

Terminal Velocity Problem

Determine the 'terminal' velocity of this skydiver after falling from rest before prior to opening the parachute



The mass of the parachutist and his parachute is 90 kg.



Iteration 1:

Linking to Prior Knowledge

An object falling under the influence of gravity from rest. Calculate its velocity after five seconds and after 10 seconds. (assuming that it has not hit the ground in this time)

Its velocity five seconds after falling from rest

$$v = u + a(t) \\ \Rightarrow v = 0 + 9.8(5) = 49 \text{ m s}^{-1}$$

Its velocity ten seconds after falling from rest

$$v = u + a(t) \\ \Rightarrow v = 0 + 9.8(10) = 98 \text{ m s}^{-1}$$

Conclusion - an object's velocity will increase by 9.8 m s^{-1} for each second that it continues to fall under the sole influence of gravity.

Can we refine our assumptions to find a more accurate solution?

Iteration
①



Evaluation: The student originally considered the force of gravity as the only force being experienced by the falling object. On evaluating the solution the student considered that there is another force in play, namely, air resistance (drag). This would influence the student re-formulating the problem and moving to their next iteration.

Iteration 2:

Determining the Terminal Velocity without Calculus

$$\begin{aligned}\text{The net force} &= \text{mass} \times \text{acceleration} \\ &= \text{weight} - \text{drag} \\ &= m \cdot g - (0.24 v^2)\end{aligned}$$

$$\Rightarrow 90 \cdot a = 90 \cdot g - 0.24 v^2$$

Multiplying both sides of the equation by 100

$$9000 \cdot a = 9000 \cdot g - 24 v^2$$

Divide by 24

$$\Rightarrow 375 \cdot a = 375 \cdot g - v^2$$

Terminal Velocity is the fastest you will fall in a given configuration. This is when air resistance balances your weight. So the parachutist net acceleration is zero.

$$v^2 = 375 \cdot g \quad \Rightarrow \quad v = 35 \cdot \sqrt{3} \text{ m s}^{-1}$$

This is the equivalent of just over 60 m s^{-1}

This value as a **second iteration** will have to be evaluated.

Can we refine our assumptions further to find a more accurate solution?

Iteration 3:

Determining the Terminal Velocity using Calculus

$$\begin{aligned}\text{The net force} &= \text{mass} \times \text{acceleration} \\ &= \text{weight} - \text{drag} \\ &= m \cdot g - (0.24 v^2)\end{aligned}$$

$$\Rightarrow 90 \cdot a = 90 \cdot g - 0.24 v^2$$

We developed this earlier to :

$$375 \cdot a = 375 \cdot g - v^2$$

$$375 \cdot \frac{dv}{dt} = 375 \cdot g - v^2$$

$$\text{Multiplying both sides by } dt \Rightarrow 375 dv = (375 \cdot g - v^2) dt$$

$$\text{Dividing both sides by } 375 \cdot g - v^2$$

$$\Rightarrow \frac{375 \cdot dv}{(375 \cdot g - v^2)} = 1 \cdot dt$$

Now we are ready to integrate both sides of this equality



If we integrate both sides of the equality with respect to their derivatives $\frac{375 \cdot dv}{(375 \cdot g - v^2)} = 1 \cdot dt$

$$\Rightarrow \int \frac{375 \cdot dv}{(375 \cdot g - v^2)} = \int 1 \cdot dt \Rightarrow 375 \int \frac{1 \cdot dv}{(375 \cdot g - v^2)} = \int 1 \cdot dt$$

The LHS of the integral is of the form

$$375 \int \frac{1 \cdot dv}{(35\sqrt{3})^2 - (v)^2} = \int 1 \cdot dt$$

$$375 \cdot \frac{1}{2(35\sqrt{3})} \ln \left| \frac{35\sqrt{3} + v}{35\sqrt{3} - v} \right| = t + c$$

How do we determine 'c'? We have one unknown 'c', so we require one condition

When $t=0, v=0$ Substituting into : $375 \cdot \frac{1}{2(35\sqrt{3})} \ln \left| \frac{35\sqrt{3} + v}{35\sqrt{3} - v} \right| = t + c$

We get $\Rightarrow 375 \cdot \frac{1}{2(35\sqrt{3})} \ln |1| = 0 + c \Rightarrow 0 = c$

$$375 \cdot \frac{1}{2(35\sqrt{3})} \ln \left| \frac{35\sqrt{3} + v}{35\sqrt{3} - v} \right| = t + c \text{ becomes } 375 \cdot \frac{1}{2(35\sqrt{3})} \ln \left| \frac{35\sqrt{3} + v}{35\sqrt{3} - v} \right| = t$$

So we now have an expression for time and velocity

$$375 \cdot \frac{1}{35\sqrt{3}} \ln \left| \frac{35\sqrt{3} + v}{35\sqrt{3} - v} \right| = t$$

To identify the terminal velocity, we must find the value of v as time tends to infinity.

RHS of the equation

As time tends to infinity:

$$\lim_{t \rightarrow \infty} (t) = \infty$$

LHS of the equation

The denominator must tend to zero:

$$35\sqrt{3} - v \rightarrow 0$$

$$\Rightarrow v = 35\sqrt{3} \sim 60 \text{ m s}^{-1}$$

$$\Rightarrow \text{Terminal Velocity} = 35\sqrt{3} \sim 60 \text{ m s}^{-1}$$

This completes our third iteration which we will have to evaluate.

Graphical Representation of Terminal Velocity

