## 1) Indices:

### a) The Laws of Indices:

1) $a^p \, x \, a^q = a^{p+q}$  *e.g.* $4^4 \, x \, 4^3 = 4^7$

2) $\frac{a^p}{a^q} = a^{p-q}$  $e.g \; \frac{5^3}{5^2} = 5^{3-2} = 5^1$

3) $(a^p)^q = a^{pq}$  $e.g \; (5^2)^3 = 5^6$

4) $a^0 = 1$  $e.g.$ $7^0 = 1$ or $(0.5)^0 = 1$

5) $a^{-p} = \frac{1}{a^p}$  $e.g.\; 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

6) $(ab)^p = a^p b^p$  $e.g.\; (3x)^2 = 3^2 x^2 = 9x^2$

7) $(\frac{a}{b})^p = \frac{a^p}{b^p}$  $e.g.\; (\frac{2}{3})^3 = \frac{2^3}{3^3} = \frac{8}{27}$

8) $a^{\frac{1}{2}} = \sqrt{a}$  $e.g.\; 9^{\frac{1}{2}} = \sqrt{9} = 3$

9) $a^{\frac{1}{3}} = \sqrt[3]{a}$  $e.g.\; 27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

See Tables pg 21

### b) Solving equations with indices:

**Steps:**

1. Try and spot which powers you're dealing with, using the table below e.g. if you see a 9 and an 27 in the question, it would be powers of 3
2. Tidy up both sides of the equation into a single power using the laws of indices above. e.g. $5^x = 5^y$
3. If the bases are the same on both sides, you can now let the powers be equal to each other. i.e. x = y
4. Solve the simple equation to find your solution.

**Example:** Solve $3^x = 27\sqrt{3}$

$3^x = 3^3 . 3^{\frac{1}{2}}$ ......using Law 8 above on the $\sqrt{3}$

$3^x = 3^{3 + {}^1/_2}$.........using Law 1

$3^x = 3^{{}^7/_2}$ ............tidying up the power into a single fraction

$=> x = {}^7/_2$ .........as the bases are equal

### c) Table of Powers:

**Note:** It can be familiar to be able to recognise some of the more common powers. A table of them is shown below.

| x | $x^1$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^6$ | $x^7$ | $x^8$ |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 |
| 3 | 3 | 9 | 27 | 81 | 243 | | | |
| 4 | 4 | 16 | 64 | 256 | | | | |
| 5 | 5 | 25 | 125 | 625 | | | | |
| 6 | 6 | 36 | 216 | | | | | |
| 7 | 7 | 49 | 343 | | | | | |
| 8 | 8 | 64 | 512 | | | | | |
| 9 | 9 | 81 | 729 | | | | | |
| 10 | 10 | 100 | 1000 | | | | | |

## 2) Surds:

**Notes:**

- A **surd** is a number in the form $\sqrt{\;}$ that **can't be written** as a **rational** number i.e. in the form $\frac{a}{b}$
  E.g. $\sqrt{2}$ and $\sqrt{3}$ are both surds but $\sqrt{9}$ is not as it can be written as $\frac{3}{1}$
- We can add/subtract similar surds together
  E.g. i) $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$
  ii) $4\sqrt{3} + 2\sqrt{2}$ .......we can't add these together as the $\sqrt{\;}$ parts are different

**Reducing Surds:**

- We can use the rule $\sqrt{ab} = \sqrt{a}\sqrt{b}$ to reduce larger surds into a simpler form:

**Example:** Simplify $\sqrt{50} + \sqrt{32}$

- We use 50 = 25 x 2 rather than 10 x 5 as 25 is a square number:

$\sqrt{50} + \sqrt{32}$
$= \sqrt{25}\sqrt{2} + \sqrt{16}\sqrt{2}$
$= 5\sqrt{2} + 4\sqrt{2}$
$= 9\sqrt{2}$