Applied Maths Exam Papers

Leaving Certificate Examination

Sample Paper 1

Applied Mathematics

Higher Level
2 hours and 30 minutes

400 marks

Examination Number

For examiner	
Question	Mark
1	/50
2	/50
3	/50
4	/50
5	/50
6	/50
7	/50
8	/50
~~9~~	~~/50~~
~~10~~	~~/50~~
Written Total	/400
Project	/100
Overall Total	/500
Overall Grade	

Sample Paper 1

Question 1

(a)

A car of mass 1200 kg starts from rest and travels along a straight horizontal road. The engine of the car exerts a constant power of 3000 W.

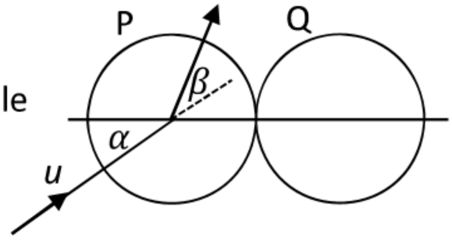
If there is no resistance to the motion of the car, find

(i) the speed of the car after 3 minutes

(ii) the average speed of the car during this time.

(b)

A smooth sphere P has mass m and speed u. It collides obliquely with a smooth sphere Q, of mass m, which is at rest. Before the collision, the direction of P makes an angle α with the line of centres, as shown in the diagram.



The coefficient of restitution between the spheres is $\frac{1}{3}$.

During the impact the direction of motion of P is turned through an angle β.

Show that $\tan\beta = \frac{2\tan\alpha}{1+3\tan^2\alpha}$.

Question 2

(a)

A train takes 40 minutes to travel from rest at station A to rest at station B. The distance between the stations is 20 km. The train left station A at 10:00. At 10:15 the speed of the train was 32 km h^{-1} and at 10:30 the speed was 48 km h^{-1}.

The speed of 48 km h^{-1} was maintained until the brakes were applied, causing a uniform deceleration which brought the train to rest at B.

During the first and second 15-minute intervals the accelerations were constant.

(i) Draw a speed-time graph of the motion.

(ii) Find the time taken for the first 16 km.

(iii) Find the deceleration of the train.

(b)

A woman takes out a loan of €120,000 to build an extension to her house. The bank agrees to a 15-year loan at a monthly percentage rate (MPR) of 0.4%.

(i) What is the annual percentage rate correct to 3 decimal places?

(ii) If D_n is the amount of debt owing after n months and A the amount she pays back each month, write down a difference equation in D_n.

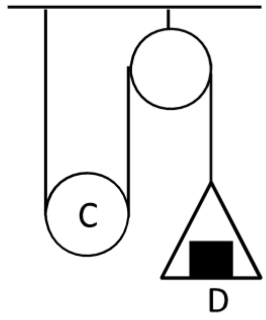
(iii) Solve the difference equation.

(iv) Find, to the nearest cent, the amount she will have to pay back every month.

Question 3

(a)

The diagram shows a light inextensible string having one end fixed, passing under a smooth movable pulley C of mass km kg and then over a fixed smooth pulley. The other end of the string is attached to a light scale pan. A bock D of mass m kg is placed symmetrically on the centre of the scale pan.
The system is released from rest. The scale pan moves upwards.



(i) Show that $k > 2$.

(ii) Find, in terms of k and m, the tension in the string.

(iii) Find, in terms of k and m, the reaction between D and the scale pan.

(b)

(i) Evaluate the following: $\int x^2 \ln x \, . dx$

(ii) An elastic constant has natural length $3\ \mathrm{m}$ and elastic constant $20\ \mathrm{N/m}$. Find the work fone in stretching the string to a length of $7\ \mathrm{m}$.

Question 4

(a)

A particle is projected from a point on horizontal ground. The speed of projection is 14 m s^{-1} at an angle α to the horizontal.

Find the two values of α that will give a range of 10 m.

(b)

A 60 gram mass is projected vertically upwards with an initial speed of 15 m s^{-1} and half a second later a 40 gram mass is projected vertically upwards from the same point with an initial speed of 22·65 m s^{-1}.

(i) Calculate the height at which the masses will collide.

The masses coalesce on colliding.

(ii) Find the greatest height which the combined mass will reach.

Question 5

(a)

Aoife is a Leaving Cert student. During her exams, she has a break of 5 days. She decides to dedicate three of these days to studying the three subjects she has left (Classical Studies, Economics and Applied Maths). She will not divide up any day between two or three subjects but will spend each day studying one particular subject. She reckons that she will improve her percentage mark by the following amounts:

Number of days	1	2	3
Increase in Classical Studies	11%	21%	26%
Increase in Economics	9%	17%	27%
Increase in Applied Maths	12%	20%	25%

How many days should Aoife allocate to each subject in order to improve her grades by the maximum amount?

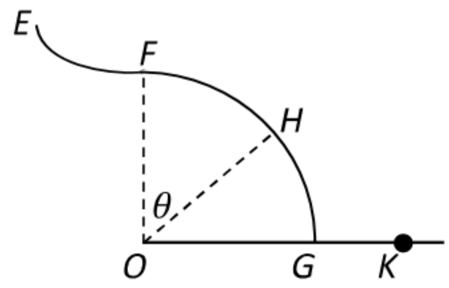
Subject	Days Available	Days Allocated	Days Left	% Increase

(b)

A smooth slide *EFG* is in the shape of two arcs, *EF* and *FG*, each of radius r. The centre O of arc *FG* is vertically below F as shown in the diagram.

Point E is at a height $\frac{r}{5}$ above point F.

A child starts from rest at E, moves along the slide past the point F and loses contact with the slide at point H. OH makes an angle θ with the vertical.



(i) Find the value of θ.

The child lands in a sandpit at point K.

(ii) Find, in terms of r, the speed of the child at K.

Question 6

(a)

A small smooth sphere A, of mass $3m$ moving with speed u, collides directly with a small smooth sphere B, of mass m moving with speed u in the opposite direction. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(i) Find, in terms of u, the speed of each sphere after the collision.

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$.
The first collision between the spheres occurred at a distance 2 metres from the wall. The spheres collide again 4 seconds after the first collision between them.

(ii) Find the value of *u*.

(b)

A particle starts from rest and moves in a straight line with acceleration $(25 - 10v)$ m s^{-2}, where v is the speed of the particle.

(i) After time t, find v in terms of t. (Note: $\int \frac{dx}{a+bx} = \frac{1}{b} ln|a + bx| + c$).

(ii) Find the time taken to acquire a speed of 2·25 m s^{-1} and find the distance travelled in this time.

Question 7

(a)

A steamboat of mass m has a power output of $12m$ watts. When the boat is travelling at speed v, the water exerts a drag on the boat of mkv newtons, where k is a constant. The maximum speed of the boat is $6\ m/s$.

(i) Find the value of k.

(ii) The maximum acceleration is $7.5\ m/s^2$ when the speed of the boat is u. Find the value of u.

(b)

A particle P travelling in a straight line has a deceleration of $4v^{n+1}$ m s^{-2}, where n (> 0) is a constant and v is its speed at time t (> 0).

P has an initial speed of u.

(i) Find an expression for v in terms of u, n and t.

(ii) When $n = 3$ obtain an expression for the speed of P when it has travelled a distance of 3 m from its initial position.

Question 8

(a)

A small smooth sphere A, of mass 1·5 kg, moving with speed 6 m s^{-1}, collides directly with a small smooth sphere B, of mass *m* kg, which is at rest.
After the collision the spheres move in opposite directions with speeds *v* and 2*v*, respectively.
80% of the kinetic energy lost by A as a result of the collision is transferred to B.
The coefficient of restitution between the spheres is *e*.

(i) Find the value of v

(ii) Find the value of e

(b)

A particle is projected vertically upwards with an initial speed of $2g\ m/s$ in a medium in which there is a resistance of $kv^2\ N$ per unit mass where v is the speed of the particle and k is a constant, where $k > 0$.

Prove that the maximum height reached is $\frac{1}{2k}\ln(1 + 4kg)$.

Leaving Certificate Examination

Sample Paper 2

Applied Mathematics

Higher Level
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400 marks

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Sample Paper 2

Question 1

(a)

A parcel rests on the horizontal floor of a van.
The van is travelling on a level road at 14 m s^{-1} .
It is brought to rest by a uniform application of the brakes.
The coefficient of friction between the parcel and the floor is $\frac{2}{5}$.
Show that the parcel is on the point of sliding forward on the floor of the van if the stopping distance is 25 m.

(b)

A particle, of mass m falls vertically downwards under gravity.
At time t, the particle has speed v and it experiences a resistance force of magnitude kmv, where k is a constant.
The initial speed of the particle is u.

(i) Show that $v = \frac{g}{k} - \left(\frac{g}{k} - u\right)e^{-kt}$, at time t.

(ii) If $u = 9{\cdot}8\ \text{m s}^{-1}$ and $k = 0{\cdot}98\ \text{s}^{-1}$, find the distance travelled by the particle in 4 seconds.

(Note: $\int \frac{dx}{a+bx} = \frac{1}{b}\ln|a + bx| + c$).

Question 2

(a)

Beatrix is going to college next year. She buys a new laptop for her studies. She will be in college for 4 years. The laptop costs €2000. She will sell whatever laptop she has at the end of the 4 years. The replacement value for this laptop each year and the maintenance costs are shown in the tables below:

Years old	1	2	3	4
Value (€)	1400	900	400	100

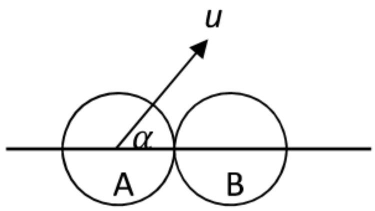
Years	1	2	3	4
Maintenance Cost(€)	50	200	300	350

(i) If Beatrix replaces her laptop after 2 years and sells again after 4 years, what will her costs amount to?

(ii) Use dynamic programming to find the minimum possible costs and the strategy which gives rise to it.

(b)

A smooth sphere, A, of mass m collides obliquely with another smooth sphere, B, of mass m.



Before impact, A is moving with speed u at an angle α to the line of centres of the spheres, where $0° < \alpha < 45°$.

B is at rest before the impact.

The coefficient of restitution for the collision is e.

(i) Find the speed of A and the speed of B after impact in terms of u, e and α.

(ii) Given that A is deflected through angle α because of the collision, show that $\tan^2 \alpha = e$.

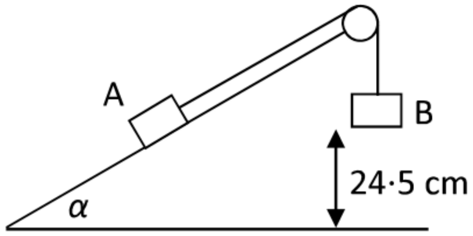
Question 3

(a)

The acceleration of a particle (in ms^{-2}) is determined by the equation $a = v^2 + 25\, m/s^2$. Find the distance travelled by the particle in this time (to 3 significant figures).

(b)

A block A of mass 10m on a smooth plane inclined at an angle α with the horizontal, where $\tan \alpha = \frac{3}{4}$, is connected by a light inextensible string which passes over a smooth pulley to a second block B of mass 10m. B is 24·5 cm above an inelastic horizontal floor, as shown in the diagram.



The system is released from rest.

Find

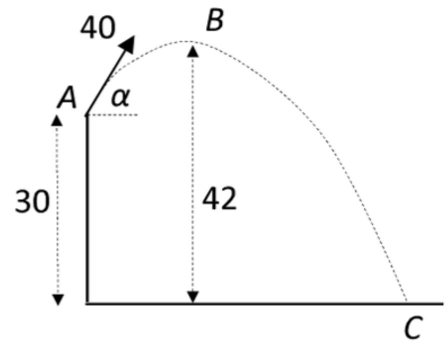
(i) the acceleration of B

(ii) the time that B remains in contact with the floor.

Question 4

(a)

A particle is projected with speed 40 m s^{-1} from a point *A* on the top of a vertical cliff of height 30 m.
The maximum height reached by the particle is 42 m above the horizontal ground, at point *B*.
It strikes the ground at *C*.



Find

(i) the value of α, the angle of projection

(ii) the horizontal range of the particle

(iii) the speed of the particle as it hits the ground at *C*.

(b)

A particle is projected horizontally along a smooth horizontal surface with initial speed 80 m s^{-1}. The particle has a retardation of $\frac{v}{100}$ m s^{-2}, where v is the speed.

Find

(i) the speed of the particle after *t* seconds

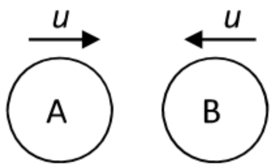
(ii) the distance travelled in t seconds

(iii) the speed v in terms of the distance travelled, s.

Question 5

(a)

A smooth sphere A of mass $4m$, moving with speed u on a smooth horizontal table collides directly with a smooth sphere B of mass m, moving in the opposite direction with speed u.



The coefficient of restitution between A and B is e.

(i) Find the speed, in terms of u and e, of each sphere after the collision.

The magnitude of the impulse on B due to the collision is T.

(ii) Show that $\frac{8mu}{5} \leq T \leq \frac{16mu}{5}$.

(b)

In a certain state in America, the population of pheasants is 25,000. The gun club release 3000 pheasant chicks into the wild every spring. The chances of a pheasant surviving through the shooting season and into the next year is 0.15.

(i) If P_n = the pheasant population in the state after n years, write down a difference equation which describes this situation.

(ii) Given that $P_0 = 25000$ find P_n in terms of n.

(iii) Estimate the pheasant population after 3 years.

(iv) Show that P_n approaches a steady state as the years go on and find that steady state.

Question 6

(a)

A particle of mass m is moving in such a way that its displacement (in metres) at time t (in seconds) from a fixed point O is given by

$$\vec{s} = (r\cos\omega t)\vec{i} + (r\sin\omega t)\vec{j}$$

(i) Show that the magnitude of its displacement from O is a constant r.

(ii) Find the acceleration vector at any time t.

(iii) Show that the force exerted on the particle is directed towards O and is of magnitude $m\omega^2 r$.

(b)

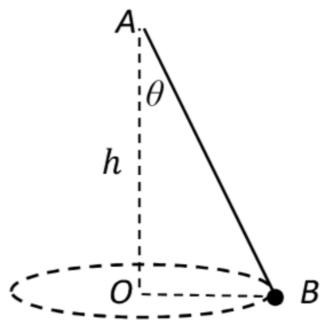
Car C, moving with uniform acceleration f passes a point P with speed u (> 0). Two seconds later car D, moving in the same direction with uniform acceleration $2f$ passes P with speed $\frac{6}{5}u$. C and D pass a point Q together. The speeds of C and D at Q are 6.5 m s^{-1} and 9 m s^{-1} respectively.

(i) Show that C travels from P to Q in $(\frac{3}{2f} + 5)$ seconds.

(ii) Find the value of f.

Question 7
(a)

One end A of a light elastic string is attached to a fixed point. The other end, B, of the string is attached to a particle of mass m. The particle moves on a smooth horizontal table in a circle with centre O, where O is vertically below A and $|AO| = h$. The string makes an angle θ with the downward vertical and B moves with constant angular speed ω about OA.



(i) Show that $\omega^2 \leq \frac{g}{h}$.

The elastic string has natural length h and elastic constant $\frac{2mg}{h}$.

(ii) Given that $\omega^2 = \frac{2g}{5h}$, find the value of θ.

(b)

A particle is projected vertically upwards with a velocity of u m s^{-1}.
After an interval of $2t$ seconds a second particle is projected vertically upwards from the same point and with the same initial velocity.

They meet at a height of h m.

Show that $h = \dfrac{u^2 - g^2t^2}{2g}$.

Question 8

(a)

One method of dyeing a piece of cloth is to immerse it in a container which has P grams of dye dissolved in a fixed volume of water.

The cloth absorbs the dye at a rate proportional to the mass of dye remaining.

$$\frac{dx}{dt} = k(P - x)$$

where t is time in seconds, x is the mass of dye absorbed by the cloth and $k = \frac{1}{50}$.

(i) Find the time taken to dye a piece of cloth if a mass of $\frac{5}{8}P$ needs to be absorbed to reach the desired colour.

(Note: $\int \frac{dx}{a+bx} = \frac{1}{b}\ln|a + bx| + c$)

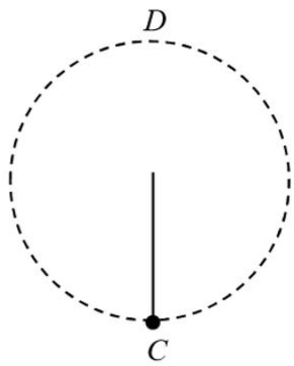
An alternative method is to keep the mass of dye present in the water constant at P grams by continuously adding dye throughout the process.

(ii) Find the time taken to dye the piece of cloth to the desired colour using this method.

(b)

A small particle hanging on the end of a light inextensible string 2 m long is projected horizontally from the point C.

(i) Calculate the least speed of projection needed to ensure that the particle reaches the point D which is vertically above C.



(ii) If the speed of projection is 7 m s^{-1} find the angle that the string makes with the vertical when it goes slack.

Leaving Certificate Examination

Sample Paper 3

Applied Mathematics

Higher Level
2 hours and 30 minutes

400 marks

Examination Number

For examiner	
Question	Mark
1	/50
2	/50
3	/50
4	/50
5	/50
6	/50
7	/50
8	/50
~~9~~	~~/50~~
~~10~~	~~/50~~
Written Total	/400
Project	/100
Overall Total	/500
Overall Grade	

Sample Paper 3

Question 1

(a)

(i) Solve $u_{n+2} - 7u_{n+1} + 10u_n = 0$, given that $u_0 = 593$ and $u_1 = 1165$.

(ii) Find the least value of n for which $u_n < 0$.

(b)

Two cars, A and B, travel along a straight level road in opposite directions. A passes point P with speed 4 m s^{-1} and uniform acceleration 2 m s^{-2}. Three seconds later B passes point Q with speed 5 m s^{-1} and uniform acceleration 4 m s^{-2}.

The distance from P to Q is 1143 m.

The cars meet t seconds after A passes P.

(i) Find the value of t.

(ii) Find the distance from P to the meeting point.

(iii) Find the distance between the cars when A is 160 m from the meeting point, before the cars meet.

Question 2

(a)

A car passes a flagpole at time $t = 0$ and drives along a straight level road. Its speed after that (at time t in seconds) is given by:

$$v(t) = 20 - \frac{1}{5}t^2$$

(i) What is the speed of the car as it passes the flagpole?

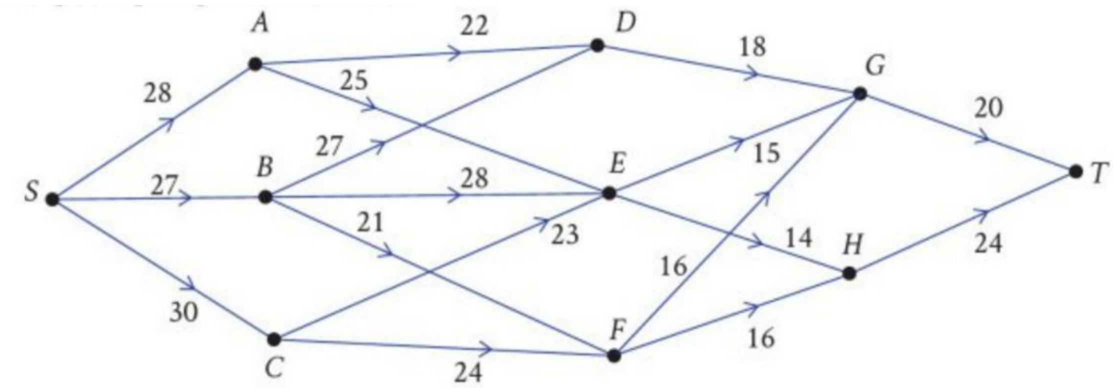
(ii) Find the deceleration of the car at $t = 4\ s$?

(iii) At what time does the car stop?

(iv) How far from the flagpole does the car stop?

(b)

The network shown represents the decisions associated with upgrading the bicycle lanes in Thures over the next four years. The numbers on each arc represents the cost (in €1000s) to Tipperary County Council (TCC) corresponding to a particular decision.



(i) Use dynamic programming to decide which decisions TCC should make over the next four years in order to minimise the overall cost.

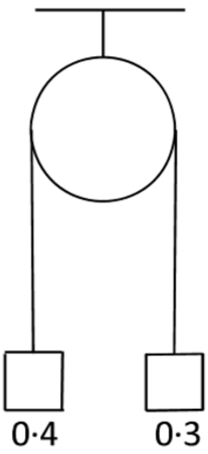
Stage	State	Action	Destination	Value

Optimal Solution to minimise costs:________________________

(ii) Calculate the average yearly cost if TCC adopts the optimal solution.

Question 3
(a)

Two particles of masses 0·4 kg and 0·3 kg are attached to the ends of a light inextensible string which passes over a light smooth fixed pulley. They are held at the same level, as shown in the diagram.



The system is released from rest.

Find

(i) the tension in the string

(ii) the speed of the 0·4 kg mass when it has descended 0·7 m.

(b)

A particle of mass m is suspended vertically from a fixed point O by a light inelastic string of length d metres.

The particle is projected horizontally with speed u, where $u^2 = 4gd$.

Show the string goes slack when it makes an angle $\cos^{-1}\frac{2}{3}$ with the upward vertical through O.

Question 4

(a)

A particle is projected from a point P with speed u m s^{-1} at an angle α to the horizontal.

(i) Show that the range of the particle is $\frac{2u^2 \sin\alpha \cos\alpha}{g}$.

The particle is 24·5 m above the horizontal ground after 5 seconds and it strikes the ground 235·2 m from P.

(ii) Find the value of u.

(b)

A car has an initial speed of u m s^{-1}. It moves in a straight line with constant acceleration f for 4 seconds. It travels 40 m while accelerating.
The car then moves with uniform speed and travels 45 m in 3 seconds.
It is then brought to rest by a constant retardation $2f$.

(i) Draw a speed-time graph for the motion.

(ii) Find the value of u.

(iii) Find the total distance travelled.

Question 5

(a)

If $\frac{dy}{dx} = 3\sin 3x + \cos 5x$ and $y = 1$ when $x = \frac{\pi}{4}$, find the value of y when $x = \frac{\pi}{2}$.

Give your answer correct to 2 decimal places.

(b)

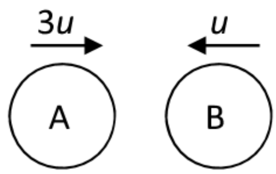
(i) Solve the difference equation $u_n = 2u_{n-1} + 2(3^n)$ with $u_0 = 1$.

(i) Hence find u_8.

Question 6

(a)

A smooth sphere A of mass m, moving with speed $3u$ on a smooth horizontal table collides directly with a smooth sphere B of mass $2m$, moving in the opposite direction with speed u.
The directions of motion of A and B are reversed by the collision.



The coefficient of restitution between A and B is e.

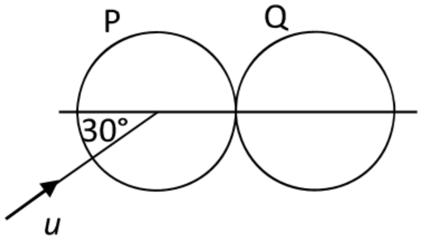
(i) Find the speed, in terms of u and e, of each sphere after the collision.

Subsequently B hits a wall at right angles to the line of motion of A and B.
The coefficient of restitution between B and the wall is $\frac{1}{2}$.
After B rebounds from the wall there is a further collision between A and B.

(ii) Show that $\frac{1}{8} < e < \frac{1}{4}$.

(b)

A smooth sphere P has mass m_1 and speed u.
It collides obliquely with a smooth sphere Q, of mass m_2, which is at rest.
Before the collision the direction of P makes an angle of 30° to the line of centres, as shown in the diagram.

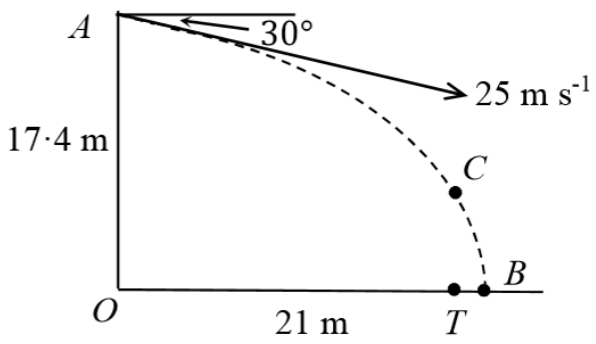


The coefficient of restitution between the spheres is e.

Prove that P will turn through a right-angle if $4m_1 = (3e - 1)m_2$.

Question 7
(a)

A ball is thrown from a point A at a target T, which is on horizontal ground. The point A is 17·4 m vertically above the point O on the ground. The ball is thrown from A with speed 25 m s^{-1} at an angle of 30° below the horizontal. The distance OT is 21 m.
The ball misses the target and hits the ground at the point B, as shown in the diagram.



Find

(i) the time taken for the ball to travel from A to B

(ii) the distance TB.

The point C is on the path of the ball vertically above T.

(iii) Find the speed of the ball at C.

(b)

P, the population of insects in a region, grows at a rate that is proportional to the current population.

$$\frac{dP}{dt} = kP$$

where k is a positive constant. In the absence of any outside factors the population will triple in 15 days.

(i) Find the value of k.

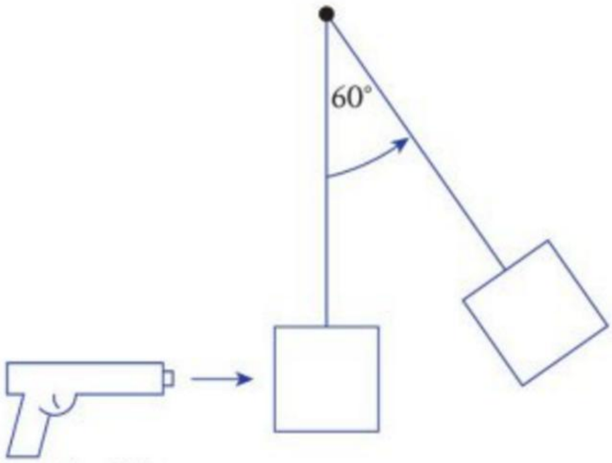
A scientist begins to remove 10 insects from the population each day.

(ii) If there are initially 120 insects in the region the population will not survive. After how many days will the population die out?

Question 8

(a)

A bullet of mass $0.1\ kg$ is fired horizontally into a block of mass $2.9\ kg$, which hangs at the end of a light $20\ m$ string. The bullet becomes embedded and the joint mass swings, finally coming to rest when the string makes an angle of $60°$ with the vertical. With what speed did the bullet enter the block?



(b)

Train A and Train B are on parallel tracks and travelling in opposite directions.
Train A starts from rest at Maynooth and accelerates uniformly at $0{\cdot}5$ m s^{-2} towards Leixlip to a speed of 25 m s^{-1}. It then continues at this constant speed.

At the same instant as train A is leaving Maynooth Train B passes through Leixlip heading towards Maynooth at a constant speed of 30 m s^{-1}.
Three minutes after leaving Leixlip train B starts to decelerate at $0{\cdot}25$ m s^{-2} and comes to rest at Maynooth.

(i) Find the distance between Maynooth and Leixlip.

(ii) At what distance from Maynooth do the trains meet?

After travelling at 25 m s^{-1} for a time, train A decelerates and comes to rest at Leixlip 36 seconds after train B stops at Maynooth.

(iii) Find the deceleration of train A.

Leaving Certificate Examination

SEC HL Sample Paper 2020

Applied Mathematics

Higher Level
2 hours and 30 minutes

400 marks

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~~10~~	~~/50~~
Written Total	/400
Project	/100
Overall Total	/500
Overall Grade	

Question 1

(a) A directed graph is represented by the adjacency matrix $M = \begin{pmatrix} 0 & 0 & 2 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

(i) Draw the graph represented by M.

(ii) Calculate M^2.

(iii) What information is provided by the elements of M^2?

(b) A gardener plants a new fruit tree which has three new branches. In a branch's first year of growth it will not produce any additional branches. Each branch will produce one additional branch every year after that.

The gardener models this growth pattern by defining U_n to be the number of branches on the tree n years after planting, with $U_0 = 3$ and $U_1 = 3$.

(i) Write down the values of U_2 and U_3.

(ii) Write down a difference equation for U_{n+2} in terms of U_{n+1} and U_n, where $n \geq 0, n \in \mathbb{Z}$.

(iii) Solve this difference equation to find an expression for U_n in terms of n.

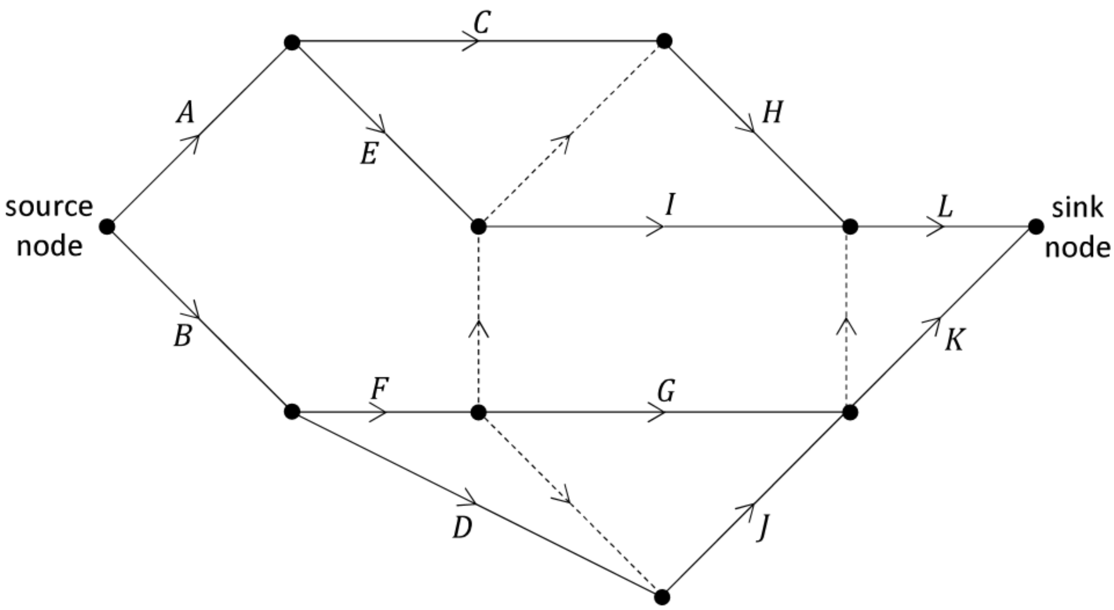
(iv) Plants must be cut back regularly to allow them room to grow. How many of the old branches should be removed at the end of year 4 to ensure that there are exactly 14 branches at the end of year 5?

Question 2

The diagram below shows the scheduling network for a project to manufacture a new chemical compound. The network provides some information about the relationships between the twelve activities that have to be completed as part of the project.

The edges of the network represent these activities and are labelled with the letters A to L. The unlabelled edges (shown with dashed lines) do not represent real activities but they help explain the order in which the activities must happen. The letters used to label the edges should **not** be taken as representing the order in which the activities happen.

The nodes of the network represent events or points in time during the project. The source node is the time when the project begins and the sink node is the time when the project ends.



(i) Complete the table on the next page by listing, for each activity, the other activities on which it depends directly. That is, for each activity $X \in \{C, D, E, \dots, L\}$, write the smallest possible list of other activities which need to be completed before activity X can begin.

Activities A and B do not depend on any prior activities, so the list is empty for these activities, as shown.

Use the space below to show relevant supporting work, if necessary.

Activity	**Depends directly on ...**
A	–
B	–
C	
D	
E	
F	
G	
H	
I	
J	
K	
L	

For each of the statements in parts **(ii)** and **(iii)** below, state whether you agree or disagree with the statement. Use the scheduling network and/or your answer to part **(i)** to justify your answer in each case.

(ii) Activity D must be completed before activity G.

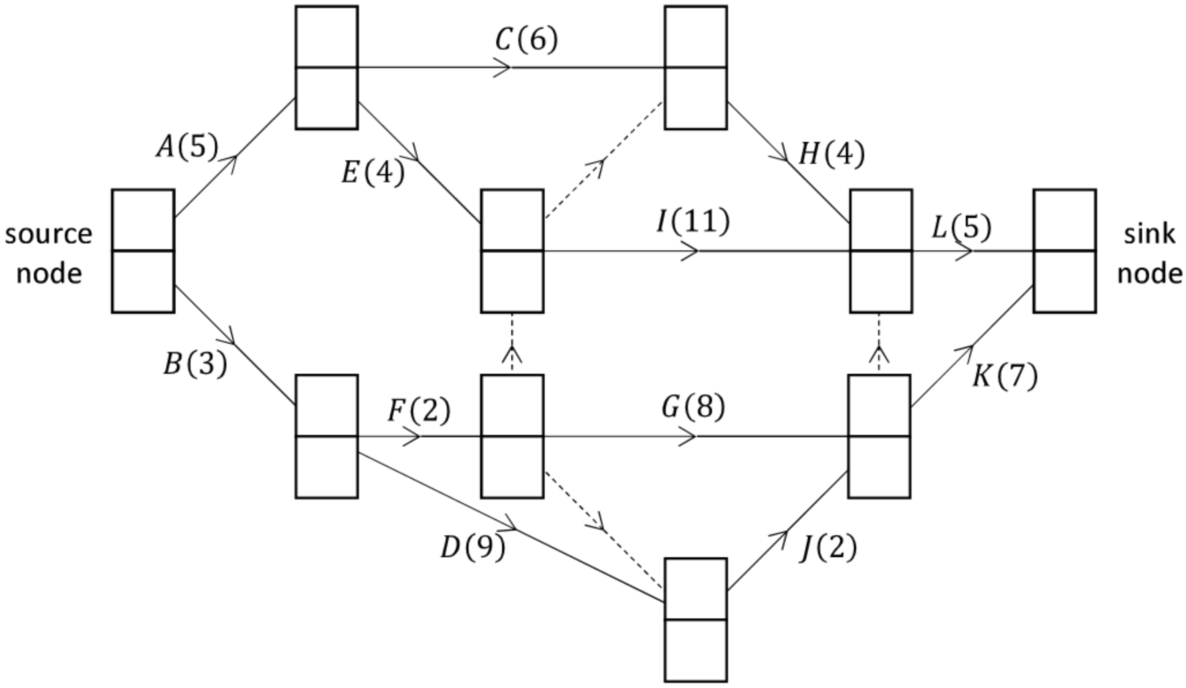
(iii) Activity E must be completed before activity H.

The time, in days, to complete each of the activities A to L is given in the table below and has also been included in the network redrawn on this page.

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Time (days)	5	3	6	9	4	2	8	4	11	2	7	5

(iv) Calculate the early time and the late time for each event.

Complete the diagram below by writing the early time (upper box) and late time (lower box) at the node representing each event.



Use the space below to show relevant supporting work, if necessary.

(v) Write down the critical path for the network.

(vi) Write down the minimum time, in days, needed to complete this project.

(vii) Select any one non-critical activity on the network and calculate its float, in days.

(viii) The project is due to begin on the morning of July 1^{st}. The key worker needed to carry out activity G will be away on holidays when the project begins.

What is the latest date on which this worker could return to work without necessarily causing the project to be delayed? Justify your answer.

Question 3

(a) A particle has initial displacement s_0 from a fixed point P. It moves away from P with initial velocity u and constant acceleration $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.

Use calculus to derive an expression for s, the displacement of the particle from P at any time t.

(b) Two athletes, Brian and Clara, are taking part in a relay race. Brian is preparing to hand over the baton to Clara. During the hand-over of the baton the athletes need to be running in the same straight line and at the same velocity.

As Brian approaches Clara's position at a constant speed of 11 m s^{-1}, Clara starts running from rest with constant acceleration f.

A short time later Brian begins to decelerate at 2 m s^{-2}.

Clara receives the baton 2.5 s after she starts running.

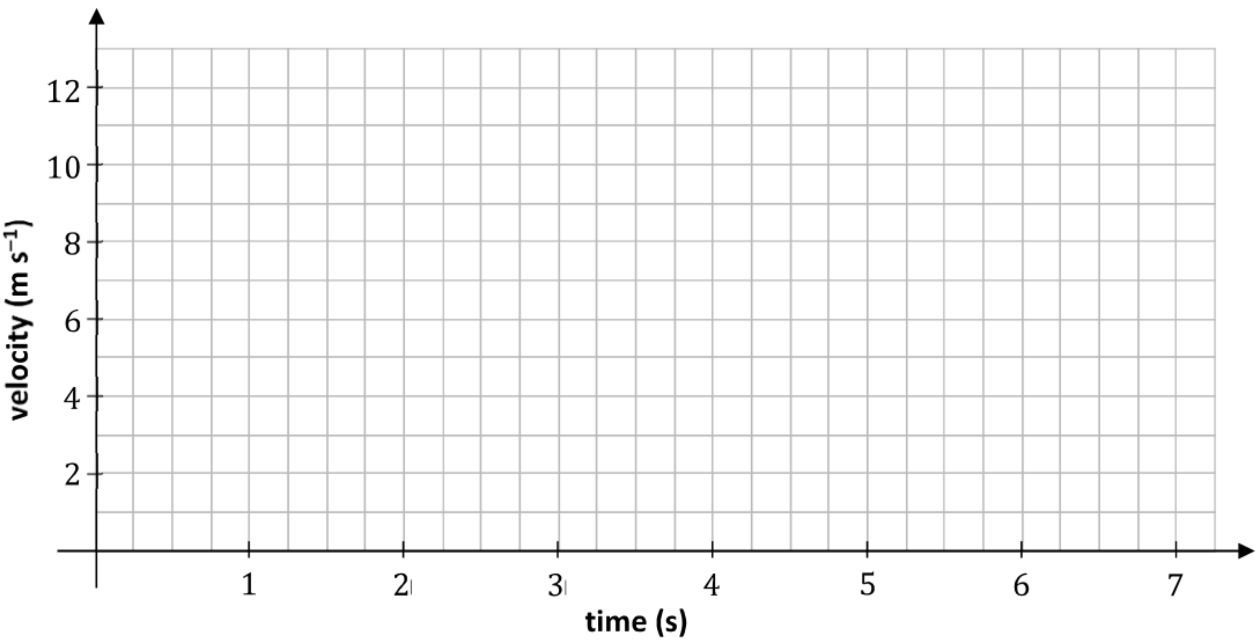
The baton is exchanged when Clara is 75 cm ahead of Brian and when both athletes have a speed of 8 m s^{-1}.

After the baton is exchanged, Brian continues to decelerate at 2 m s^{-2} until he comes to rest. Clara continues to accelerate at f until she reaches her maximum speed of 12 m s^{-1}, which she then maintains.

(i) Calculate the time it takes for Brian to decelerate before he exchanges the baton.

(ii) Using the axes below, draw an *accurate* velocity-time graph for the motion of each runner. Time is measured from the instant that Clara begins to run.

Relevant calculations should be shown in the space below.

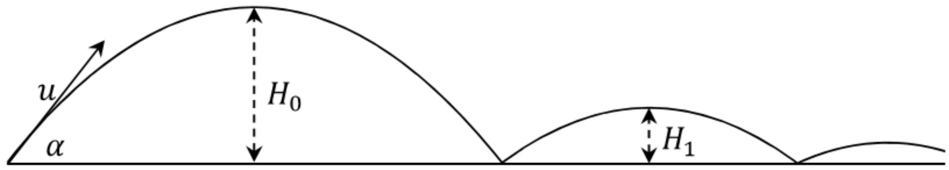


(iii) Calculate the distance between the two athletes when Clara begins to run.

Question 4

(a) A ball is projected from a point on horizontal ground, with initial speed u and at an angle α to the horizontal. The ball reaches a maximum height of H_0 above the horizontal.

Upon landing, the ball bounces with a maximum height of H_1.



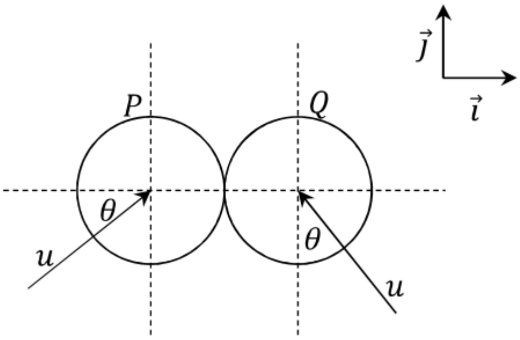
The coefficient of restitution between the ball and the ground is e.

(i) Calculate $\frac{H_0}{H_1}$.

(ii) The ball continues bouncing. Find an expression (in terms of e and H_0) for H_5, the maximum height of the ball after it lands on the ground for the fifth time.

(b) Two identical smooth spheres, P and Q, each moving with speed u, collide obliquely. The line joining their centres at the point of impact is along the $\vec{\imath}$ axis.

Before the collision, the velocity of sphere P makes an angle θ with the $\vec{\imath}$ axis and the velocity of sphere Q makes an angle θ with the $\vec{\jmath}$ axis, as shown in the diagram.



The coefficient of restitution between the spheres is e, where $0 \leq e \leq 1$.

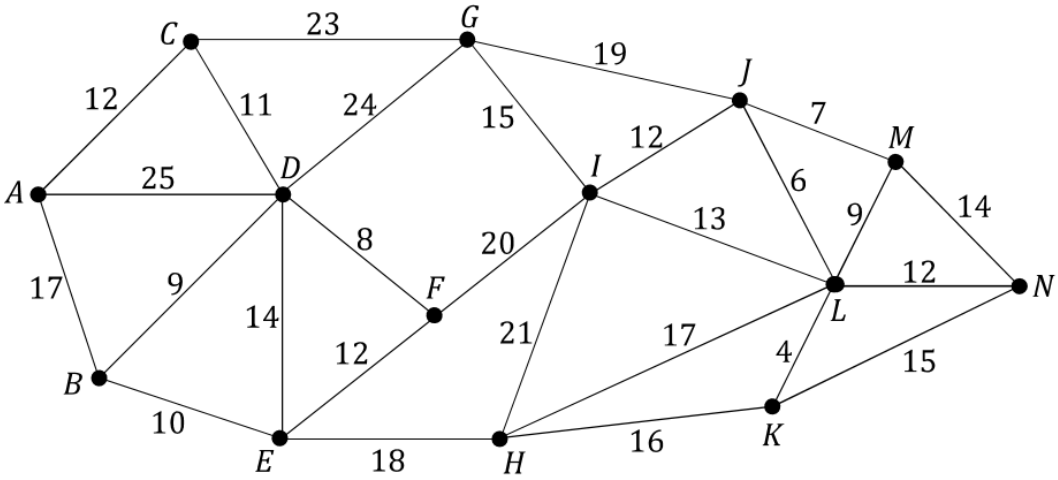
After the collision sphere Q moves off parallel to the $\vec{\jmath}$ axis.

(i) Show that $e = \frac{\tan\theta - 1}{\tan\theta + 1}$.

(ii) If 25% of the spheres' total kinetic energy is lost during the collision, calculate θ and e.

Question 5

(a) In the network shown below, the edges represent roads and the nodes represent the junctions of two or more roads, labelled with the letters A to N. The weight of each edge represents the distance (in km) between a pair of junctions.



(i) Use Dijkstra's algorithm to find the shortest path from junction A to junction N. Calculate the length of the shortest path. Relevant supporting work must be shown.

(ii) A group of engineers want to close down some of the roads to carry out maintenance work. They wish to close down as much of the road network as possible while still allowing a person to drive between any two junctions on the network.

Using an appropriate algorithm, find the minimum spanning tree for the network. Name the algorithm you used. Relevant supporting work must be shown.

(b) A rumour may be spread when a person who has heard the rumour interacts with a person who has not heard the rumour.

Therefore, the rate of spread of a rumour within a group can be modelled as being proportional to the product of the number of people in the group who have heard the rumour and the number of people in the group who have not heard it.

A student models the rate at which a certain rumour spreads within a school population of 1200 students using the differential equation:

$$\frac{dR}{dt} = kR(1200 - R)$$

where $R(t)$ is the number of students of that school who have heard the rumour at time t, measured in days, and where k is a positive constant.

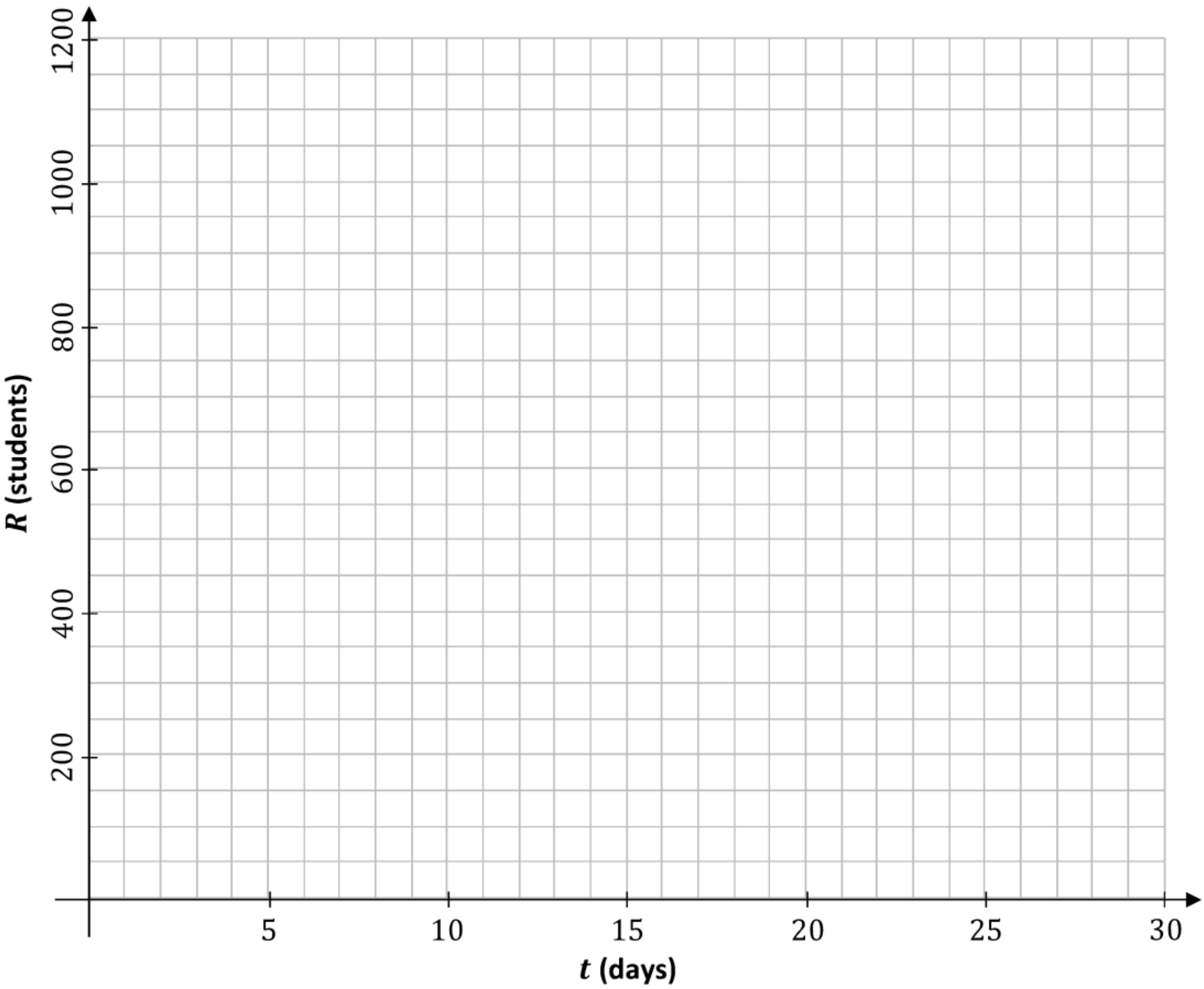
On Monday morning ($t = 0$), 100 students had heard the rumour.

(i) Solve the differential equation to find an expression that relates R, k and t.

Note that $\frac{1}{y(x-y)} = \frac{1}{x}\left(\frac{1}{y} + \frac{1}{x-y}\right)$.

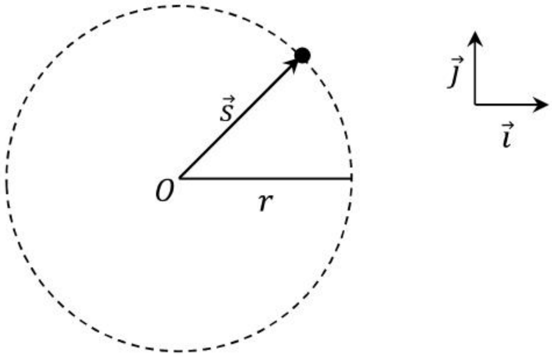
(ii) By Wednesday morning 250 students had heard the rumour. Calculate the value of k.

(iii) Sketch the shape of a graph of R against t to show how the model predicts the spread of the rumour.



Question 6

A learner driver is practising driving around a roundabout.



The motion of the car may be modelled as horizontal circular motion around centre O, with radius r and constant angular speed ω, as in the diagram above.

(i) Write an expression for $\vec{s}$, the displacement of the car relative to O at any time t, in terms of r, ω and t. Your expression should use the unit vectors $\vec{\imath}$ and $\vec{\jmath}$.

Note that $t = 0$ when $\vec{s}$ is along the $\vec{\imath}$ axis.

(ii) Derive an expression for $\vec{v}$, the velocity of the car at any time t.

(iii) Use a dot product calculation to show that the car's velocity and displacement are always perpendicular to each other.

(iv) Show that the acceleration of the car is always directed towards O.

(v) Derive an expression for the maximum velocity the car could have as it travels around the roundabout, without slipping. Your expression should be written in terms of r, g and μ, the coefficient of friction between the car and the road.

(vi) Use dimensional analysis to show that the units for the expression you derived in part **(v)** are equivalent to the units for velocity.

(vii) Do you think the assumptions made in developing this model were appropriate? Explain your answer.

Question 7

(a) A bungee jumper of mass 75 kg jumps from a height of 35 m above water. The jumper is tied to an elastic rope of natural length 12 m and elastic constant 100 N m^{-1}.

(i) Derive an expression for the work done when a spring of elastic constant k N m^{-1} is stretched by x m.

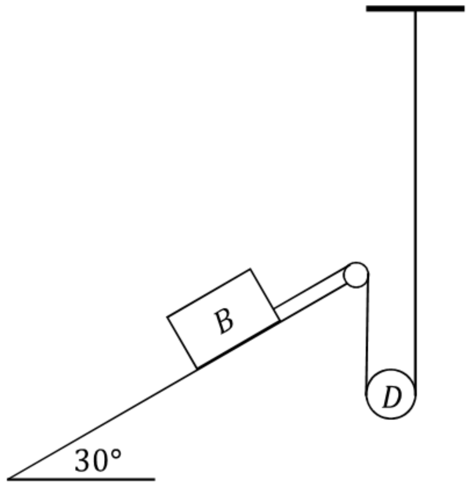
(ii) The motion of the bungee jumper may be modelled using the principle of conservation of energy. Using this model, calculate the distance between the water and the bungee jumper when the bungee jumper is at the lowest point of their motion.

(b) A small smooth moveable disk D, of mass 0.2 kg, rests on a light inextensible string. One end of the string is connected to block B, of mass 4 kg, which rests on a rough plane inclined at $30°$ to the horizontal.
The other end of the string is connected vertically to a fixed point.

The coefficient of friction between block B and the inclined plane is $\frac{1}{10}$.

When the system is released from rest, D moves upwards with acceleration a.

The tension in the string is T.



(i) Show, on separate diagrams, the forces acting on block B and disk D while they are moving.

(ii) Explain why the acceleration of B is $2a$.

(iii) Calculate a and T.

Question 8

A group of scientists are investigating the population, P, of rabbits on a certain island. They estimate that there are 8000 rabbits on the island and that the population is growing at a constant rate of 3% per year.

The scientists plan to remove a number of rabbits from the island every year, to help populate another habitat. They develop mathematical models to predict how P will change if B rabbits are removed from the island every year.

The first model which the scientists develop uses a difference equation to express the population of rabbits in year $n + 1$ in terms of the population in year n.

The difference equation is:

$$P_{n+1} = 1.03P_n - B$$

where $n \geq 0$, $n \in \mathbb{Z}$ and $P_0 = 8000$.

(i) Solve this difference equation to find an expression for P_n in terms of n and B.

The second model which the scientists develop uses a differential equation to express the rate of change of P with respect to n, time measured in years.

The differential equation is:

$$\frac{dP}{dn} = 0.03P - B$$

where $n \geq 0$, $n \in \mathbb{R}$ and $P(0) = 8000$.

(ii) Solve this differential equation to find an expression for P in terms of n and B.

The scientists want to know what each model predicts the rabbit population on the island will be after 50 years, if 200 rabbits are removed each year.

(iii) Calculate P_{50} using the first model and $P(50)$ using the second model, when $B = 200$.

(iv) Each of these models makes a different assumption about the removal of the rabbits from the island. What are the two different assumptions?

(v) The scientists want to know what value of B should be chosen so as to keep the rabbit population on the island constant. Calculate this value of B using either model.

Leaving Certificate Examination 2023

Applied Mathematics

Higher Level

Tuesday 27 June   Afternoon 2:00 - 4:30

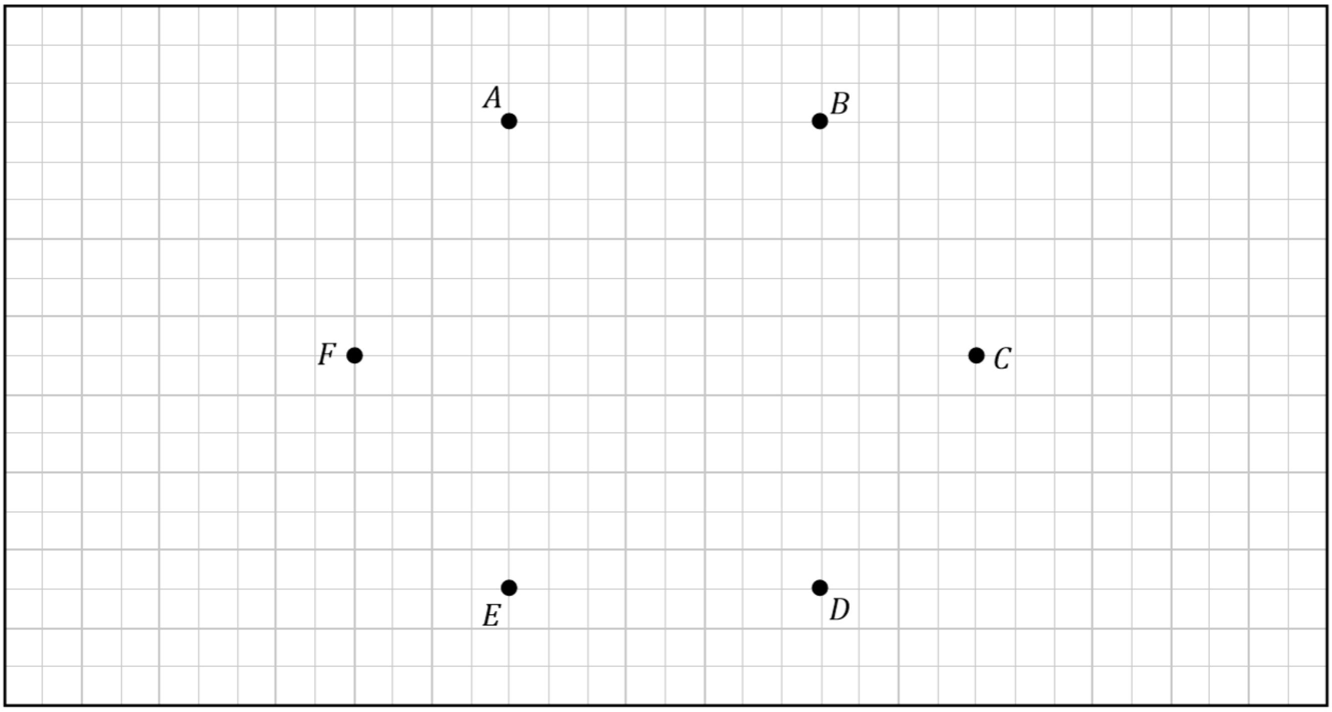
400 marks

Question 1

(a) A directed graph is represented by the adjacency matrix M, where

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

(i) Use the nodes below to draw a graph represented by M.



(ii) Write down a cycle which starts at node B.

(iii) How does a directed graph differ from an undirected graph?

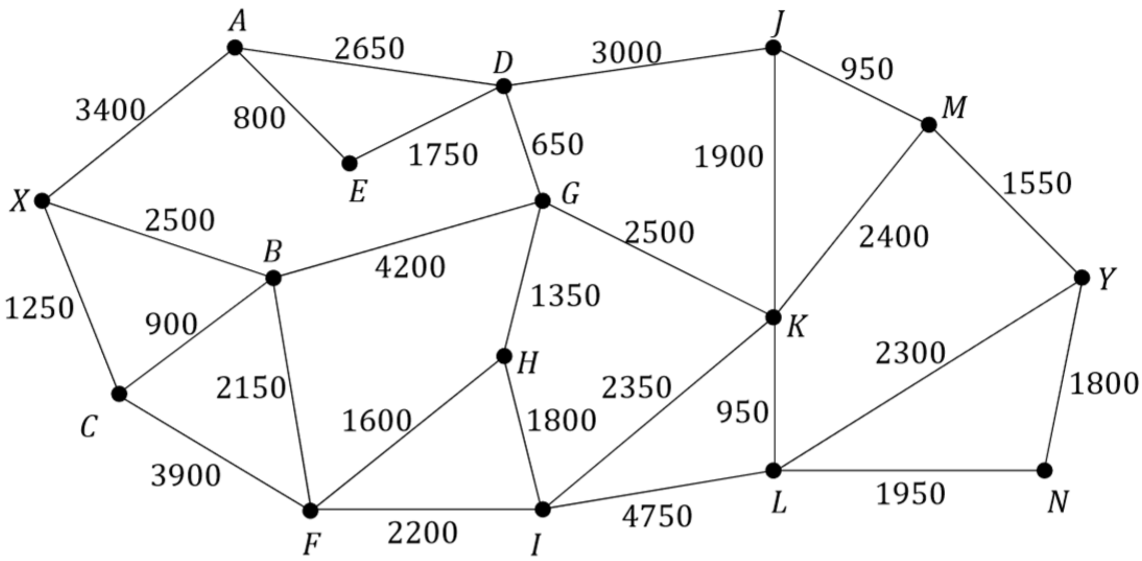
(b) A particle moving along a straight line has velocity $v = \frac{ds}{dt} = 2te^{-t}, t \geq 0$.

(i) Using integration by parts or otherwise, derive an expression for $s(t)$, the displacement of the particle at any time t, given that $s(0) = 0$.

(ii) Calculate $s(3)$.

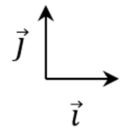
Question 2

(a) A university has decided to improve the paths on its campus. In the network shown below the nodes labelled with the letters X and Y represent the two entrances to the campus and the nodes labelled with the letters A to N represent the key buildings on the campus. The edges represent the paths, with the weight of each edge representing the cost (in €) of carrying out the improvement work for that path.



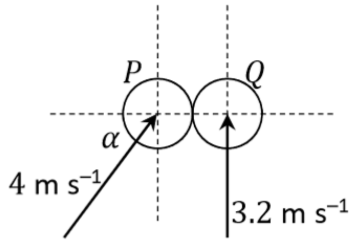
The university decides that the first part of the work will be to provide an improved route between entrance X and entrance Y. Use Dijkstra's algorithm to find the route between X and Y that is cheapest to improve. Calculate the cost of carrying out such improvements. Relevant supporting work must be shown.

(b) Two smooth spheres, P and Q, have equal radius and are of mass m and $2m$ respectively. P and Q collide obliquely. The line joining their centres at the point of impact lies along the $\vec{\imath}$ axis.



Before the collision, sphere P moves with a velocity of 4 m s^{-1} at an angle α with the $\vec{\imath}$ axis, where $\sin \alpha = \frac{4}{5}$.

Before the collision, sphere Q moves with a velocity of 3.2 m s^{-1} perpendicular to the $\vec{\imath}$ axis.



The coefficient of restitution between the spheres is e, where $0 \le e \le 1$.

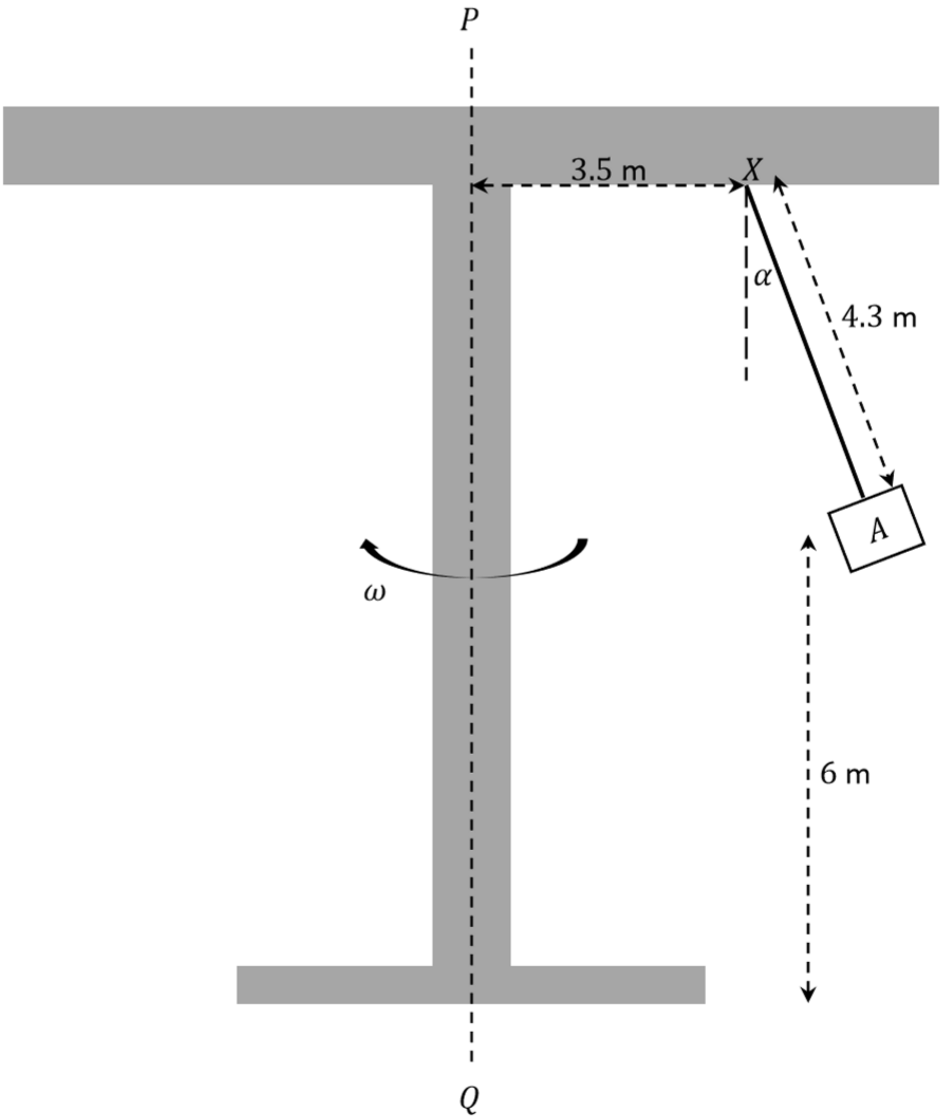
Calculate, in terms of e, the velocity of each sphere immediately after they collide

Question 3

The photograph on the right is of a chain swing ride in an amusement park. The disk at the top of the ride is rotating in a horizontal plane. People sit in seats which are attached freely by inextensible chains of length 4.3 m to fixed points on the disk.

The chain attaching seat A hangs from point X on the ride and makes an angle α with the vertical. X is 3.5 m from the axis of rotation, which is the vertical line PQ, as shown in the diagram below. The chain is free to swing in or out relative to PQ.

The ride rotates about PQ with constant angular velocity ω. Seat A moves in a horizontal circular path which is 6 m above the ground.



(i) Draw a diagram to show the external forces acting on seat A.

(ii) Show that $\omega = \sqrt{\frac{g \tan \alpha}{3.5+4.3 \sin \alpha}}$.

(iii) Use dimensional analysis to show that the units for the expression $\sqrt{\frac{g \tan \alpha}{3.5+4.3 \sin \alpha}}$ are equivalent to the units for ω.

It is found by measurement that $\alpha = 25°$.

(iv) Calculate how many complete revolutions the ride makes in one minute.

The person sitting in seat A throws a small orange into the air. The person imparts an upward vertical velocity component of 4 m s^{-1} to the orange.

(v) Calculate the time from when the orange is thrown until it hits the ground.

Question 4

A ball of mass m kg is projected with initial velocity 15 m s^{-1} vertically downwards into a tank of water. The ball travels through the water against an upward buoyancy force that is 4 times the magnitude of the weight of the ball and a drag force of mv^2 N.

(i) Draw a diagram to show the forces acting on the ball while it is moving downwards through the water.

(ii) Show that, while the ball is moving downwards, the rate of change of its velocity v with respect to its distance s below the surface of the water can be expressed by the differential equation:

$$v\frac{dv}{ds} = -29.4 - v^2$$

(iii) Solve this differential equation to find an expression for v in terms of s.

(iv) The ball is at its maximum depth, D, when $v = 0$. Calculate D.

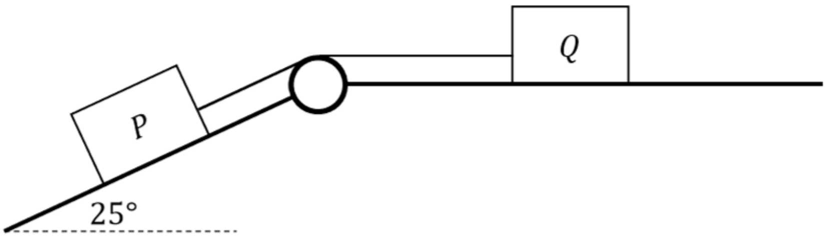
After reaching its maximum depth the ball changes direction and begins to move upwards through the water.

(v) Draw a diagram to show the forces acting on the ball while it is moving upwards through the water.

(vi) Write down a differential equation for the rate of change of the velocity v of the ball while it moves upwards through the water.

Question 5

(a) Block P (of mass 6.3 kg) and block Q (of mass 2.5 kg) are held at rest on a rough surface. They are connected by a light inextensible string which passes over a smooth fixed pulley. Block Q lies on the horizontal part of the surface and block P lies on the part of the surface that is inclined at $25°$ to the horizontal, as shown in the diagram.



The coefficient of friction between each block and the surface is 0.2.

The blocks begin to move when they are released.

(i) Show, on separate diagrams, the forces acting on the blocks while they are moving.

(ii) Calculate the acceleration of the blocks.

(b) Áine travels by car from her house to work each morning. On Monday morning she starts her car and accelerates uniformly for 40 s to a speed of 22.5 m s^{-1}. Áine then travels at this speed for 8 minutes until decelerating uniformly to rest at her work. She reaches her work at exactly $08:30$.

On Tuesday morning Áine leaves her house 140 s later than the day before. She takes the same route to work. She starts her car and accelerates at 1.5 m s^{-2} for 20 s, then maintains this steady speed for 6 minutes before decelerating uniformly to rest at her work. She again reaches her work at exactly $08:30$.

Calculate the time when Áine leaves her house on Tuesday morning.

Question 6

Spider plants (*Chlorophytum comosum*) can reproduce asexually, producing new plants called ‘spiderettes’ or ‘pups’. The manager of a garden centre is told that a one year old spider plant produces two pups each year, that a two year old spider plant produces three pups each year, and that spider plants which are less than one year old or more than two years old do not produce any pups.

The manager predicts that U, the number of pups produced in the garden centre in any year can be expressed by the second-order homogeneous difference equation:

$$U_{n+2} = 2U_{n+1} + 3U_n$$

where $n \geq 0, n \in \mathbb{Z}, U_0 = 1$ and $U_1 = 2$.

(i) Write down the values of U_2 and U_3.

(ii) Solve the difference equation to find an expression for U_n in terms of n.

(iii) Calculate U_{10}.

The manager realises that this model does not take into account the sale of any of the spider plants produced in the garden centre. The manager decides that the garden centre will not sell any of the spider plants in either of the first two years, but that $2n$ of the new pups will be sold in each year n after that.

As part of an improved model, the manager now predicts that V, the number of pups produced and retained (not sold) in the garden centre in any year can be expressed by the second-order inhomogeneous difference equation:

$$V_{n+2} = 2V_{n+1} + 3V_n - 2(n+2)$$

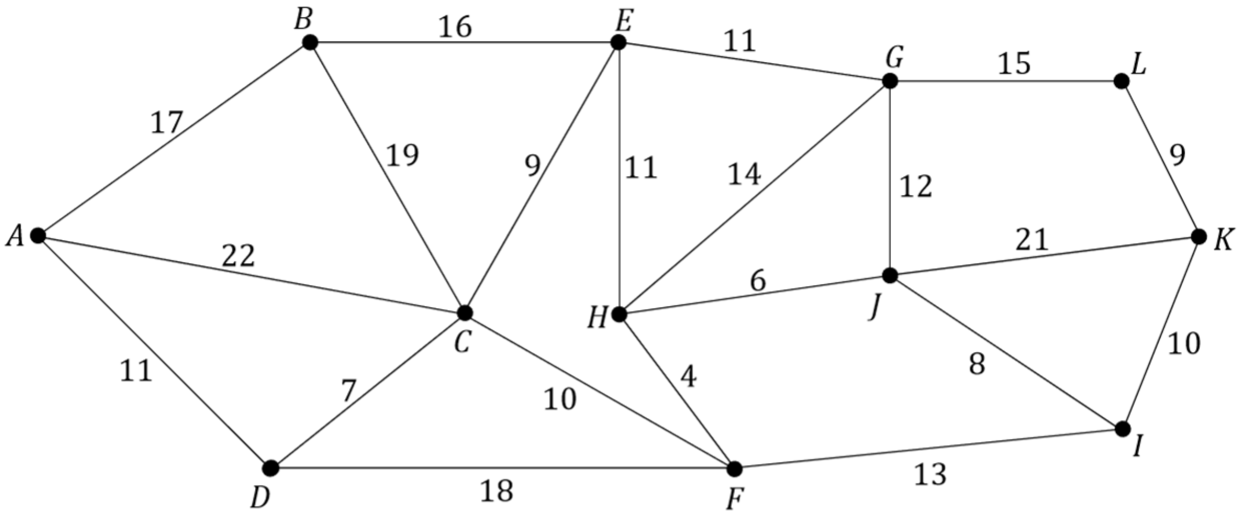
where $n \geq 0$, $n \in \mathbb{Z}$, $V_0 = 1$ and $V_1 = 2$.

(iv) Solve this new difference equation to find an expression for V_n in terms of n.

(v) Calculate V_{10}.

Question 7

(a) There are 12 waterfalls in a certain national park. Paths allow visitors to walk from one waterfall to another. In the network shown below, the edges represent the paths and the nodes represent the waterfalls, labelled with the letters A to L. The weight of each edge represents the time (in minutes) taken to walk between a pair of waterfalls.



The park authorities wish to plan a route along the paths which allows visitors to see every waterfall while moving through the park without wasting time. The paths that are not on this route will be closed.

(i) Using an appropriate algorithm, find the minimum spanning tree for the network. Name the algorithm you used. Relevant supporting work must be shown.

(ii) The park entrance is at waterfall A and the park exit is at waterfall L. Using your minimum spanning tree, calculate the time needed to enter the park at waterfall A, visit every waterfall, and leave the park at waterfall L.

(b) A *learning curve* is a graphical representation of how a person's ability to perform a certain task increases with the time the person spends learning or practicing that task.

A student wishes to be able to spell 2000 difficult words. The rate of the student's learning may be modelled by the differential equation:

$$\frac{dN}{dt} = k(2000 - N)$$

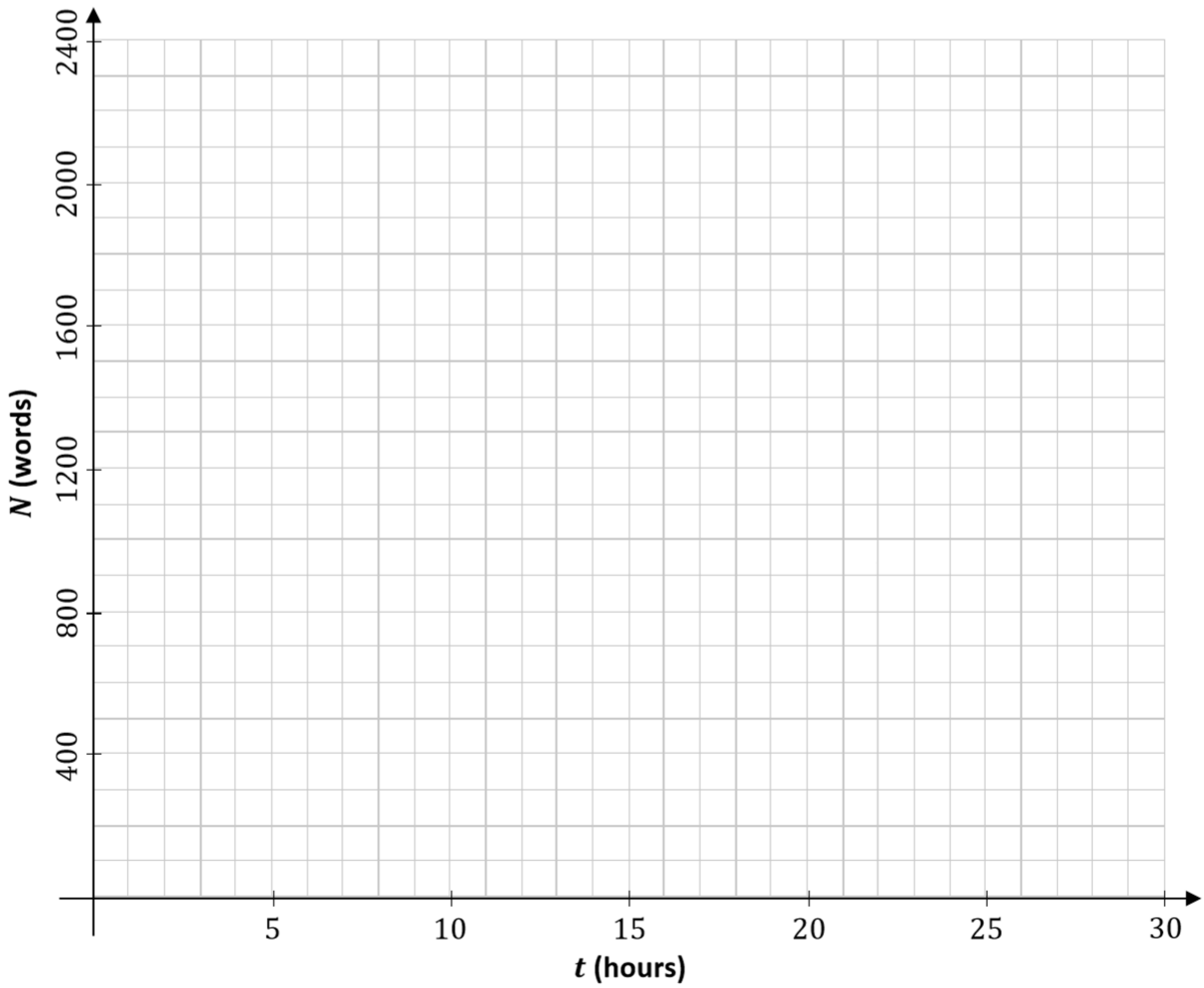
where $N(t)$ is number of these words the student is able to spell after t hours of learning, and where k is a positive constant.

At the start of their learning the student is already able to spell 250 of these words, i.e. $N(0) = 250$.

(i) Solve the differential equation to find an expression for N in terms of k and t.

(ii) After 6 hours of learning, the student is able to spell 1500 of these words. Calculate k.

(iii) Sketch the shape of a graph of N against t to show the model's prediction for the student's learning curve.

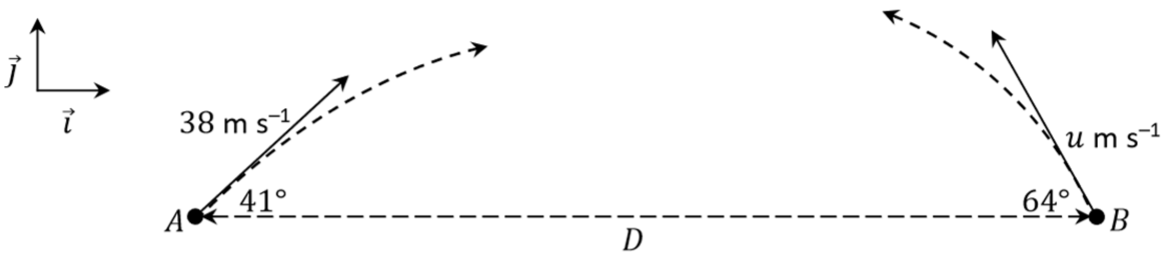


Question 8

Two balls, P and Q, are projected into the air from points A and B, which are a distance D apart along the horizontal $\vec{i}$ axis. The motion of the balls may be modelled as projectile motion in a vertical plane, ignoring the effects of air resistance.

P is projected from point A at time $t = 0$ s with initial velocity 38 m s^{-1} at $41°$ to AB.

Q is projected from point B at time $t = 1$ s with initial velocity u m s^{-1} at $64°$ to BA.



P and Q collide in mid-air when $t = 3$ s.

(i) Show that $u = 28$ m s^{-1} to the nearest whole number.

(ii) Calculate D.

(iii) In terms of $\vec{\imath}$ and $\vec{\jmath}$, calculate $\overrightarrow{v_P}$, the velocity of P, and $\overrightarrow{v_Q}$, the velocity of Q, when the balls collide, i.e. when $t = 3$ s.

(iv) Calculate the dot product of $\overrightarrow{v_P}$ and $\overrightarrow{v_Q}$ when $t = 3$ s.

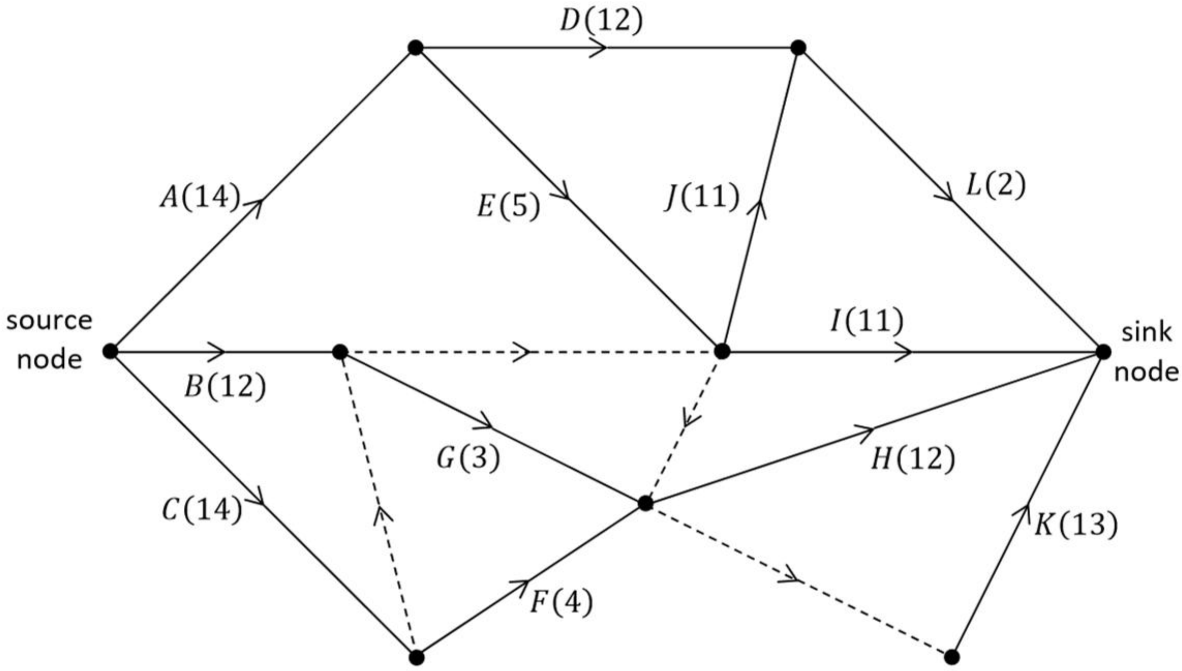
(v) Hence or otherwise calculate the acute angle between $\overrightarrow{v_P}$ and $\overrightarrow{v_Q}$ when $t = 3$ s.

Question 9

The manager of a regional hospital decides to arrange for the refurbishment of one of the hospital wards. The diagram below shows the scheduling network for the project.

The edges of the network represent the activities that have to be completed as part of the project and are labelled with the letters A to L. The duration, in days, of each activity is represented by the number in brackets. The unlabelled edges (shown with dashed lines) do not represent real activities but they help explain the order in which the activities must happen. The letters used to label the edges should **not** be taken as representing the order in which the activities happen.

The nodes of the network represent events or points in time during the project. The source node is the time when the project begins and the sink node is the time when the project ends.



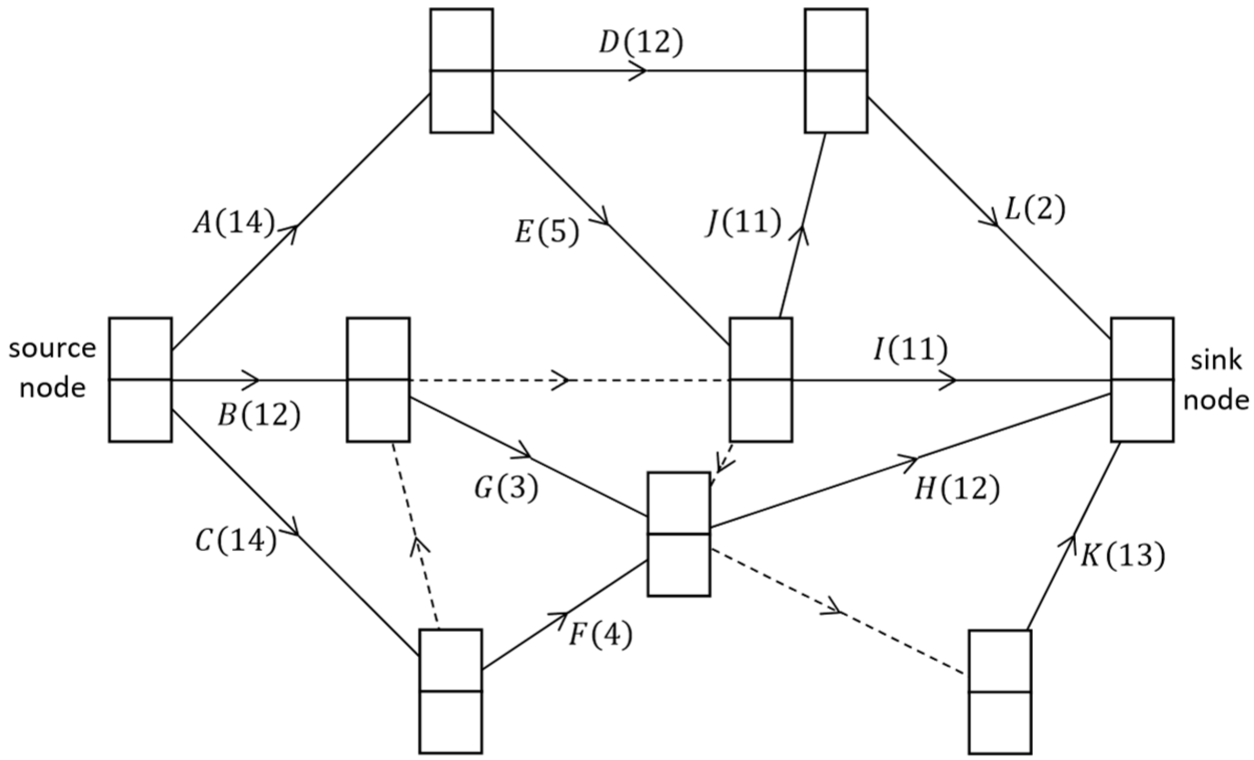
(i) Complete the table on the next page by listing, for each activity, the other activities on which it depends directly. That is, for each activity $X \in \{A, B, C, \dots, L\}$, write the smallest possible list of other activities which need to be completed before activity X can begin.

Use the space below to show relevant supporting work, if necessary.

Activity	Depends directly on ...	Activity	Depends directly on ...
A		G	
B		H	
C		I	
D		J	
E		K	
F		L	

(ii) Calculate the early time and the late time for each event.

Complete the diagram below by writing the early time (upper box) and late time (lower box) at the node representing each event.



Use the space below and on the next page to show relevant supporting work, if necessary.

(iii) Write down the critical path(s) for the network.

A cascade chart (Gantt chart) is a type of bar chart which may be used to represent a project's schedule. The duration of each activity is represented by the width of the horizontal bar for that activity, with time on the horizontal axis. The float time for an activity is represented by a rectangle drawn using dotted lines to the right of the bar for that activity. The top row of a cascade chart is used for a critical path.

(iv) Draw a cascade chart or similar bar chart to represent the schedule for this project.

day

2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34

The hospital manager visits the project on day 18 to check the progress of the work, which is on schedule.

(v) Write down the activities which may be happening on day 18.

Question 10

(a) An entomologist (a scientist who studies insects) maintains a population of grasshoppers in her laboratory.

The entomologist's research tells her that the population of this species of grasshopper should increase by a factor of 1.2 each month if they are left undisturbed. However the entomologist removes 30 grasshoppers from the population each month, to carry out research on them.

The entomologist develops a difference equation model to predict U_n, the number of grasshoppers present at the beginning of month n.

At the start of the first month the entomologist has 175 grasshoppers, i.e. $U_0 = 175$.

(i) Calculate the values of U_1 and U_2.

(ii) Write down a difference equation to express U_{n+1} in terms of U_n, where $n \geq 0$, $n \in \mathbb{Z}$.

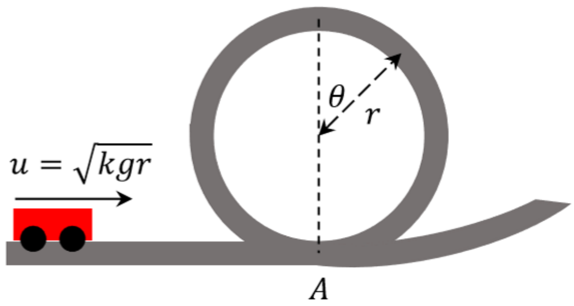
(iii) Solve this difference equation to find an expression for U_n in terms of n.

(iv) Calculate U_{12}, the number of grasshoppers which the model predicts will be in the population after one year.

(b) A toy car track consists of a series of components that connect to make a closed circuit. Part of the track makes a vertical circular loop.

To model the motion of a car on this track, its velocity at the base of the loop (point A) is expressed as $u = \sqrt{kgr}$, where r is the radius of the loop, g is the acceleration due to gravity, and k is a constant.

The model ignores the effects of friction.



(i) Draw a diagram to show the forces acting on the car at the instant when the radius to the car makes an angle θ with the upward vertical.

(ii) If the car loses contact with the track at the instant when the radius to the car makes an angle θ with the upward vertical, show that $\cos\theta = \frac{k-2}{3}$.

(iii) Calculate the minimum value of k such that the car successfully completes the loop without losing contact with the track.

Leaving Certificate Examination 2023

Applied Mathematics

Higher Level

Deferred Exam

400 marks

Question 1

(a) A and B are two 3×3 matrices.

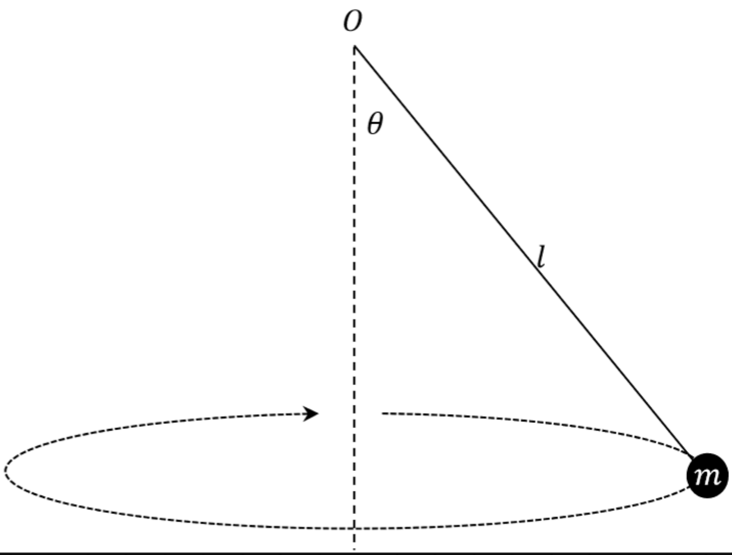
$$A = \begin{pmatrix} -1 & 0 & 2 \\ 4 & -3 & 0 \\ 1 & 2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 & 2 \\ 3 & -2 & 0 \\ 1 & 4 & 0 \end{pmatrix}$$

(i) Calculate AB.

(ii) Verify that $AB \neq BA$.

(b) A spherical bob of mass m is attached to a light inextensible string of fixed length l. It is suspended from support point O.

The string makes an angle θ with the vertical. The bob moves through a horizontal circle which has its centre on the vertical. The bob has constant linear speed v.



(i) Show on a diagram the forces acting on the bob.

(ii) Derive an expression for v in terms of l, θ and g, the acceleration due to gravity.

(iii) Derive an expression for T, the period of rotation of the bob, in terms of l, θ and g.

(iv) Use dimensional analysis to show that the units for the expression you derived in part **(iii)** are equivalent to the units for period.

Question 2

(a) Seven computers, A, B, C, D, E, F and G, are part of a computer network. Each computer is connected to one or more of the other computers on the network. The time (in ms) for communication between each of the connected computers is given in the table below.

	A	B	C	D	E	F	G
A	–	42	33	–	–	–	–
B	42	–	20	54	–	–	–
C	33	20	–	–	105	78	–
D	–	54	–	–	29	–	94
E	–	–	105	29	–	41	49
F	–	–	78	–	41	–	71
G	–	–	–	94	49	71	–

A computer scientist wishes to model this information using a weighted graph, where the nodes represent computers $A - G$ and the weights of the edges represent the communication times between the connected computers.

(i) Use the table above to draw a weighted graph to represent the computer network.

(ii) Calculate the shortest time for a message to travel from computer A to computer G. List the computers that the message travelled through, in order. Name the algorithm you used. Relevant supporting work must be shown.

(b) In another, larger computer network, a message travels from one computer to another in the network. Each time a message travels from one computer to the next the number of errors in the message, E, increases by 15%. However C errors are corrected each time the message travels. The number of computers the message travels to is counted using the number n.

A message starts at computer $n = 0$ and travels on a linear path through the computer network.

E, the number of errors in the message, may be modelled by the difference equation:

$$E_{n+1} = 1.15E_n - C$$

where $n \geq 0$, $n \in \mathbb{Z}$.

There are 101 errors in the message when it leaves computer 0, i.e. $E_0 = 101$.

(i) Solve this difference equation to find an expression for E_n in terms of n and C.

(ii) It is found that the message contains zero errors after it reaches the 21^{st} computer, i.e. $E_{21} = 0$. Calculate the value of C to the nearest whole number.

(iii) E may also be modelled using a differential equation. Write a differential equation for $\frac{dE}{dn}$, the rate of change of E with respect to n, in terms of E and C.

Question 3

In an economic model, the gross national income G of a country, consists of three separate contributions:

$$G = P + I + S$$

where P represents private spending by citizens, I represents investment in the economy, and S represents government spending.

G can be modelled using a difference equation, where P and I change each year n and where S is assumed to be constant. That is:

$$G_n = P_n + I_n + S$$

In any year, P is proportional to the value of G for the previous year. That is:

$$P_{n+1} = aG_n$$

where $n \geq 0$, $n \in \mathbb{Z}$ and $a = \frac{15}{16}$.

In any year, I is proportional to the change in the value of P between that year and the previous one. That is:

$$I_{n+1} = b(P_{n+1} - P_n)$$

where $n \geq 0$, $n \in \mathbb{Z}$ and $b = \frac{3}{5}$.

(i) Use this information to form a second-order inhomogeneous difference equation for G and express it in the form:

$$G_{n+2} + cG_{n+1} + dG_n = S$$

for the constants c, d which are to be determined.

Calculate the values for c and d.

(ii) Assuming the government spends no money (i.e. assuming $S = 0$ euro), G can be expressed by the second-order homogeneous difference equation:

$$G_{n+2} + cG_{n+1} + dG_n = 0$$

Using $G_0 = 840$ and $G_1 = 820$ in billions of euros, solve this difference equation to find an expression for G_n in terms of n.

Calculate G_6 to the nearest billion euros.

(iii) Assuming the government spends 40 billion euros each year (i.e. assuming $S = 40$ in billions of euros), G can be expressed by the second-order inhomogeneous difference equation:

$$G_{n+2} + cG_{n+1} + dG_n = 40$$

Again using $G_0 = 840$ and $G_1 = 820$ in billions of euros, solve this difference equation to find an expression for G_n in terms of n.

Again calculate G_6 to the nearest billion euros.

Question 4

In 1838 the Belgian mathematician Pierre François Verhulst published a differential equation to model rate of change of population P with respect to time t:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

where r and K are constants for a given population.

For a certain species of insect in an environment it is known that the population can increase by up to 8% per week, i.e. $r = 0.08$.

At $t = 0$ weeks there are 20 insects in the population.

When the population P is small relative to K, the ratio $\frac{P}{K}$ is also small and Verhulst's model can be approximated by the simplified differential equation:

$$\frac{dP}{dt} = rP$$

(i) Solve this simplified differential equation to find an expression for P in terms of t.

(ii) Calculate P to the nearest whole number when $t = 12$ weeks.

(iii) Explain why this approximation of Verhulst's model is not practical for predicting the long-term behaviour of the population of insects.

(iv) Solve the differential equation for Verhulst's model:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

to find an expression that relates P, K and t.

Note that $\frac{1}{y(x-y)} = \frac{1}{x}\left(\frac{1}{y} + \frac{1}{x-y}\right)$.

(v) $P = 39$ insects when $t = 12$ weeks. Calculate the value of K to the nearest whole number.

(vi) Explain the significance of K in the Verhulst model.

Question 5

(i) A ball is thrown vertically upwards from the edge of a building that is 24.5 m high. The ball reaches its maximum height 2 s after it is thrown. Using a model that neglects the effects of air resistance, calculate the time from when the ball is thrown to when it lands on the ground at the bottom of the building.

A more sophisticated model for the motion of a ball that is thrown vertically upwards includes the effects of air resistance. The rate of change of the velocity v of the ball in terms of time t during the upward part of its journey can be modelled by the following differential equation:

$$\frac{dv}{dt} = -g - kv$$

where $k > 0$ is a constant. Take the initial upward velocity of the ball to be 20 m s^{-1}.

(ii) Solve this differential equation to find an expression for v in terms of t and k.

(iii) Using $k = 0.1225$, calculate the time the ball takes to reach its maximum height.

(iv) Write down a differential equation for the rate of change of the velocity of the ball on the downward part of its journey.

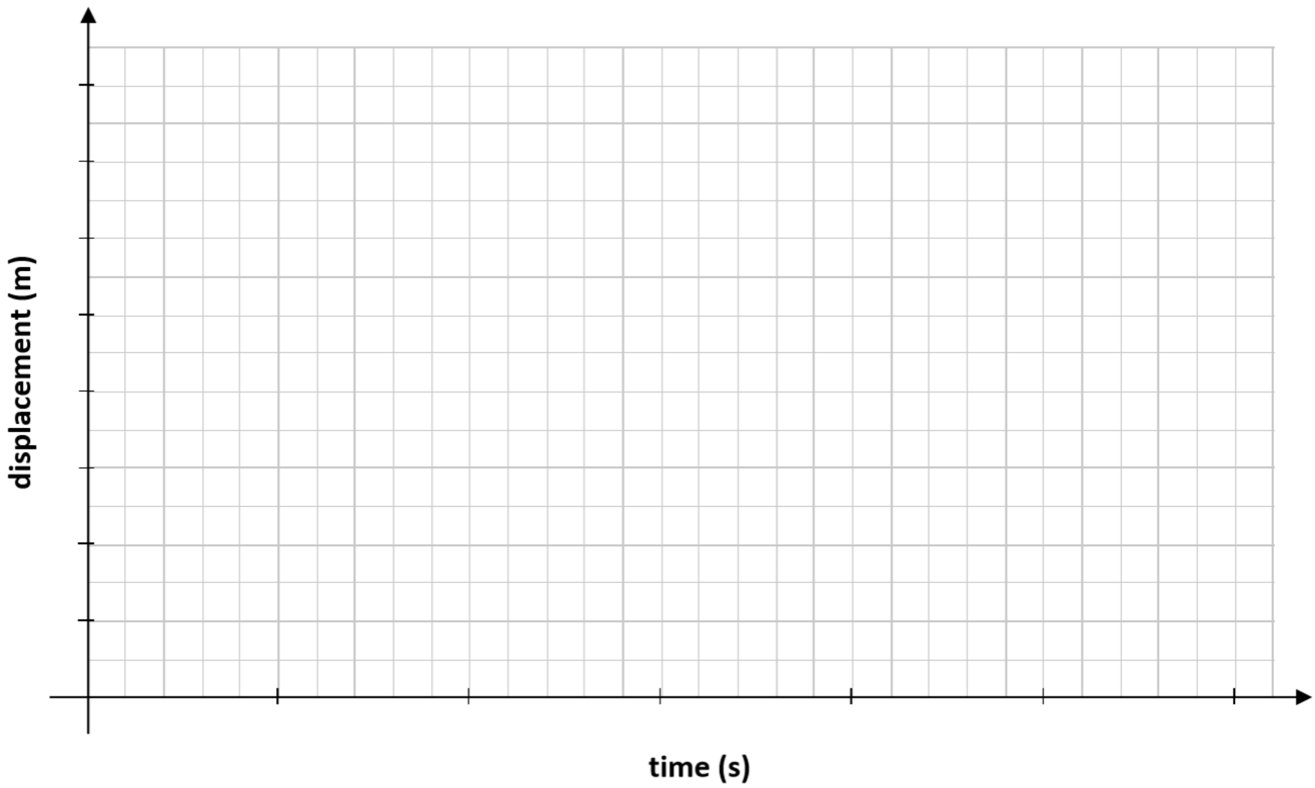
Question 6

(a) Motorbike B travelling with speed 5.5 m s^{-1} and constant acceleration 0.5 m s^{-2} on a straight stretch of road is overtaken, at a road sign S, by car C travelling with speed 11 m s^{-1} and constant acceleration 0.125 m s^{-2}.

(i) Calculate the greatest distance that car C is ahead of motorbike B.

(ii) Calculate the distance from S to the point where B overtakes C.

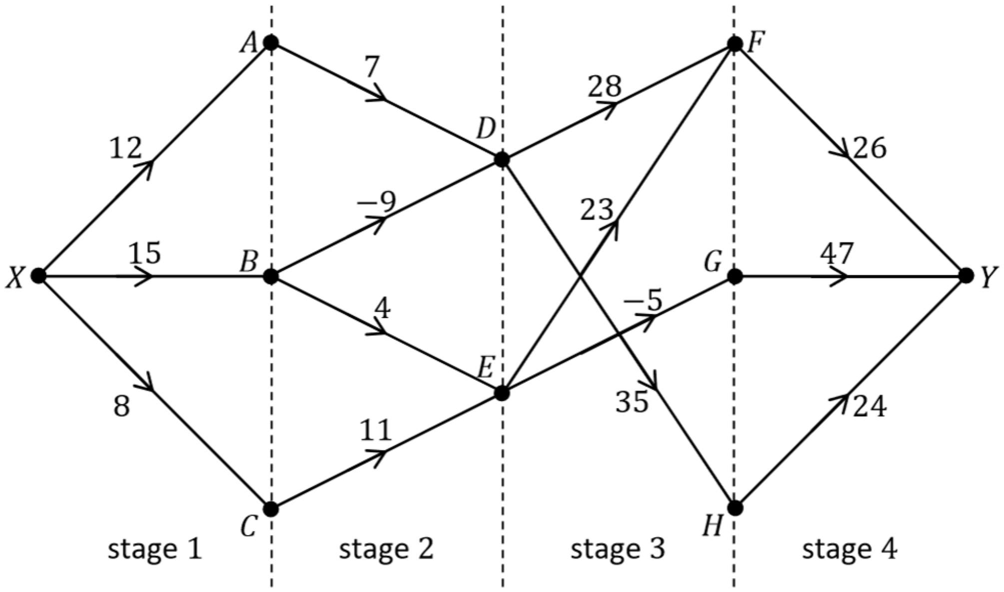
(iii) Using the axes below, sketch the shape of the displacement-time graph for the displacement of B relative to S for the first 30 s of its motion after it passes S. Using the same axes, sketch the shape of the displacement-time graph for the displacement of C relative to S for the same period of time.
Include scales on your axes.



(b) A project manager is responsible for maintaining a portion of road. The maintenance work is carried out in four stages. For each stage, the manager has a number of options regarding how to complete the work and which sub-contractors to use.

The options available to the manager may be modelled as a network. The options are represented by edges, where the weight of the edge represents the cost of that option in thousands of euros. Some of the weights are negative because of discounts offered by sub-contractors. The manager wishes to choose the optimal policy for maintaining the road, i.e. the cheapest overall plan.

The nodes X and Y represent the initial and final states of the decision problem. Each of the other nodes represents a possible state of the decision problem.



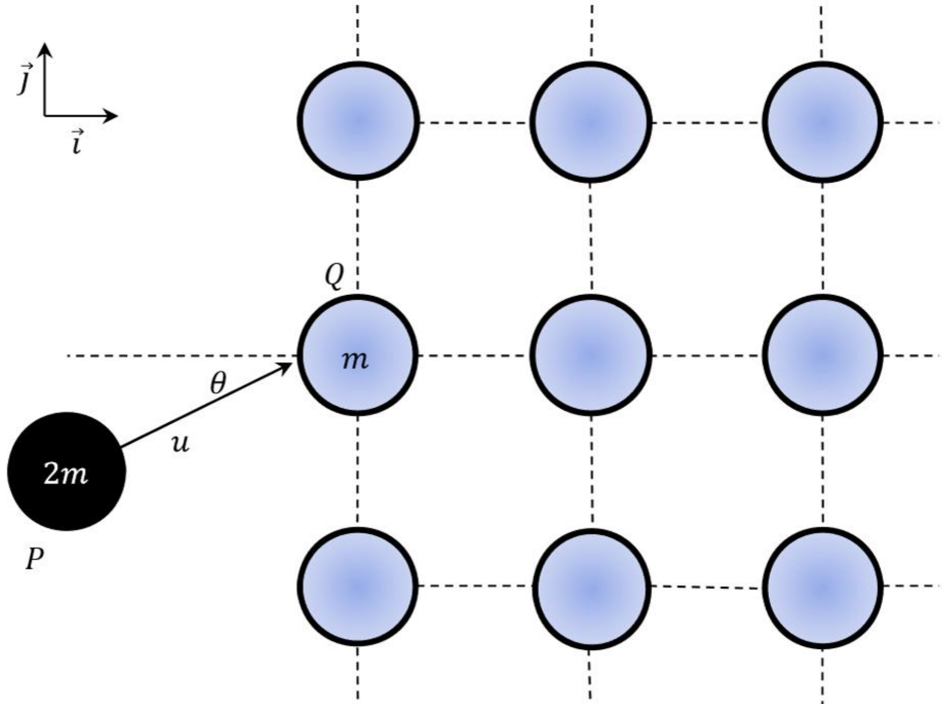
Calculate the plan which minimises the cost of maintaining the road. Relevant supporting work must be shown.

Question 7

A solid can be modelled as a two-dimensional lattice of identical particles of mass m. A particle of the solid may be moved temporarily out of its position but it is quickly returned to that position by the forces that hold the solid together.

An incoming particle P of mass $2m$, moving with speed u, collides obliquely with particle Q which is at rest on the outer surface of the solid. The line joining the centres of the particles at the point of impact is along the $\vec{\imath}$ axis.

Before the collision, the direction of P makes an angle θ with the $\vec{\imath}$ axis.



After the collision the direction of P has turned through an angle θ, such that it now makes an angle 2θ with the $\vec{\imath}$ axis.

The coefficient of restitution for the collision is $\frac{2}{3}$.

(i) Show that $\tan\theta = \frac{1}{3}$.

(ii) Calculate the $\vec{\imath}$ and $\vec{\jmath}$ components of the velocity of Q immediately after the collision in terms of u.

After the collision Q experiences a restoring force F which is proportional to x, the displacement of Q from its initial position, where k is the constant of proportionality. (That is, the restoring force may be modelled as being equivalent to the restoring force exerted on a particle by a spring of spring constant k stretched through displacement x.)

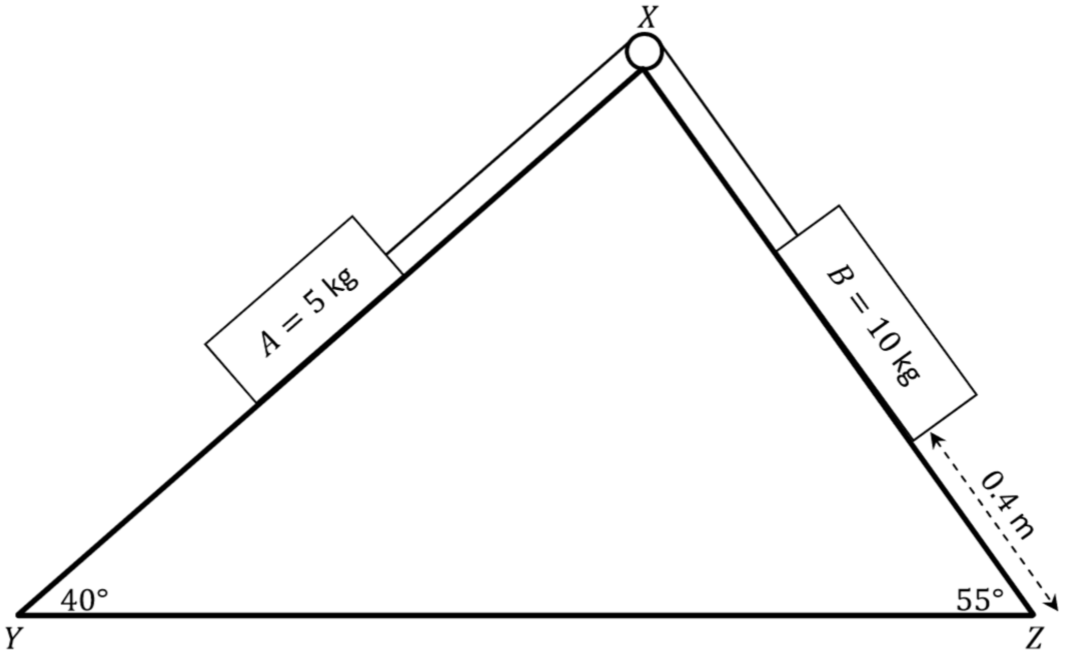
(iii) Derive an expression for the work done when Q moves through displacement x.

(iv) Find the maximum displacement of Q from its initial position in terms of m, k and u.

Question 8

Two rectangular blocks A and B, of mass 5 kg and 10 kg, rest on two sides of fixed triangular wedge XYZ, with side YZ lying on the horizontal ground, as shown in the diagram. The blocks are connected by a light inextensible string passing over a smooth pulley at X.

The edge of block B is a distance of 0.4 m from the ground along side XZ. The angles of inclination of sides YX and ZX with the horizontal ground are $40°$ and $55°$ respectively.



μ_1, the coefficient of friction between side YX and block A, is $\frac{1}{8}$.

μ_2, the coefficient of friction between side ZX and block B, is $\frac{1}{5}$.

The wedge does not move when the system is released from rest.

(i) Show, on separate diagrams, the forces acting on blocks A and B while they are moving.

(ii) Calculate the common acceleration of blocks A and B and the tension in the string.

Block B hits the ground and does not rebound.

(iii) Calculate the speed of block B when it touches the ground.

After block B hits the ground, block A continues to move up side YX.

(iv) Calculate the new acceleration of block A as it continues to move up side YX.

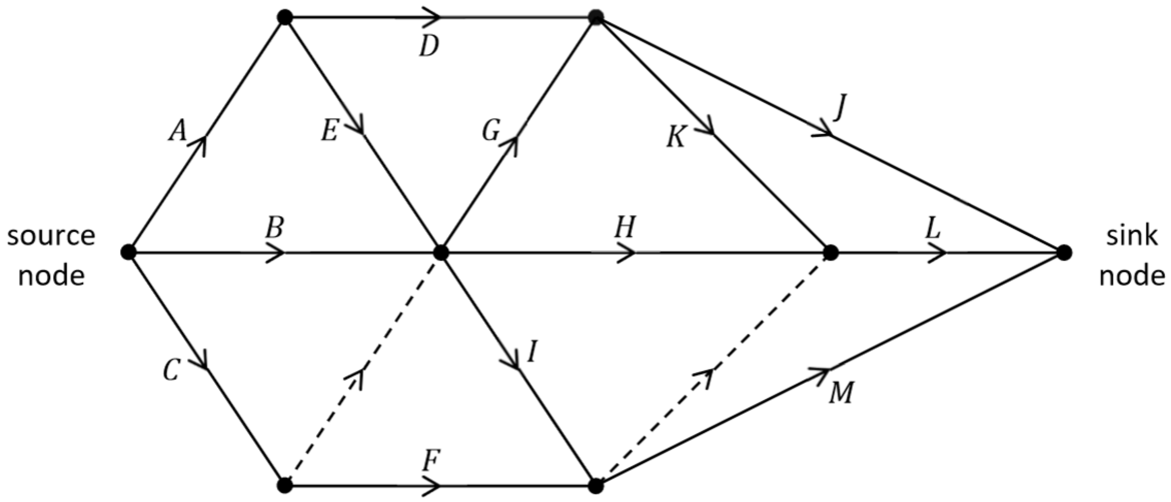
(v) Calculate the total displacement of block A from its initial position when it is at its greatest height.

Question 9

The diagram below shows the scheduling network for a project to construct a stage for a music performance. The network provides some information about the relationships between the thirteen activities that have to be completed in the project.

The edges of the network represent these activities and are labelled with the letters A to M. The letters used to label the edges should **not** be taken as representing the order in which the activities happen.

The nodes of the network represent events or points in time during the project. The source node is the time when the project begins and the sink node is the time when the project ends.



(i) Explain the significance of the edges represented by dotted lines.

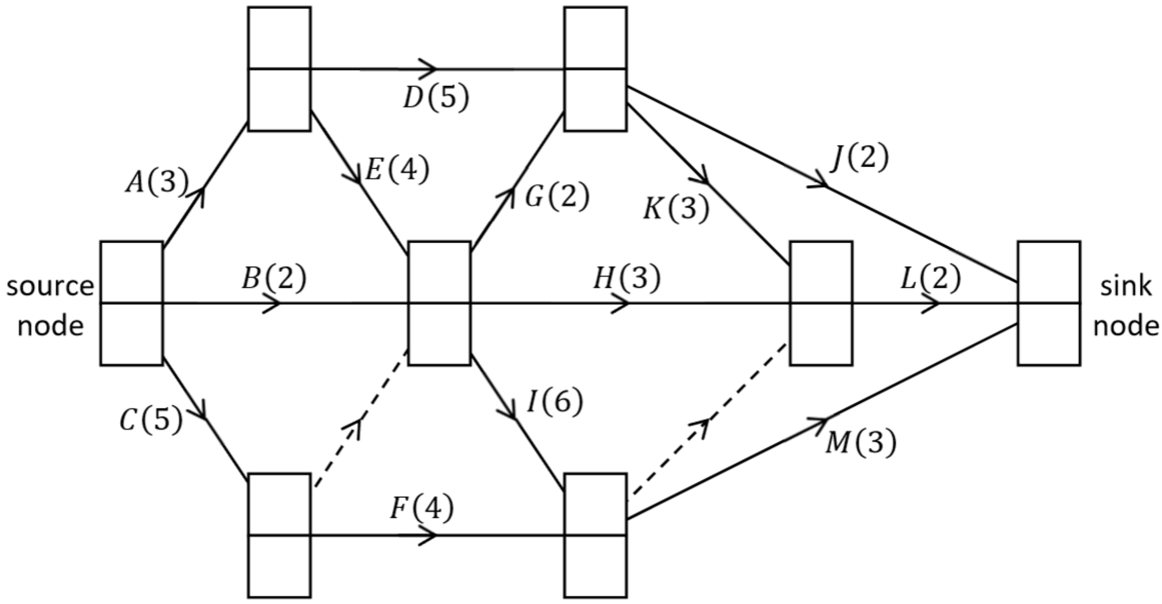
(ii) Complete the table below by listing, for each activity, the other activities on which it depends directly. That is, for each activity $X \in \{A, B, C, \dots, M\}$, write the smallest possible list of other activities which need to be completed before activity X can begin.

Use the space below to show relevant supporting work, if necessary.

Activity	**Depends directly on …**	**Activity**	**Depends directly on …**
A		H	
B		I	
C		J	
D		K	
E		L	
F		M	
G			

(iii) The time, in hours, to complete each of the activities is represented by the number in brackets. Calculate the early time and the late time for each event.

Complete the diagram below by writing the early time (upper box) and late time (lower box) at the node representing each event.



Use the space below to show relevant supporting work, if necessary.

(iv) Write down the critical path(s) for the network.

(v) Calculate the minimum time needed to complete the project.

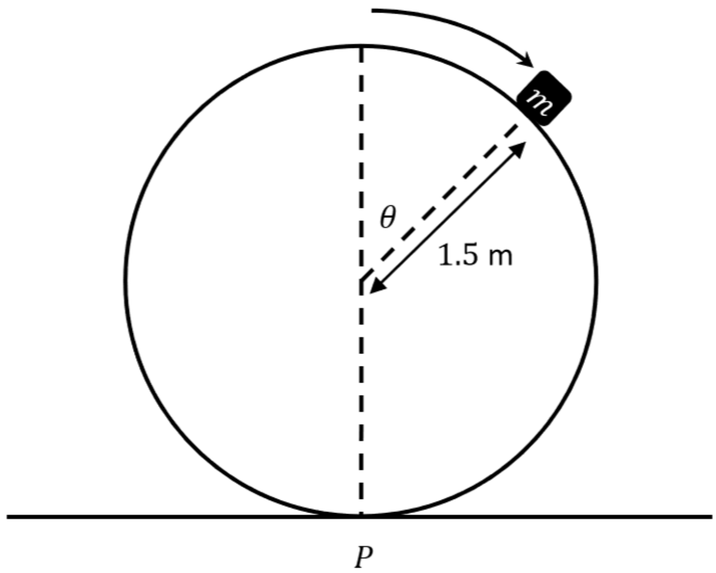
(vi) If activity D takes 7 hours instead of 5 hours, what effect will this have on the critical path(s) and the time it takes to complete the project? Explain your answer.

(vii) If activity J takes 7 hours instead of 2 hours, what effect will this have on the critical path(s) and the time taken to complete the project? Explain your answer.

Question 10

A smooth cylinder of radius 1.5 m lies on its side in a fixed position on horizontal ground.
A diagram of a circular cross-section of the cylinder is shown below. The point of contact of the cylinder with the ground is fixed at point P.

A small object of mass m rests on the highest point of the cylinder, vertically above P. The object is slightly disturbed from rest so that it begins to slide down the cylinder. As it slides it makes an angle θ with the vertical, as shown in the diagram.



A student wishes to model the initial motion of the object as motion in a vertical circle.

(i) Outline the assumptions made by the student's model.

(ii) Calculate the value of θ when the object leaves the surface of the cylinder.

(iii) Calculate the velocity of the object when it leaves the surface of the cylinder.

The student models the motion of the object after it leaves the surface of the cylinder as projectile motion in a uniform gravitational field.

(iv) Calculate the time between when the object leaves the surface of the cylinder and when it lands on the ground.

(v) Calculate the horizontal distance between point P and the point where the object lands on the ground.

Leaving Certificate Examination 2024

Applied Mathematics

Higher Level

Tuesday 25 June Afternoon 2:00 - 4:30

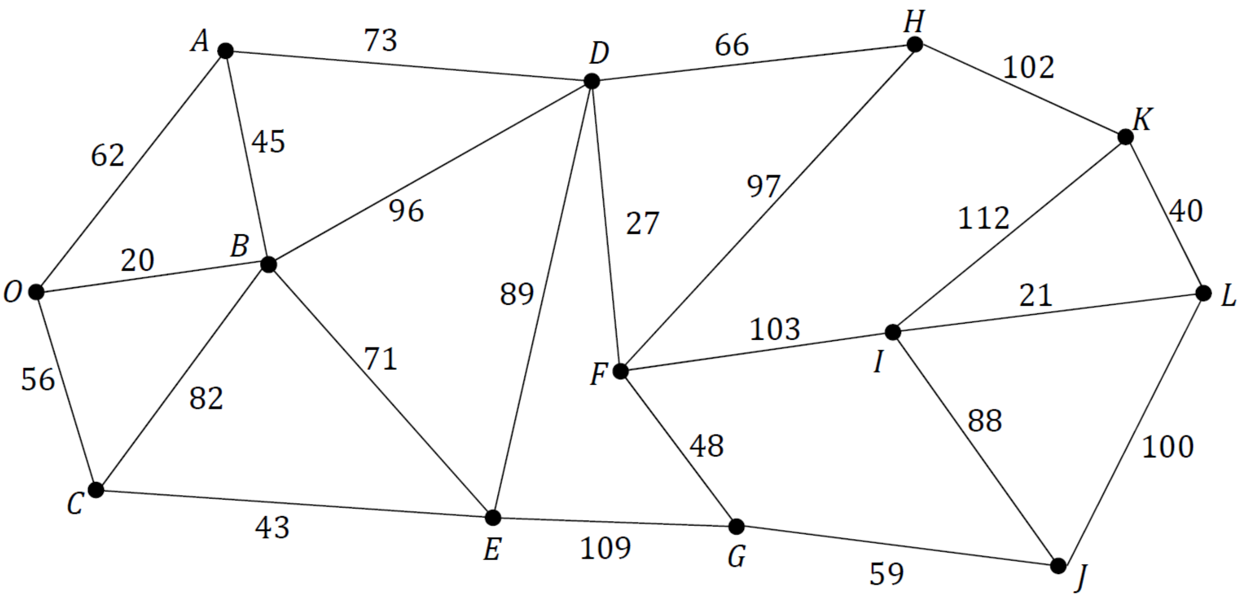
400 marks

Question 1

(a) Matrix $A = \begin{pmatrix} -1 & 0 & 2 \\ 3 & 1 & -1 \\ 0 & 2 & -3 \end{pmatrix}$. Matrix $B = \begin{pmatrix} 5 & -1 & -2 \\ 0 & 4 & 3 \\ 2 & -1 & 0 \end{pmatrix}$. Calculate AB.

(b) A security manager wishes to install cameras in the public areas of a shopping centre. She wishes to locate these cameras so as to minimise the total length of the cables between them.

In the network shown below node O represents the manager's office. The nodes A to L represent the locations where the security cameras are to be installed. The weight of each edge represents the distance (in meters) between each location.



(i) Using an appropriate algorithm, find the minimum spanning tree for the network. Name the algorithm you used. Relevant supporting work must be shown.

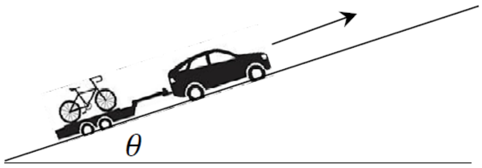
(ii) Calculate the shortest total distance the security manager would have to walk if she started at her office (O) and followed the minimum spanning tree to visit each of the camera locations in the shopping centre.

(c) John is cycling on a straight horizontal road at a constant velocity of 10 m s^{-1} and is 21 m behind another cyclist, Kevin, who is cycling at a constant velocity of 4 m s^{-1} in the same direction. John begins to accelerate at 2 m s^{-2}. One second later, Kevin begins to accelerate at 4 m s^{-2}.

Calculate the times when John and Kevin overtake each other.

Question 2

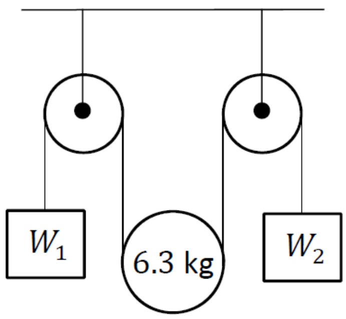
(a) A jeep of mass 2375 kg pulls a trailer of mass 350 kg up a hill inclined at angle θ. The jeep and trailer move at a constant speed. The engine of the jeep exerts a force of 4400 N up the hill. The forces due to friction on the jeep and the trailer are 1525 N and 375 N respectively.



(i) Draw a diagram to show the forces acting on the jeep.

(ii) Calculate θ.

(b) A student sets up the pulley arrangement shown in the diagram.



A light inextensible string passes over two smooth fixed pulleys and under a smooth movable pulley of mass 6.3 kg.

Weight $W_1 = 24.5$ N is attached to one end of the string and weight $W_2 = 44.1$ N is attached to the other end.

The system is released from rest.

(i) Show, on separate diagrams, the forces acting on the pulley and on each of the weights while they are moving.

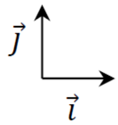
(ii) Calculate the tension in the string.

Question 3

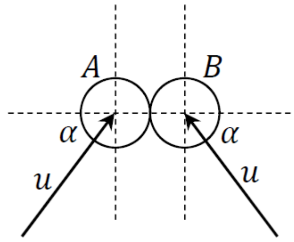
(a) A particle moving along a straight line has acceleration $a = \frac{dv}{dt} = t^2 \sin 2t$ where $t \geq 0$ and $v = 0$ when $t = 0$.

Using integration by parts or otherwise, calculate v when $t = \frac{\pi}{2}$.

(b) Two identical smooth spheres, A and B, each moving with speed u, collide obliquely. The line joining their centres at the point of impact is along the $\vec{\imath}$ axis.



Before the collision the velocity of A makes an acute angle α with the *positive* direction of the $\vec{\imath}$ axis and the velocity of B makes an acute angle α with the *negative* direction of the $\vec{\imath}$ axis, as shown in the diagram.



The coefficient of restitution between the spheres is e, where $0 \leq e \leq 1$.

(i) Calculate, in terms of e and u, the velocity of each sphere after the collision.

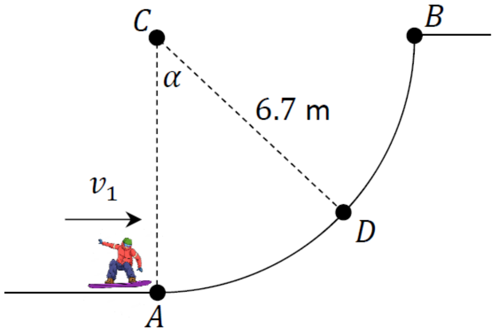
(ii) A and B move perpendicularly to each other after the collision.
Show that $e = \tan \alpha$.

Question 4

A snowboarder of mass m is travelling with velocity v_1 when he enters circular arc AB at point A.

AB has radius 6.7 m and centre C.
A, B and C lie in a vertical plane.
AC is vertical and BC is horizontal.

A student models the motion of the snowboarder, ignoring the effects of friction and air resistance.



(i) Draw a diagram to show the forces acting on the snowboarder when he is at point D, where CD makes angle α with the downward vertical.

(ii) Express the reaction force on the snowboarder at point D in terms of m, v_1 and α.

The snowboarder reaches point B with vertical velocity v_2.

(iii) Express v_2 in terms of v_1.

At point B the snowboarder leaves the arc and moves vertically upwards through the air.
The student wishes to calculate v, the velocity of the snowboarder t seconds after he leaves the arc. The force due to air resistance is now modelled as mkv.

(iv) Draw a diagram to show the forces acting on the snowboarder while he is moving upwards through the air.

(v) Show that while the snowboarder is moving upwards through the air, the rate of change of v with respect to t can be expressed by the differential equation:

$$\frac{dv}{dt} = -(g + kv)$$

(vi) Solve this differential equation to find an expression for v in terms of t, k and v_1.

Question 5

(a) Smooth sphere S_1 of mass $2m$ and speed $2u$ collides directly with smooth sphere S_2 of mass $3m$ which is moving in the opposite direction with speed u.

The coefficient of restitution between the spheres is e, where $0 \leq e \leq 1$.

(i) Calculate, in terms of e and u, the speed of each sphere after the collision.

(ii) Calculate the range of values for e such that after the collision the spheres both move in the same direction.

(b) A train departs from Connolly Station and accelerates uniformly from rest for t_1 seconds until it reaches a speed of v. The train maintains this speed for t_2 seconds, where $t_2 = 2t_1$. The train then decelerates to rest at Pearse Station in a time of t_3 seconds.

The total time taken for the journey is $T = t_1 + t_2 + t_3$.

(i) Show that $T = \frac{2d - t_2 v}{v}$, where d is the distance between Connolly Station and Pearse Station.

(ii) If the average speed for the entire journey is $\frac{2v}{3}$, show that $T = 6t_1$.

Question 6

(a) A statistician is monitoring the population of a small island.

Due to the birth rate being greater than the death rate, the population of the island naturally increases by 1.2% per year. However a fixed number of people, x, emigrate from the island every year.

The statistician develops a difference equation to model P_n, the population at the beginning of the year n. The model assumes that emigration occurs at the end of each year.

At the start of the year 2022 there were 13 500 people on the island, i.e. $P_0 = 13\,500$.

(i) Write down a difference equation to express P_{n+1} in terms of P_n and x, where $n \geq 0$, $n \in \mathbb{Z}$.

(ii) Solve this difference equation to find an expression for P_n in terms of n and x.

At the start of the year 2023 there were $13\,514$ people on the island, i.e. $P_1 = 13\,514$.

(iii) Calculate the value of x.

(iv) Calculate the predicted population at the start of the year 2029.

(b) The acceleration of a particle moving in a straight line may be expressed in terms of its velocity v in m s^{-1} and its displacement s in m by the differential equation:

$$v\frac{dv}{ds} = e^{\frac{v^2}{4}}$$

(i) Solve the differential equation to find an expression for v in terms of s, given that $v = 0$ when $s = 0$.

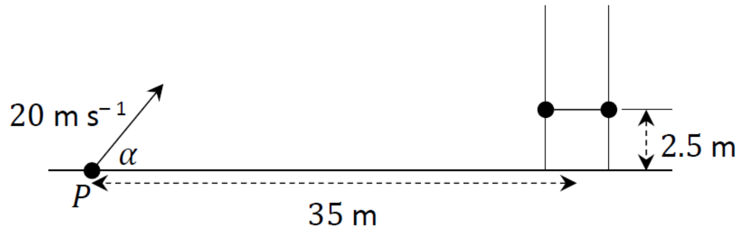
(ii) Calculate the velocity of the particle when $s = 0.3$ m.

Question 7

(a) Fiona plays camogie and wishes to improve her accuracy in scoring points.

She stands at P, 35 m in front of the goal line and strikes a sliotar with an initial velocity of 20 m s^{-1} at an angle α to the horizontal ground.

In order to successfully score a point the sliotar must pass over the cross bar which is 2.5 m above the ground, as shown in the diagram.



(i) Derive an expression, in terms of α, for the time taken for the sliotar to reach the goal line.

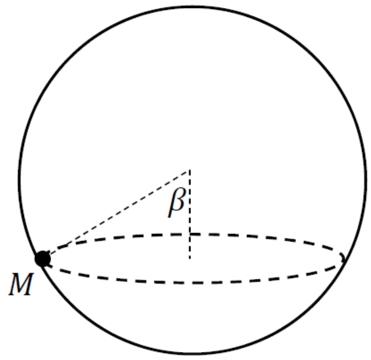
(ii) Calculate the values of α such that the sliotar hits the cross bar.

(b) In the stunt cage of a circus, a motorcyclist performs acrobatic stunts in a hollow sphere of diameter 20 m.

The motorcyclist and motorbike, labelled M in the diagram, have a combined mass of m kg.

M moves with uniform horizontal circular motion.

The line from M to the centre of the sphere makes an angle $\beta = \tan^{-1}\frac{4}{3}$ with the downward vertical.



A student models the motion of M. In the student's first modelling iteration friction is ignored.

(i) Draw a diagram to show the forces acting on M.

(ii) Calculate the velocity of M.

The student's second modelling iteration introduces a coefficient of friction, μ.

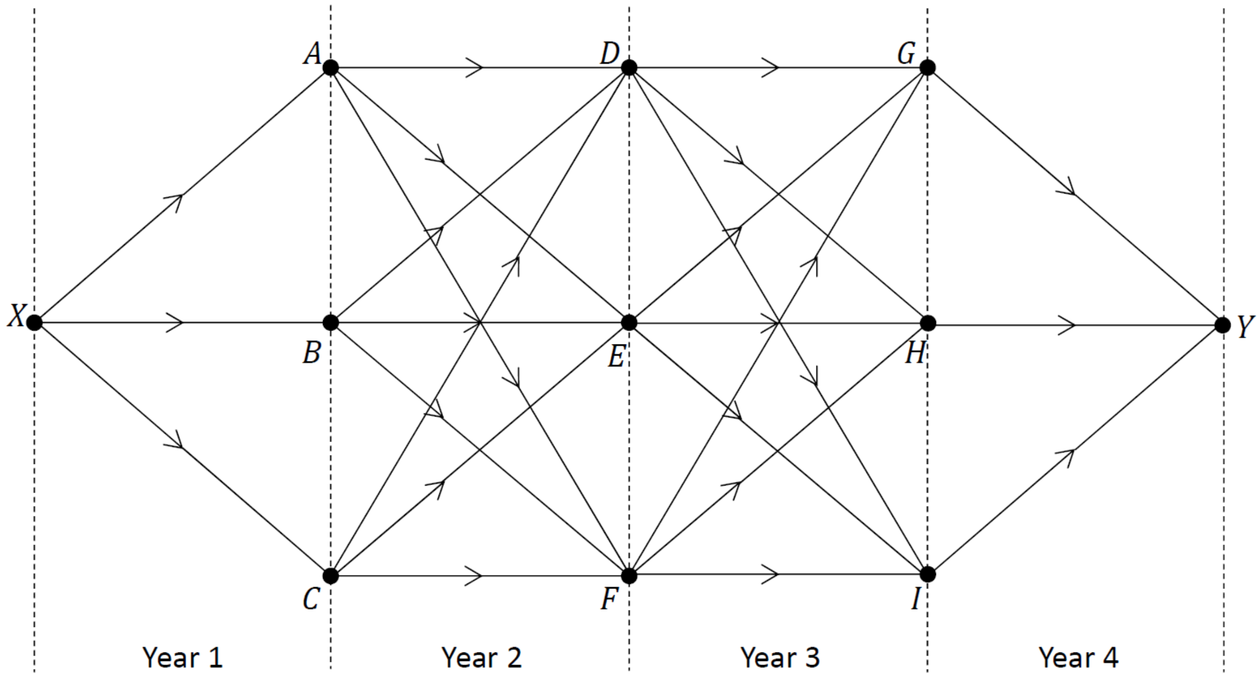
(iii) Draw a diagram to show the forces acting on M when it is on the point of slipping downwards.

(iv) M has velocity 7.5 m s^{-1} when it is on the point of slipping downwards.
Calculate the value of μ.

Question 8

(a) A company director wishes to design a business plan to promote her brand over four years. Each year she chooses from a number of different promotion strategies. She estimates the profit (positive value) or loss (negative value) of each strategy (in €1000's).

She draws the network shown below to help design the most profitable plan, where the edges represent the different strategies and the nodes represent the possible states associated with the plan at a given point in time. X and Y represent the start point and the end point of the plan respectively.



The table below shows the estimated profit (positive value) and the estimated loss (negative value) of each strategy.

Strategy	Estimated profit/loss	Strategy	Estimated profit/loss
X to A	-11	D to G	-5
X to B	-13	D to H	-3
X to C	-9	D to I	3
A to D	5	E to G	-2
A to E	-2	E to H	5
A to F	-5	E to I	6
B to D	7	F to G	4
B to E	-3	F to H	3
B to F	4	F to I	2
C to D	5	G to Y	6
C to E	-4	H to Y	5
C to F	-1	I to Y	7

(i) Use Bellman's Principle of Optimality to calculate the business plan that maximises profit. Relevant supporting work must be shown.

(ii) State one difference between Bellman's Principle of Optimality and Dijkstra's algorithm.

(b) The algebraic formula below is written in terms of force F, mass m, displacement s and angular velocity ω.

$$\sqrt{\frac{4Fs}{m\omega^2}}$$

Use dimensional analysis to show that this formula has the same units as the units for displacement.

Question 9

A geologist is carrying out a survey of an opal mine.

During the first month of mining, a mass of 200 kg of opal was removed. During the second month of mining, a mass of 245 kg of opal was removed.

The geologist predicts that M, the mass of opal removed in any month, can be expressed by the second-order homogeneous difference equation:

$$M_{n+2} = M_{n+1} + \frac{3M_n}{4}$$

where $n \geq 0, n \in \mathbb{Z}, M_0 = 200$ and $M_1 = 245$.

(i) Write down the values of M_2 and M_3.

(ii) Solve the difference equation to find an expression for M_n in terms of n.

(iii) Calculate the total mass of opal that is predicted to be removed during the first six months of mining.

After the government introduces stricter mining laws, the geologist changes their predictions by estimating that the mass of opal mined in month n will be reduced by 2^n kg.

The geologist predicts that P, the mass of opal removed in any month, can now be expressed by the second-order inhomogeneous difference equation:

$$P_{n+2} = P_{n+1} + \frac{3P_n}{4} - 2^{n+2}$$

where $n \geq 0$, $n \in \mathbb{Z}$, $P_0 = 199$ and $P_1 = 243$.

(iv) Solve this new difference equation to find an expression for P_n in terms of n.

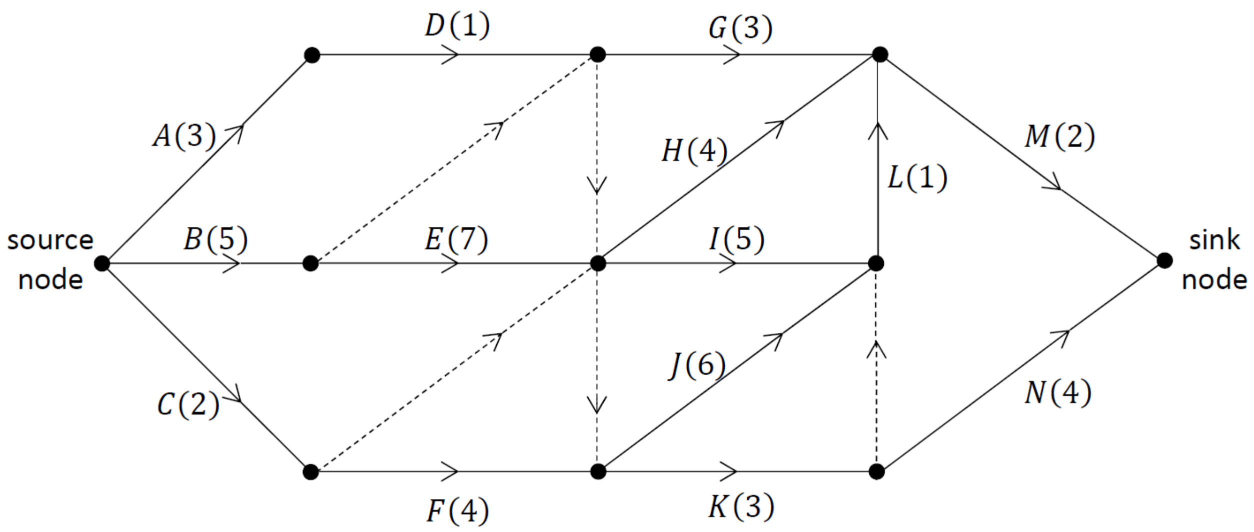
(v) Calculate the total mass of opal that is now predicted to be removed during the first six months of mining.

Question 10

(a) The diagram below shows the scheduling network for manufacturing a car.

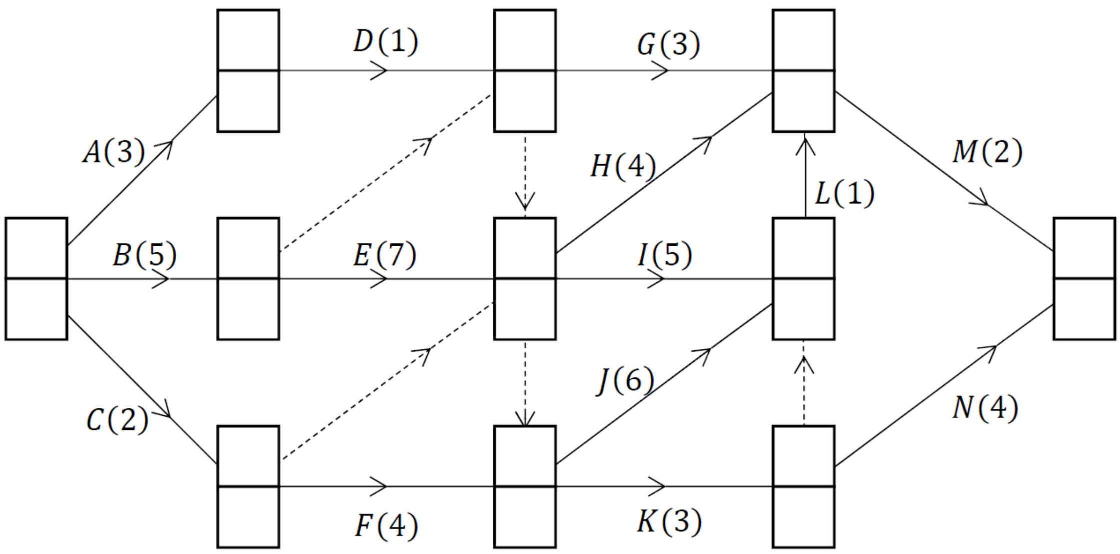
The edges of the network represent the activities that have to be completed as part of the overall manufacture of the car and are labelled with the letters A to L. The duration, in weeks, of each activity is represented by the number in brackets. The unlabelled edges (shown with dashed lines) do not represent real activities but they help explain the order in which the activities must happen. The letters used to label the edges should **not** be taken as representing the order in which the activities happen.

The nodes of the network represent events or points in time during the project. The source node is the time when the project begins and the sink node is the time when the project ends.



(i) Calculate the early time and the late time for each event.

Complete the diagram below by writing the early time (upper box) and late time (lower box) at the node representing each event.



(ii) Write down the critical path(s) for the network.

(iii) What is a critical path?

(iv) If activity K takes 5 weeks instead of 3 weeks, what effect will this have on the critical path(s) and the time it takes to complete the project? Explain your answer.

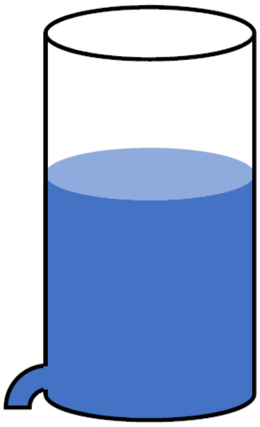
(b) A tank of uniform cross-sectional area contains water of height x m.

Water leaves the tank through a pipe at its base.

The rate at which the height of the water decreases is proportional to x.

At time $t = 0$, $x = H$. At $t = 45$ s, $x = \frac{H}{3}$.

Calculate t when $x = \frac{H}{8}$.



Leaving Certificate Examination 2025

Applied Mathematics

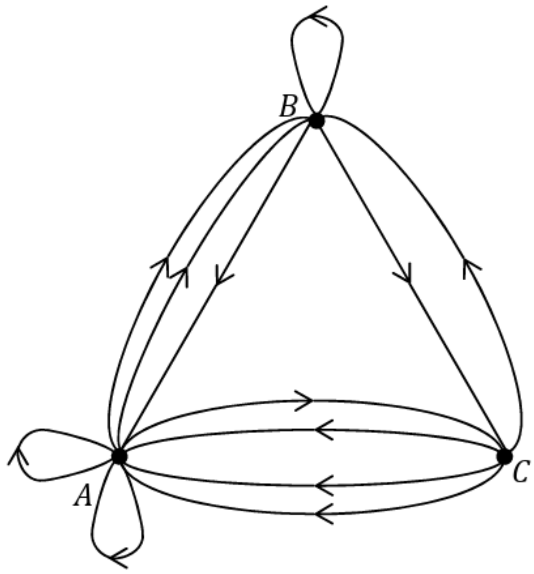
Higher Level

Tuesday 24 June   Afternoon 2:00 - 4:30

400 marks

Question 1

(a) The diagram shows a directed graph.

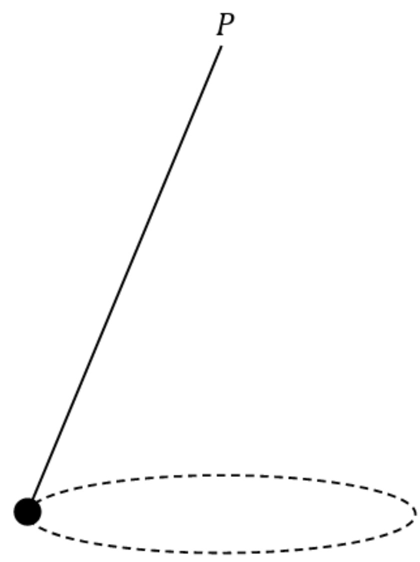


(i) Write the adjacency matrix, M, for this graph.

(ii) Calculate M^2.

(iii) What information is provided by the elements of M^2?

(b) A conical pendulum consists of a particle of mass 2 kg attached by a light inextensible string of length 1.3 m to a fixed point P. The particle moves in a horizontal circle of radius 0.5 m. The centre of the circle is vertically below P.



(i) Draw a diagram showing the forces acting on the particle.

(ii) Calculate the tension in the string.

(iii) Calculate the angular velocity of the particle.

(iv) The particle is given an increased velocity such that its period changes to 1.5 s.
Calculate the new radius.

Question 2

A particle of mass m moves vertically upwards through the air with displacement s and velocity v. Its motion may first be modelled by ignoring air resistance, so that it has constant acceleration a. At time $t = 0$ the particle has velocity $v_0 = 4$ m s^{-1} and displacement $s_0 = 0$.

(i) Using the chain rule, show that $a = v\frac{dv}{ds}$.

(ii) Use calculus to derive an expression for v in terms of a and s.

The model may be improved by including the force due to air resistance as $\frac{1}{40}mv^2$.

(iii) Show that the upward motion can now be expressed by the differential equation:

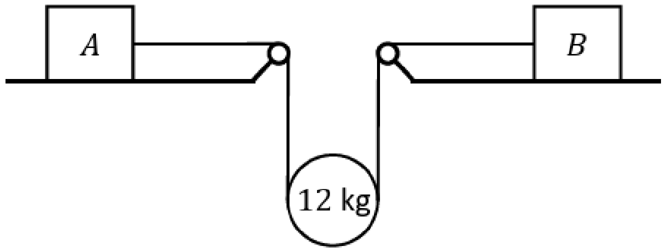
$$\frac{2v}{v^2 + 392}\,dv = -\frac{1}{20}\,ds$$

(iv) Solve this differential equation to find an expression for v in terms of s.

(v) Calculate the greatest height the particle will reach.

(vi) By using $a = \frac{dv}{dt}$ solve a differential equation to find an expression that relates v and t.

Question 3



(a) Block A of mass 8 kg and block B of mass 4 kg lie at rest on two rough horizontal tables.

The coefficient of friction between A and the table it lies on is μ_A.
The coefficient of friction between B and the table it lies on is μ_B.

$\mu_B = 2\mu_A$

The blocks are connected by a light inextensible string which passes under a smooth movable pulley of mass 12 kg.

After the system is released from rest the blocks and the pulley move and the string has a tension of 32 N.

(i) Calculate μ_B.

(ii) Calculate the acceleration of the movable pulley.

(b) An electrician needs to link 7 landline telephones in an office. The telephones, A to G, need to be connected by cables within the same electrical network.

The table below shows the distance, in metres, between the different telephones.

	A	B	C	D	E	F	G
A	–	9	14	13	12	15	18
B	9	–	13	14	12	9	15
C	14	13	–	11	13	14	16
D	13	14	11	–	10	9	14
E	12	12	13	10	–	11	12
F	15	9	14	9	11	–	8
G	18	15	16	14	12	8	–

(i) The electrician wishes to use the minimum length of cable required to connect every telephone within the same electrical network. Starting at telephone A, use Prim's algorithm to calculate the minimum length of cable required.

(ii) Draw the electrical network that uses the minimum length of cable.

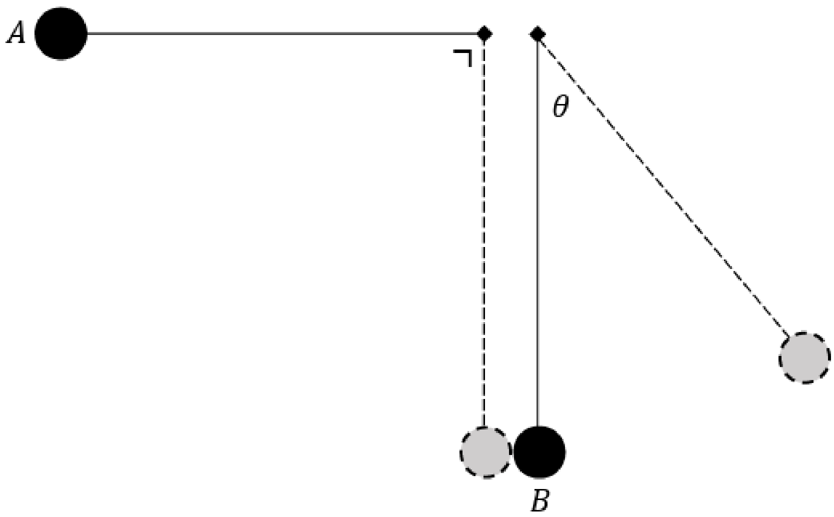
(iii) Name another algorithm that could be used to calculate the minimum length of cable required.

Question 4

Two small smooth spheres, A of mass 2 kg and B of mass 1 kg, are attached to pegs of equal height by two light inextensible strings, each of length 30 cm.

The spheres are initially at rest. A is released when the string is taut and horizontal.

A collides with B, which is suspended vertically.



The coefficient of restitution between the spheres is $\frac{3}{50}$.

(i) Show that A strikes B with a speed of $\frac{7\sqrt{3}}{5}$ m s^{-1}.

(ii) Calculate the speed of each sphere immediately after the collision.

(iii) Show in a diagram the forces acting on B when the string makes an angle θ with the downward vertical.

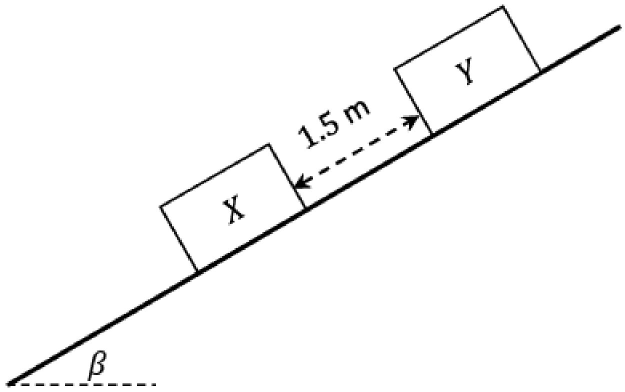
(iv) Write an expression, in terms of θ, for the height B has moved through when the string makes an angle θ with the downward vertical.

(v) Write an expression, in terms of θ, for the speed of B when the string makes an angle θ with the downward vertical.

(vi) Calculate the value of θ such that the string attaching B has a tension of 15 N.

Question 5

(a) Block X and block Y are held separately at rest on a rough plane inclined at β to the horizontal. X and Y are 1.5 m apart, as shown.



The coefficient of friction between X and the plane is $\frac{1}{2}$ and the coefficient of friction between Y and the plane is $\frac{2}{5}$. The blocks are released from rest.

(i) Show, on separate diagrams, the forces acting on the blocks while they are moving.

(ii) Calculate the acceleration of each block in terms of β.

The blocks collide 2 s after they are released.

(iii) Calculate β.

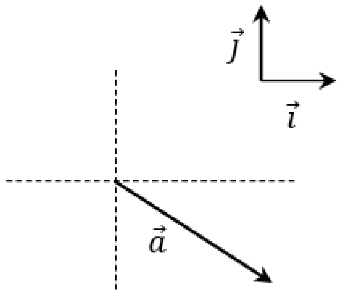
(b) $\vec{a} = 14\vec{i} - 9\vec{j}$

$\vec{b} = p\vec{i} + q\vec{j}$

$|\vec{b}| = 21$

$\vec{a} \perp \vec{b}$

Calculate the possible values of p and q.



Question 6

(a) Éanna wishes to purchase a new campervan. He wants to have the use of a campervan for five years and to minimise how much money this will cost him.

He does not wish to own a campervan at the end of the five years.

The cost of a new campervan is €80 000.

A campervan must be serviced each year.
The cost of servicing a campervan during its first year is €500.
The cost of servicing a campervan during its second year is €800.
The cost of servicing a campervan during its third year is €1400.

A one year old campervan has a resale value of €74 000.
A two year old campervan has a resale value of €70 000.
A three year old campervan has a resale value of €61 500.

Éanna wishes to own a campervan that is no more than three years old at all times during the five year period. Any time he purchases a campervan, it is a new one.

(i) Use Dynamic Programming to find Éanna's optimal strategy. Calculate how much it will cost Éanna if he uses this optimal strategy. Relevant supporting work must be shown.

(ii) In the context of this question, distinguish between the concepts of *stage* and *state*.

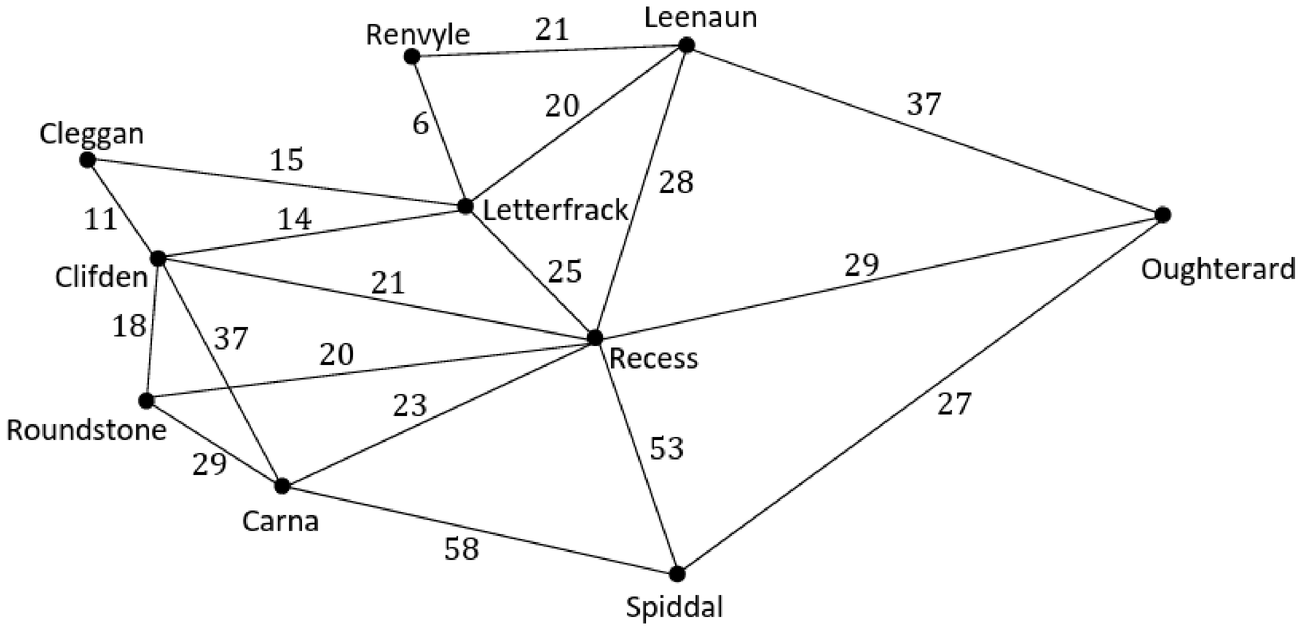
(b) Two students, Áine and Brody, are investigating the properties of an elastic resistance band in their school gym. The band has a natural length of 1 m and an elastic constant of 650 N m^{-1}. One end of the band is attached to a fixed pole.

(i) Use integration to calculate the work done by Áine in extending the band horizontally to a length of 1.2 m.

(ii) Brody takes the extended band and extends it further. He claims to have done twice the work that Áine did. If Brody is correct, calculate the new length of the band.

Question 7

(a) Shauna wishes to tour Connemara. While planning her route, she uses a road map to draw the network shown below. The weight of each edge represents the distance (in km) between each location.



(i) Use Dijkstra's algorithm to find the shortest path from Oughterard to Cleggan. Calculate the length of the shortest path. Relevant supporting work must be shown.

(ii) Describe how a network differs from a road map.

(b) Chioma is throwing a basketball against a wall. The ball leaves her hands at chest height, 1.35 m above the ground. The ball hits the wall at a height of 2 m. Chioma is standing 3 m from the wall when she throws the basketball with an initial speed of 6.3 m s^{-1} at an angle α to the horizontal.

(i) Calculate the two possible values of α.

(ii) For the smaller value of α, calculate the velocity of the basketball as it hits the wall.

The coefficient of restitution between the basketball and the wall is $\frac{3}{7}$.

(iii) For the smaller value of α, calculate the horizontal distance between the wall and where the basketball lands on the ground for the first time.

Question 8

(a) On Monday morning, the symptoms of a specific infection were detected in 5 students. On Tuesday morning, the symptoms were detected in an additional 12 students.

A local doctor predicts that the increase in the number of new infections detected on a given day will be twice the increase in the number of new infections detected on the previous day.

The doctor wishes to model this prediction as a difference equation so as to determine U_n, the number of new infections detected on day n.

(i) Write down a difference equation to express U_{n+2} in terms of U_{n+1} and U_n, where $n \geq 0$, $n \in \mathbb{Z}$.

(ii) Is this a homogeneous difference equation or an inhomogeneous difference equation? Explain your answer.

(iii) Solve the difference equation to find an expression for U_n in terms of n.

(iv) Calculate the total number of infections detected between Monday morning and Friday evening.

(b) The population growth in a certain region can be modelled by the following differential equation:

$$\frac{dP}{dt} = t^2 e^t$$

where P is the population and t is the time measured in years.

(i) Using integration by parts or otherwise, derive an expression for $P(t)$, the population at any time t, given that $P(0) = 2200$.

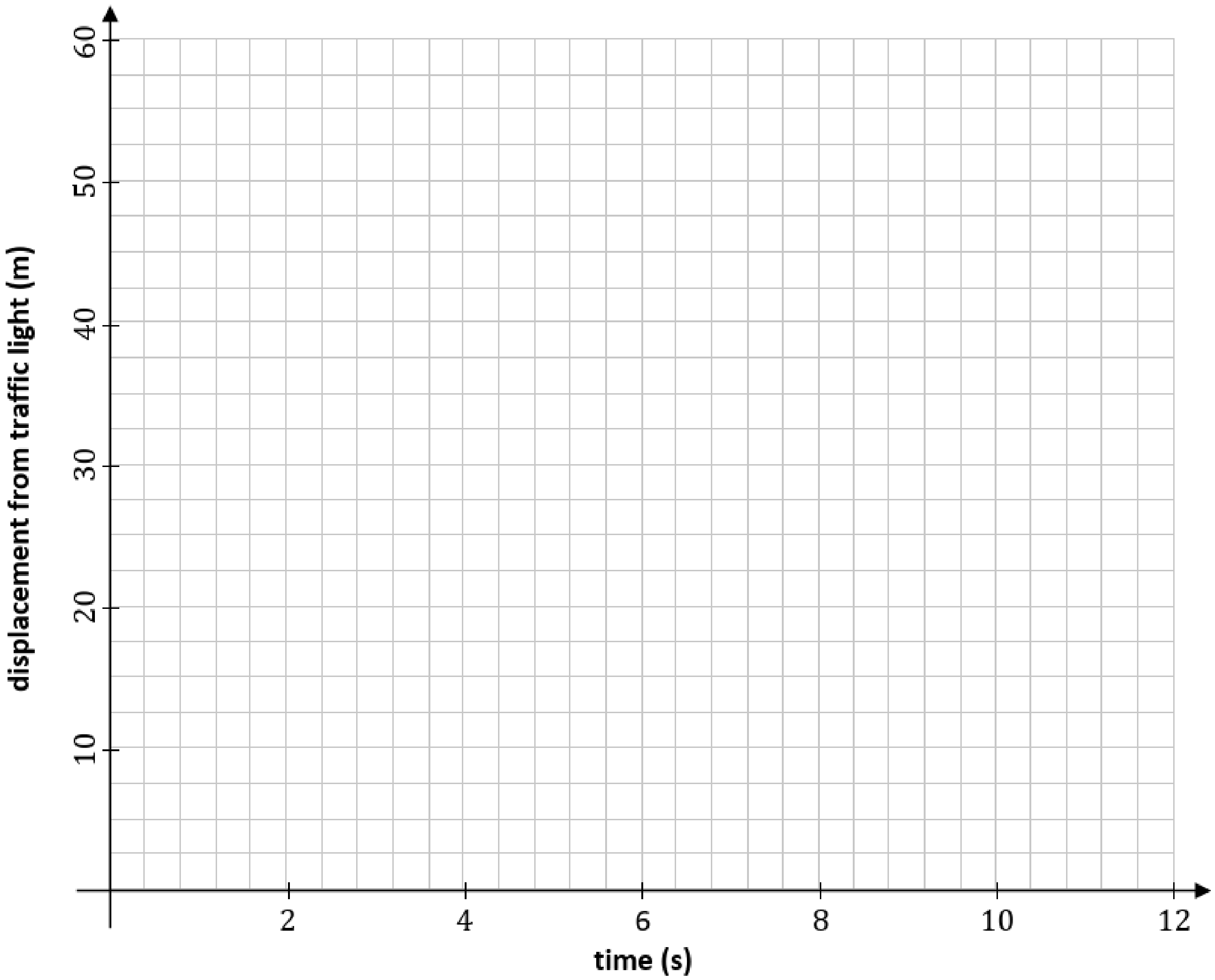
(ii) Calculate $P(5)$, the population after 5 years.

Question 9

(a) A car, using cruise control, is travelling with a constant velocity of 50 km hr^{-1}.
The driver sees a traffic light 60 m ahead change from green to orange.
The driver takes half a second to react to this.

(i) Calculate the constant deceleration required to stop the car at the light.

(ii) Using the axes below, draw a displacement-time graph for the motion of the car from when the driver sees the traffic light change. Relevant calculations should be shown in the space provided.



(b) Two small smooth spheres A and B, of masses $3m$ and m respectively, collide obliquely. The velocity of A before the collision is $3\vec{\imath} + 4\vec{\jmath}$ and the velocity of B before the collision is $-2\vec{\imath} + \vec{\jmath}$, where the $\vec{\imath}$ axis is along the line joining the centres of the spheres at the point of impact.

The coefficient of restitution is $\frac{1}{3}$.

Show that the fraction of kinetic energy lost as a result of the collision is $\frac{5}{24}$.

Question 10

(a) A new streaming media service is trying to increase its total number of customers. The company introduces a strategy to attract new customers.

The company develops a difference equation to predict S_n, the total number of customers in month n. The difference equation is:

$$S_{n+2} = \frac{1}{4}(S_{n+1} + 5S_n) - 15n + 40$$

where $n \geq 0, n \in \mathbb{Z}$.

Immediately before the introduction of the new strategy, the company has 2016 customers, i.e. $S_0 = 2016$. After one month the company has 3500 customers, i.e. $S_1 = 3500$.

(i) Write down the values of S_2 and S_3.

(ii) Solve the difference equation to find an expression for S_n in terms of n.

(iii) The strategy is deemed successful if the service has at least 35 000 customers after one year. Is the strategy successful? Justify your answer.

(b) A particle is projected from a point P with initial speed u. After 6 s its horizontal displacement and vertical displacement are 75 m and 30 m respectively.

Calculate u.

Exam Papers Answers:

Sample Paper 1:

Q1. (a) (i) $v = 30\ m/s$ (ii) $20\ m/s$

Q2. (a) (ii) $32.5\ mins$ (iii) $\frac{2}{45}\ m/s^2$ (b) (i) 4.907% (ii) $D_n = 1.004D_{n-1} - A$

(iii) $250A + (120000 - 250A)(1.004)^n$ (iv) €936.50

Q3. (a) (ii) $T = \left(\frac{3k}{k+4}\right)mg$ (iii) $R = \left(\frac{3k}{k+4}\right)mg$ (b) (i) $\frac{x^3}{9}(3\ln x - 1) + c$ (ii) $160\ J$

Q4. (a) $15°$ or $75°$ (b) (i) $10.1\ m$ (ii) $15.4\ m$

Q5. (a) 2 days Classical, 1 day App Maths (33%) (b) (i) $36.87°$ (ii) $\sqrt{\frac{12}{5}gr}$ or $\frac{14\sqrt{3r}}{5}$ or $4.85\sqrt{r}$

Q6. (a) (i) $\frac{u}{4}, \frac{5u}{4}$ (ii) $0.93\ m/s$ (b) (i) $v = 2.5(1 - e^{-10t})$ (ii) $0.35\ m$

Q7. (a) (i) $k = \frac{1}{108}$ (ii) $u = \frac{3}{2}\ m/s$ (b) (i) $v = \frac{u}{(4ntu^n+1)^{\frac{1}{n}}}$ (ii) $v = \frac{u}{\sqrt{1+24u^2}}$

Q8. (a) (i) $v = \frac{12}{7}\ m/s$ (ii) $e = \frac{6}{7}$

Sample Paper 2:

Q1. (a) $25\ m$ (b) (ii) $39.8\ m$

Q2. (a) (i) €2700 (ii) €2600; Sell at end of years 1, 2, 3, 4

(b) (i) $v_a = \sqrt{\{\left(\frac{1-e}{2}\right)u\cos\alpha\}^2 + \{u\sin\alpha\}^2}$, $v_b = \left(\frac{1+e}{2}\right)u\cos\alpha$

Q3. (a) $0.228\ m$ (b) (i) $1.96\ m/s^2$ (ii) $0.33\ s$

Q4. (a) (i) $22.5°$ (ii) $165.9\ m$ (iii) $46.79\ m/s$

(b) (i) $v = 80e^{-\frac{1}{100}t}$ (ii) $s = 8000(1 - e^{-\frac{1}{100}t})$ (iii) $v = 80 - \frac{1}{100}s$

Q5. (a) (i) $\frac{u(3-2e)}{5}, \frac{u(3+8e)}{5}$ (b) (i) $P_{n+1} = 0.15P_n + 3000$

(ii) $\frac{60000}{17} + \frac{365000}{17}(0.15)^n$ (iii) 3602 (iv) 3529

Q6. (a) (ii) $a = (-\omega^2 r\cos\omega t)\vec{i} + (-\omega^2 r\sin\omega t)\vec{j}$ (b) (ii) $f = 0.5$

Q7. (a) $36.87°$

Q8. (a) (i) $49\ s$ (ii) $31.25\ s$ (b) (i) $7\sqrt{2}\ m/s$ (ii) $\alpha = 80.41°$

Sample Paper 3:

Q1. (a) (i) $u_n = 600(2)^n - 7(5)^n$ (ii) $n = 5$ (b) (i) $20\ s$ (ii) $480\ m$ (iii) $420\ m$

Q2. (a) (i) $20\ m/s$ (ii) $1.6\ m/s^2$ (iii) $10\ s$ (iv) $\frac{400}{3}\ m$ (b) (i) $SBFGT$ (ii) €21000

Q3. (a) (i) $3.36\ N$ (ii) $1.4\ m/s$

Q4. (a) (ii) $49\ m/s$ (b) (ii) $5\ m/s$ (iii) $107.5\ m$

Q5. (a) (i) 0.63 (b) (i) $u_n = -5(2)^n + 6(3)^n$ (ii) 38086

Q6. (a) (i) $\frac{u(8e-1)}{3}, \frac{u(1+4e)}{3}$

Q7. (a) (i) $1\ s$ (ii) $0.65\ m$ (iii) $30.87\ m/s$ (b) (i) $k = 0.07324$ (ii) $28.8\ days$

Q8. (a) $420\ m/s$ (b) (i) $7200\ m$ (ii) $2931.75\ m$ (iii) $0.54\ m/s^2$

SEC HL Sample Paper 2020:

Q1. (a) (ii) $\begin{pmatrix} 0 & 2 & 0 \\ 1 & 1 & 2 \\ 1 & 1 & 0 \end{pmatrix}$ (b) (i) $u_2 = 6, u_3 = 9$ (ii) $u_{n+2} = u_{n+1} + u_n$ (iii) $u_n = 2.171\left(\frac{1+\sqrt{5}}{2}\right)^n +$ $0.829\left(\frac{1-\sqrt{5}}{2}\right)^n$ (iv) $5\ older\ ones$

Q2. (ii) $Disagree$ (iii) $Agree$ (v) $AEIL$ (vi) $25\ days$ (vii) $5\ days$ (viii) $Morning\ of\ July\ 11th$

Q3. (a) $s = ut + \frac{1}{2}at^2 + s_o$ (b) (i) $1.5\ s$ (iii) $16\ m$

Q4. (a) (i) $\frac{1}{e^2}$ (ii) $e^{10}H_0$ (b) (ii) $e = 0.577, \theta = 75°$

Q5. (a) (i) $ACGJLN, 72\ km$ (ii) $140\ km$ (b) (i) $R = \frac{1200.e^{1200kt}}{11+e^{1200kt}}$ (ii) 0.000443

Q6. (i) $\vec{s} = r\cos\omega t\,\vec{\imath} + r\sin\omega t\,\vec{\jmath}$ (ii) $\vec{v} = -\omega r\sin\omega t\,\vec{\imath} + \omega r\cos\omega t\,\vec{\jmath}$ (v) $v_{max} = \sqrt{\mu g r}\ m/s$

Q7. (a) (i) $W = \frac{kx^2}{2}$ (ii) $0.47\ m$ (b) (iii) $T = 1.17\ N, a = 1.9\ m/s^2$

Q8. (a) (i) $P_n = (\frac{24000-100B}{3})(1.03)^n + \frac{100}{3}B$ (ii) $P = \frac{e^{0.03n}(240-B)+B}{0.03}$ (iii) $Model\ 1{:}\ 12512, Model\ 2{:}\ 12642$ (v) 240

SEC HL Paper 2023:

Q1. (a) (ii) $BCAB$ or $BCDFB\ldots\ldots$ (b) (i) $s(t) = 2(1 - e^{-t} - 2e^{-t})$ (ii) 1.6017

Q2. (a) $Cheapest\ = XAEDJMY, €11450$ (b) $P = \frac{4-8e}{5}\vec{\imath} + \frac{16}{5}\vec{\jmath}, Q = \frac{4+4e}{5}\vec{\imath} + 3.2\vec{\jmath}$

Q3. (iv) $8\ complete\ revolutions$ (v) $1.5876\ s$

Q4. (iii) $v = \sqrt{\frac{1272e^{-2s}-147}{5}}$ (iv) $1.1\ m$ (vi) $\frac{dv}{ds} = \frac{29.4-v^2}{v}$

Q5. (a) (ii) $1.14\ m/s^2$ (b) $08{:}23$

Q6. (i) $u_2 = 7, u_3 = 20$ (ii) $u_n = \frac{1}{4}(-1)^n + \frac{3}{4}(3)^n$ (iii) 44287 (iv) $v_n = \frac{1}{8}(3)^n - \frac{1}{8}(-1)^n + \frac{1}{8}n + 1$ (v) 7387

Q7. (a) (i) $Min\ Weight\ = 101$ (ii) $137\ mins$ (b) (i) $N = 2000 - 1750e^{-kt}$ (ii) $k = 0.20879$

Q8. (i) $28\ m/s$ (ii) $110.53\ m$ (iii) $\vec{p} = 28.68\vec{\imath} - 4.47\vec{\jmath}, \vec{Q} = -12.32\vec{\imath} + 5.6\vec{\jmath}$ (iv) -378.3696 (v) $15.57°$

Q9. (iii) $AEJL$ or $AEK\ including\ 2\ dummies$ (v) $D, E, F\ or\ G$

Q10. (a) (i) $U_1 = 180, U_2 = 186$ (ii) $U_{n+1} = 1.2U_n - 30$ (iii) $U_n = 25(1.2)^n + 150$ (iv) 373 (b) (iii) $k = 5$

SEC HL Deferred Paper 2023:

Q1. (a) (i) $\begin{pmatrix} 2 & 7 & -2 \\ -9 & 10 & 8 \\ 7 & 1 & 2 \end{pmatrix}$ (b) (ii) $v = \sqrt{lg\sin\theta\tan\theta}$ (iii) $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

Q2. (a) (ii) $Dijsktra's, ABDEG, 174\ ms$ (b) (i) $E_n = (101 - \frac{20C}{3})(1.15)^n + \frac{20C}{3}$ (ii) 16 (iii) $\frac{dE}{dn} = 0.15E - C$

Q3. (i) $c = \frac{3}{2}, d = \frac{9}{16}$ (ii) $420\ billion$ (iii) $G_n = 200\left(\frac{3}{4}\right)^n + 40n\left(\frac{3}{4}\right)^n + 640, G_6 = 718\ billion$

Q4. (i) $P = 10e^{0.08t}$ (ii) $52\ weeks$ (iv) $P = \frac{20Ke^{0.08t}}{K-20+20e^{0.08t}}$ (v) 95

Q5. (i) $5\ s$ (ii) $v = \frac{1}{k}((g + 20k)e^{-kt} - g)$ (iii) $1.82\ s$ (iv) $\frac{dv}{dt} = g - kv$

Q6. (a) (i) $40.33\ m$ (ii) $376.44\ m$ (b) $Optimal{:}\ XBDFY, €60{,}000$

Q7. (ii) $\frac{u\sqrt{10}}{3}\vec{\imath} + 0\vec{\jmath}$ (iii) $W = \frac{kx^2}{2}$ (iv) $x = \frac{u}{3}\sqrt{\frac{10m}{k}}\ m$

Q8. (ii) $a = 2.19\ m/s^2, T = 47.14\ N$ (iii) $1.32\ m/s$ (iv) $-7.24\ m/s^2$ (v) $0.52\ m$

Q9. (iv) $AEIM$ (v) $16\ hours$ (vi) No

Q10. (ii) $48.19°$ (iii) $3.13\ m/s$ (iv) $0.46\ s$ (v) $2.08\ m$

SEC HL Paper 2024:

Q1. (a) $\begin{pmatrix} -1 & -1 & 2 \\ 13 & 2 & -3 \\ -6 & 11 & 6 \end{pmatrix}$ (b) (i) $586\, km$ (b) (ii) $751\, m$ (c) $6.41s\ or\ 3.59s$

Q2. (a) (ii) $5.37°$ (b) (ii) $\frac{343}{11}N\ or\ 31.18N$

Q3. (a) $\frac{\pi^2-4}{8}\ m/s\ or\ 0.73\ m/s$ (b) (i) $\overrightarrow{V_a} = -eu\cos\alpha\,\vec{i} + u\sin\alpha\,\vec{j}, \overrightarrow{V_B} = eu\cos\alpha\,\vec{i} + u\sin\alpha\,\vec{j}$

Q4. (ii) $R = \frac{mv_1{}^2 - 13.4mg + 20.1mg\cos\alpha}{6.7}$ (iii) $v_2 = \sqrt{v_1{}^2 - 13.4g}$ (vi) $v = \frac{1}{k}\left(g\ +\ k(\sqrt{v_1{}^2 - 13.4g})\right)e^{-kt} - \frac{g}{k}$

Q5. (a) (i) $\frac{u-9eu}{5}, \frac{u+6eu}{5}$ (ii) $0 < e < \frac{1}{9}$

Q6. (a)(i) $P_{n+1} = 1.0012P_n - x$ (ii) $P_n = \left(13500 - \frac{250x}{3}\right)(1.012)^n + \frac{250x}{3}$ (iii) 148 (iv) 13602

(b) (i) $v = \sqrt{4\ln(\frac{2}{2-s})}$ (ii) $0.806\, m/s$

Q7. (a) (i) $\frac{7}{4\cos\alpha}\ s$ (ii) $\alpha = 58.12°\ or\ 35.96°$ (b) (ii) $10.22\ m/s$ (iv) 0.315

Q8. (a)(i) $XCDIY\ giving\ €6000\ profit$

Q9. (i) $M_2 = 395, M_3 = 578.75$ (ii) $M_n = 172.5(\frac{3}{2})^n + 27.5(-\frac{1}{2})^n$ (iii) $3602.8125\ kg$ (iv) $P_n = 175.25(\frac{3}{2})^n + 26.95(-\frac{1}{2})^n - \frac{4}{5}(2)^{n+2}$ (v) $3458\ kg$

Q10. (a) (i) $Finishing\ Time = 21\ days$ (ii) $BEJLM$ (b) $85.175\ s$

SEC HL Paper 2025:

Q1. (a)(i) $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & 0 \end{pmatrix}$ (ii) $\begin{pmatrix} 9 & 7 & 4 \\ 6 & 4 & 2 \\ 7 & 7 & 4 \end{pmatrix}$ (iii) Number of walks of length 2 between nodes (b) (ii) $21.23\ N$

(iii) $2.86\ rads/s$ (iv) $1.17\ m$

Q2. (ii) $v = \sqrt{16 + 2as}$ (iv) $v = \sqrt{408e^{\frac{-1}{20}s} - 392}$ (v) $0.8\ m$ (vi) $v = 14\sqrt{2}\tan(0.2 - \frac{7\sqrt{2}}{20}t)$

Q3. (a) (i) 0.208 (ii) $4.47\ m/s^2$ (b) (i) $56\ m$ (ii) Kruskal's

Q4. (ii) $1.57\ m/s, 1.71\ m/s$ (iv) $0.3 - 0.3\cos\theta$ (v) $v = \sqrt{5.88\cos\theta - 2.96}$ (vi) $32.24°$

Q5. (a) (ii) $g\sin\beta - \frac{g\cos\beta}{2}, g\sin\beta - \frac{2g\cos\beta}{5}$ (iii) $40.1°$

Q6. (a) (i) €29,100, 2/1/2 or 2/2/1 or 1/2/2 (b) (i) $13\ J$ (ii) $1.35\ m$

Q7. (i) $61\ km$ (b) (i) $61.44°, 40.79°$ (ii) $4.77\vec{i} - 2.05\vec{j}$ (iii) $0.94\ m$

Q8. (a) (i) $U_{n+2} - 3U_{n+1} + 2U_n = 0$ (ii) Homogeneous (iii) $u_n = 7(2)^n - 2$ (iv) 207

(b) (i) $P(t) = e^t(t^2 - 2t + 2) + 2198$ (ii) 4721

Q9. (a) (i) $1.82\ m/s^2$

Q10. (a) (i) $3435, 5258.75$ (ii) $S_n = 2416(\frac{5}{4})^n - 425(-1)^n + 30n + 25$ (iii) $Yes, S_{12} = 35117.43$

(b) $36.6\ m/s$

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