Topic 3: Patterns/Sequences

1) Arithmetic Sequences/Series:

a) Linear(Arithmetic) Sequences:

- A list of numbers where the **difference** between **each term** is the **same** every time.
  E.g. 3, 8, 13, 18, ………
- In Senior Cycle, we refer to these sequences as **Arithmetic** Sequences.
- The General Term for an Arithmetic sequence can be found using:

$$T_n = a + (n-1)d$$

See Tables pg 22

where 'a' is the first term and 'd' is the common difference between the terms.

Example: i) Find the General Term for the sequence 3, 8, 13, 18…..
ii) Find the 50[th] term, T_{50}.

a = 3 and d = 5

i) => $T_n = 3 + (n-1)5$ => $T_n = 3 + 5n - 5$ => $T_n = 5n - 2$	ii) $T_n = 5n - 2$ => $T_{50} = 5(50) - 2$ => $T_{50} = 250 - 2 = 248$

b) Arithmetic Series:

- If we add the terms of an arithmetic sequence together, then we get an arithmetic **series**.
- We need to be able to find the sum of the first n terms of such a series, which we can find using:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

See Tables pg 22

where 'a' is the first term and 'd' is the common difference between the terms of the series.

Example: Find the sum of the first 20 terms of the series 2 + 6 + 10 + 14 + …….

a = 2 and d = 4

=> $S_{20} = \frac{20}{2}\{2(2) + (20-1)4\}$

=> $S_n = 10\{4 + (19)4\}$

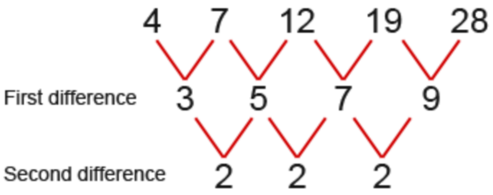
=> $S_n = 10\{80\}$

=> $S_n = 800$

2) Non-Linear Sequences:

a) Quadratic Sequences:

- A sequence where the **second difference** is the **same** every time.
  E.g. 4, 7, 12, 19, 28……. (see below)



Steps to find General Term:

1. Let General Term = $T_n = an^2 + bn + c$
2. Find 2[nd] difference and let = 2a….solve for a.
3. Use any 2 terms to form two equations in b and c.

Example: Find the General Term of the sequence 4, 7, 12, 19, 28

Step 1: Let the General Term $T_n = an^2 + bn + c$.

Step 2: Second difference = 2a = +2 => a = +1.

Step 3: Use two of the terms in the sequence to make two simultaneous equations, which we solve to find 'b' and 'c'……

$T_n = an^2 + bn + c$

$T_2 = (2)^2 + b(2) + c = 7$ => 4 + 2b + c = 7 => 2b + c = 3…..Eqn 1	$T_3 = (3)^2 + b(3) + c = 12$ => 9 + 3b + c = 12 => 3b + c = 3……Eqn 2

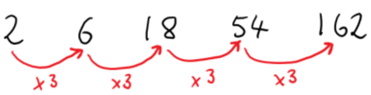
Step 4: Solving Equations 1 and 2 gives b = 0 and c = 3

=> $T_n = n^2 + (0)n + 3$

=> $T_n = n^2 + 3$

b) Exponential Sequences:

- A sequence where each term is found by multiplying the previous term by the same number every time.
  E.g. 2, 6, 18, 54, 162………



c) Cubic Sequences

- A sequence where the **third difference** is the **same** every time.
  E.g. 4, 14, 40, 88, 164……. (see below)