Topic: Integration in Book 1 (Topics 102 to 106)

Q1. Integrate the following:

(i) $\int 2x^3 \, . \, dx$ (ii) $\int \frac{1}{x^3} . \, dx$ (iii) $\int \sqrt{x} \, . \, dx$

(iv) $\int \frac{x^3 - 2}{x^2} . \, dx$ (v) $\int \sin 5x \, . \, dx$ (vi) $\int e^{-2x} \, . \, dx$

(vii) $\int (1 + \sqrt{x})^2 \, . \, dx$

Ans: (i) $\frac{1}{2}x^4 + c$ (ii) $\frac{-1}{2x^2} + c$ (iii) $\frac{2}{3}x^{1.5} + c$

(iv) $\frac{1}{2}x^2 + \frac{2}{x} + c$ (v) $-\frac{1}{5}\cos 5x + c$

(vi) $-\frac{1}{2}e^{-2x} + c$ (vii) $\frac{4}{3}x^{1.5} + \frac{1}{2}x^2 + x + c$

Q2. Integrate and evaluate the following:

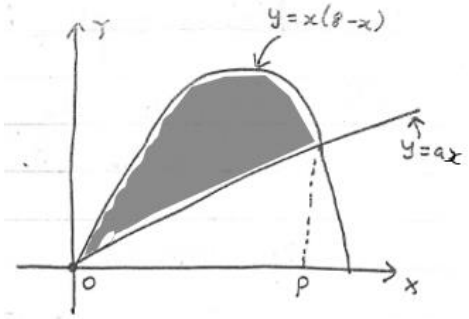
(i) $\int_1^2 \frac{1}{x} . \, dx$ (ii) $\int_0^{\frac{\pi}{4}} \sin 5x \, . \cos 3x \, . \, dx$

(iii) $\int_0^{\frac{1}{3}} \frac{1}{1 + 9x^2} . \, dx$

Ans: (i) $\ln 2$ (ii) $\frac{1}{4}$ (iii) $\frac{\pi}{12}$

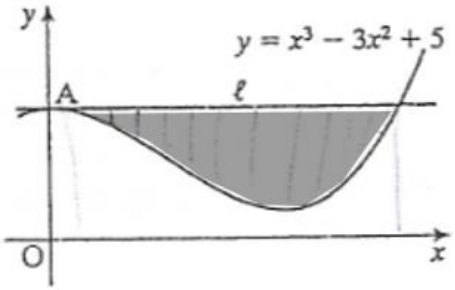
Q3. Find the area of the bounded region enclosed by the curve $y = x^2 - 3x + 3$ and the line $y = 2x - 1$. **Ans:** 4.5 units^2

Q4. 'a' is a real number such that $0 < a < 8$. The line y = ax intersects the curve $y = x(8 - x)$ at x = 0 and x = p.



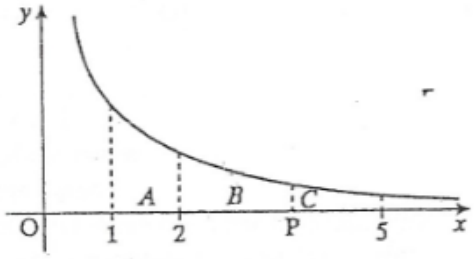
(i) Show that p = 8 – a (ii) Show that the area between the curve and the line is $\frac{p^3}{6}$.

Q5. Part of the curve $y = x^3 - 3x^2 + 5$ is given. The point A is a local maximum point of the curve and l is a tangent to the curve at A. Find the coordinates of A and hence find the area between the line l and the curve.



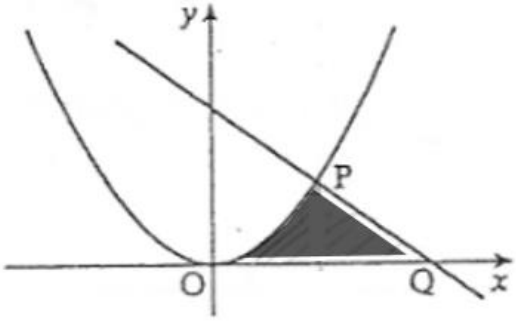
Ans: $\frac{27}{4}$ units^2

Q6. The figure shows the part of the curve $y = \frac{10}{x^2}$. (i) Find the area of region A. (ii) Find the value of P for which the regions B and C are of equal area.



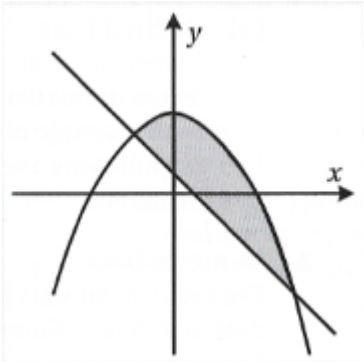
Ans: (i) 0.05 units^2 (ii) $\frac{20}{7}$

Q7. The figure shows the curve $y = x^2$ and the straight line $2x + y = 15$. (i) Find the coordinates of P and Q. (ii) Find the area of the shaded region.

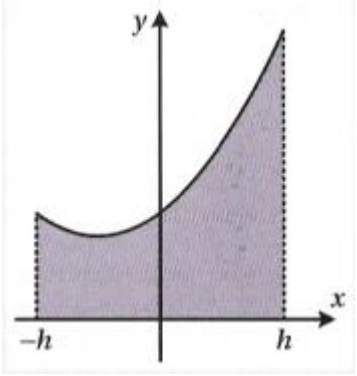


Ans: (i) P = (3, 9), Q = $(\frac{15}{2}, 0)$ (ii) 29.25 units^2

Q8. The diagram shows the curve $y = 4 - x^2$ and the line $2x + y - 1 = 0$. Calculate the area of the shaded region enclosed by the curve and the line. **Ans:** $\frac{32}{3}$



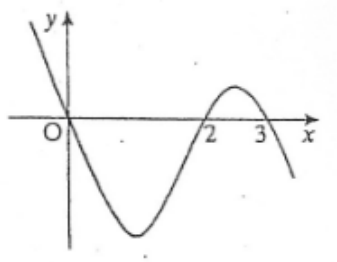
Q9. The graph of the function $f(x) = ax^2 + bx + c$ from x = -h to x = h is shown in the diagram. Show that the area of the shaded region is $\frac{h}{3}[2ah^2 + 6c]$.



Q10. Find the average value of the function f(x) = (x + 3)(2x – 5) in the interval [1, 5].
Ans: $\frac{26}{3}$

Q11. Find the average value of the function f(x) = $2x^2 - x$ over the interval [0, 4]. **Ans:** $\frac{26}{3}$

Q12. Find the exact area of the regions enclosed by the graph y = x(2 – x)(x – 3) and the x-axis.



Ans: $\frac{37}{12}$ units^2

Q13. A particle is moving in a straight line such that after t seconds, its velocity is v m/s, where v = 6t + $12t^2$. Find (i) the average velocity during the first 2 seconds of motion (ii) the average acceleration between t = 1 and t = 5.
Ans: (i) 22 m/s (ii) 78 m/s^2

Q14. If $f(x) = x \sin 2x$, find $f'(x)$. Use your result to find $\int 2x \cos 2x \, . dx$.
Ans: $x \sin 2x + \frac{\cos 2x}{2} + c$

Q15. If $\int_0^a (9x^2 - 4x) \, . dx = 0$, find the possible values of a for when $a > 0$. **Ans:** a = $\frac{2}{3}$

Q16. A particle starts from rest 3 m from a fixed point O and moves in a straight line with an acceleration a given by a = 6t + 10, where t is the time in seconds. (i) Find the velocity v after 5 seconds (ii) Express its position, s, from the fixed point O in terms of t. (iii) How many metres is the particle from O after 3 seconds? (iv) Find the average speed from t = 1 to t = 4.
Ans: (i) 125 m/s (ii) s = $t^3 + 5t^2 + 3$ (iii) 46 m/s

Q17. The volume, V cm^3, of water in a hemispherical bowl is given by $V = \frac{1}{3}\pi(30h^2 - h^3)$, where h cm is the depth of the water. Find the average volume of water in the bowl as the depth increases from 0 to 4 cm.
Ans: 48π cm^3

Q18. The area between the curve y = x^2 + 2x and the x-axis, for $0 \leq x \leq 3$, is equal to the area of a rectangle of base 3 and height k. Find the value of k. **Ans:** k = 6

Q19. Show that the average value of the function f(x) = x^2 over the interval [0, a] is $\frac{1}{3}f(a)$.