(c) Solve the following equation for $n \in \mathbb{R}$, where $n > 0$:

$$\log_2(8 - 4n) - 2\log_2 n = 2$$

(b) **(i)** Solve the equation $\frac{1}{y} + 2y = 3$, where $y \in \mathbb{R}, y \neq 0$.

(ii) Hence, or otherwise, find the values of $x \in \mathbb{R}$ for which:

$$4^{-x} + 2(4^x) = 3$$

(b) p is a positive constant.
Use the laws of logs to write the expression:

$$\ln[(e^3 p)^5]$$

in the form $c + d \ln p$, where $c, d \in \mathbb{Z}$ are constants.

(c) The following is an expression in x, where $x > 0, x \in \mathbb{R}$:

$$\log_{100} x + \log_{100} 18 - \log_{100} 3$$

Write this expression in the form $p \log_{10} k\,x$, where $p, k \in \mathbb{R}$.

(b) t is a positive real number, with:

$$\log_3 t + \log_9 t + \log_{27} t + \log_{81} t = 10$$

Find the value of t. Give your answer in the form 3^r, where $r \in \mathbb{Q}$.

Hint: use the formula $\log_a b = \dfrac{\log_c b}{\log_c a}$.

(c) **(i)** Explain what $\log_6 m$ means, where m is a positive real number.

(ii) m is a real number, and $m > 6$.

What information does this give about the value of $\log_6 m$?

(b) Two functions, $f(x)$ and $g(x)$, are defined as follows, for $x \in \mathbb{R}$, $x > 0$:

$$f(x) = e^{9x}$$
$$g(x) = \ln \sqrt{x}$$

Use these functions to answer parts **(b)(i)**, **(b)(ii)**, and **(b)(iii)**.

(i) Find the value of $f(1{\cdot}2)$.

Give your answer in the form $a \times 10^n$, where $a \in \mathbb{R}$, $1 \leq a < 10$, $n \in \mathbb{N}$, and a is correct to 1 decimal place.

(ii) Find the value of x for which $g(x) = 3{\cdot}5$.
Give your answer in the form e^p, where $p \in \mathbb{R}$.

The number of bacteria in the early stages of a growing colony of bacteria can be approximated using the function:

$$N(t) = 450e^{0{\cdot}065t}$$

where t is the time, measured in hours, since the colony started to grow, and $N(t)$ is the number of bacteria in the colony at time t.

(a) **(i)** Find the number of bacteria in the colony after 4·5 hours.
Give your answer correct to the nearest whole number.

(ii) Find the time, in **hours**, that it takes the colony to grow to 790 bacteria.
Give your answer correct to 1 decimal place.

(b) **(i)** Write down a pair of real values of a and b, if $\log_3 4 = \frac{\log_2 a}{\log_2 b}$.

$a =$ $b =$

(ii) Show that $(\log_2 3)(\log_3 4) = 2$.

(iii) Hence, or otherwise, solve for $n \in \mathbb{N}$, the equation

$$(\log_2 3)(\log_3 4)(\log_4 5) \ldots (\log_{n-1} n)(\log_n (n+1)) = 11.$$

Scientists can estimate the age of certain ancient items by measuring the proportion of carbon−14, relative to the total carbon content in the item. The formula used is $Q = e^{-\frac{0{\cdot}693t}{5730}}$, where Q is the proportion of carbon−14 remaining and t is the age, in years, of the item.

(a) An item is 2000 years old. Use the formula to find the proportion of carbon−14 in the item.

(b) The proportion of carbon−14 in an item found at Lough Boora, County Offaly, was 0·3402. Estimate, correct to two significant figures, the age of the item.

(b) Given $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 0$, write each of the following in terms of p and q:

(i) $\log_a \frac{8}{3}$

(ii) $\log_a \frac{9a^2}{16}$.

If we learn a skill, and then don't practise it, how well we can do it usually decreases over time. For example, if you learn to play the guitar, and then don't play the guitar for a number of months, you will probably not be as good at playing the guitar the first time you try it again.

This effect can be modelled by the following equation:

$$A = B(t+1)^c$$

where A is a measure of how well the skill can be done at a certain time ($t = 0$);
B is a measure of how well the skill can be done t months later, without practising;
c is a constant; and $A, B, c, t \in \mathbb{R}$.

(e) **(i)** Write c in terms of $\log_{10} A$, $\log_{10} B$, and $\log_{10}(t+1)$.

(ii) A student got 80% on a guitar exam.
After two **years** of not playing the guitar, the student got 47% on the same exam.

Use this to find the value of c in the model above, correct to 3 decimal places.

In a science experiment, a quantity $Q(t)$ was observed at various points in time t. Time is measured in seconds from the instant of the first observation. The table below gives the results.

t	0	1	2	3	4
$Q(t)$	2·920	2·642	2·391	2·163	1·957

Q follows a rule of the form $Q(t) = Ae^{-bt}$, where A and b are constants.

(a) Use any two of the observations from the table to find the value of A and the value of b, correct to three decimal places.

(b) Use a different observation from the table to verify your values for A and b.