}{y+3}$

Q15. Express $\frac{x+y^2}{x^2} + \frac{x-1}{x} - 1$ as a single fraction in its simplest form. $\rightarrow$ $\frac{y^2}{x^2}$

Q16. Evaluate $(2\sqrt{5} + \sqrt{7})(2\sqrt{5} - \sqrt{7})$, without a calculator. 13

Q17. Find the values of p and q if $3x+5 = p(x+1) + q(x-1)$ $p=4 \quad q=-1$

Q18. Show that $x=2$ is a root of $x^3 + 7x^2 - 4x - 28 = 0$ and hence find the other two roots. $-2, -7$ Sketch! the curve

Q19. If $(x-a)^2$ is a factor of $x^3 + 4px + 8q$, show that
i) $p = -\frac{3a^2}{4}$ and ii) $q = \frac{a^3}{4}$

Q20. An airplane flies 520 km against the wind and 680 km with the wind, in a total time of 4 hours. The speed of the plane in still air is 300 km/h. Calculate the speed of the wind. 40 km/h

Q21. The length of a rectangle is 8 cm less than twice its width. If the area of the rectangle is 120 cm^2 , what is the length of the diagonal, leaving your answer in surd form? $2\sqrt{61}$

Q22. Show that $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 5+2\sqrt{6}$.

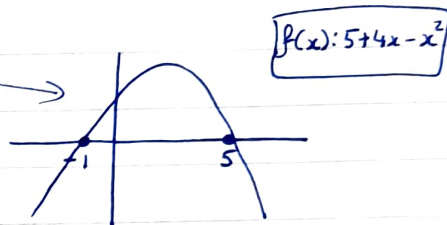
Q23. Express $\frac{1-\sqrt{2}}{1+\sqrt{2}}$ in the form $a\sqrt{2} - b$. $2\sqrt{2} - 3$

Q24. $17 \left(1 - \frac{1}{17^2}\right)^{\frac{1}{2}} = n\sqrt{2}$. Find the value of n . $n=12$

Q25. Express $x^2 + 8x + 14$ in the form $(x+b)^2 + c$ and hence find the minimum value of the function. -2

Q26. Construct a quadratic equation that has roots of $-\frac{1}{3}$ and $\frac{5}{2}$. $6x^2 - 13x - 5 = 0$

Q27. Express the following function in the form $ax^2 + bx + c = f(x)$



Q28. If $px^3 + 8x^2 + qx + 6$ is exactly divisible by $x^2 - 2x - 3$, find the value of p and the value of q . $p = -5, q = 19$

Q29. Given that $x+3$ and $x-1$ are both factors of the expression $x^3 + ax^2 - bx - a$, calculate the values of the constants a and b . Hence find the third factor of the expression. $a = 3, b = 1$ Factor: $(x+1)$

Q30. If $x^2 + ax + b$ is a factor of $x^3 + cx + d$, prove that
i) $d + ab = 0$ ii) $a^2 = b - c$

Q31. A spacecraft is travelling along a line $2x + y + 6 = 0$. Ahead lies a large asteroid field in the shape of an ellipse represented by the equation $4x^2 + 9y^2 = 36$. It is not safe to travel through an asteroid field. Should the spacecraft alter its course? Justify your answer. $(-2.4, -1.2), (-3, 0)$

Q32. Draw a rough sketch of the graph $f(x) = 2x^3 - 4x^2 - 22x + 24$, showing where it crosses the x -axis. $-3, 1, 4$

Worksheet 2: Algebra (Chaps 4 → 6)

- Q1. Solve the following equations:
- i) $5^x = \frac{1}{5\sqrt{5}}$ $\boxed{x = -\frac{3}{2}}$
 - ii) $(x^{\frac{1}{3}} + 1)^{\frac{1}{2}} = 2$ $\boxed{x = 27}$
 - iii) $2^{2x+2} - 33(2^x) + 8 = 0$ $\boxed{x = -2, 3}$
 - iv) $\log_4(6x+1) - 2 = 2\log_4 x$ $\boxed{x = \frac{1}{2}}$
 - v) $8(2^{x^2}) = 16^x$ $\boxed{x = 1, 3}$
 - vi) $\frac{4x-1}{x-3} < 2, x \in \mathbb{R}$ $\boxed{-2.5 < x < 3}$
 - vii) $\log_2(x+2) + \log_2(x-2) = 5$ $\boxed{x = 6}$
 - viii) $3^{2x} - 3^{x+1} - 3^x + 3 = 0$ $\boxed{x = 0, 1}$
 - ix) $|1+3x| = 1$ $\boxed{x = -\frac{2}{3}, 0}$
 - x) $2^x + 2^{-x} = 2$ $\boxed{x = 0}$
 - xi) $\log_2(3x-1) = 3$ $\boxed{x = 3}$
 - xii) $3^x = 1000$ $\boxed{x = 6.29}$
 - xiii) $6\log_x 2 + \log_2 x = 5$ $\boxed{x = 4, 8}$
 - xiv) $|x+1| \leq 4, x \in \mathbb{R}$ $\boxed{-5 \leq x \leq 3}$
 - xv) $2^x + 2^{2-x} = 4$ $\boxed{x = 1}$

Q2. Simplify $\frac{x^{\frac{3}{2}} - x^{-\frac{1}{2}}}{x^{\frac{1}{2}} - x^{-\frac{1}{2}}}$ $\boxed{x+1}$

- Q3. Find the range of values of k for which the quadratic equation $x^2 + (k-4)x + (k-1) = 0$ has real roots.
- $\boxed{k \leq 2 \text{ or } k \geq 10}$

Q4. Solve the simultaneous equations:

$$2\log y = \log 2 + \log x \quad \text{and} \quad 2^y = 4^x \quad \boxed{x = \frac{1}{2}, y = 1}$$

Q5. Write $\log_x 4$ as an expression containing $\log_4 x$. $\boxed{\frac{1}{\log_4 x}}$

Q6. If for all integers n , $U_n = (n-20)2^n$, verify that $U_{n+2} - 4U_{n+1} + 4U_n = 0$.

$$a=18$$

Q7. If $4(2^{n+2}) + 2^{n+1} = a \times 2^n$, find the value of a .

Q8. Given that $3e^x - 7 + 2e^{-x} = 0$, find two values for e^x . $\boxed{\frac{1}{3}, 2}$

Hence find two values for x , expressing your answers in terms of $\log_e$. $\boxed{\log_e \frac{1}{3}, \log_e 2}$

Q9. If $U_n = 2^{n-1} - 1$, show that $U_{n+2} = 2^{n+1} - 1$.
Hence show that $U_{n+2} - 3U_{n+1} + 2U_n = 0$.

Q10. Find $x \in \mathbb{R}$ if $\left| \frac{x-3}{x+1} \right| < 2$. $\boxed{x < -5, x > \frac{1}{3}}$

Q11. Find the range of values of x for which $\frac{3^{x^2}}{3^{3x}} > \frac{1}{9}$. $\boxed{x < 1 \text{ or } x > 2}$

Q12. Solve for x $\frac{5-x}{x-2} < 1$, $x \in \mathbb{R}$. $\boxed{x < 2, x > \frac{7}{2}}$

Q13. i) Factorise $q^3 - p^3$.

ii) If p and q are distinct positive real numbers, prove that $\frac{q^3 - p^3}{pq^2 - qp^2} > 3$.

Q14. Prove that $a^3 + b^3 > a^2b + ab^2$.

Q15. Show for all real numbers $a, b > 0$ that $a+b \geq 2\sqrt{ab}$.

Q16. The amount of radioactive tracer remaining after t days is given by $A = A_0(e^{-0.058t})$, where A_0 is the starting amount at the beginning of the time period. How many days, to the nearest day, will it take for one half of the original amount to decay? $\boxed{12 \text{ days}}$