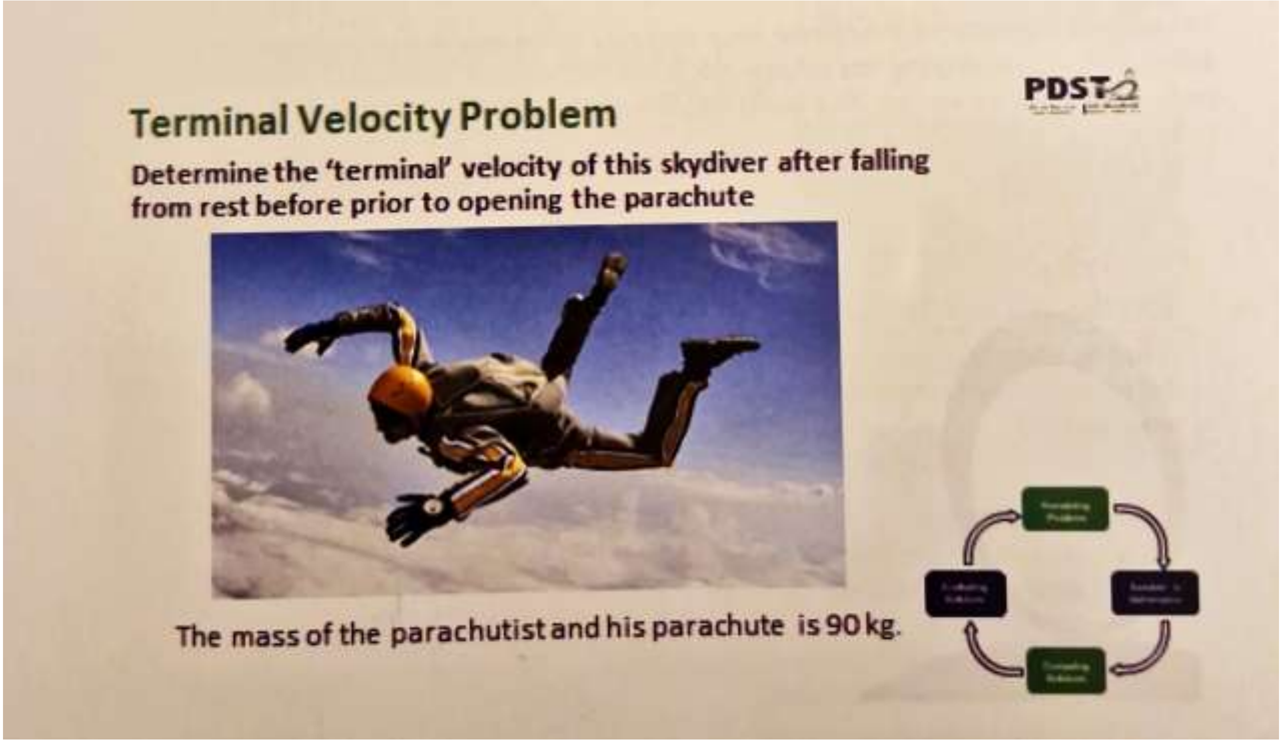
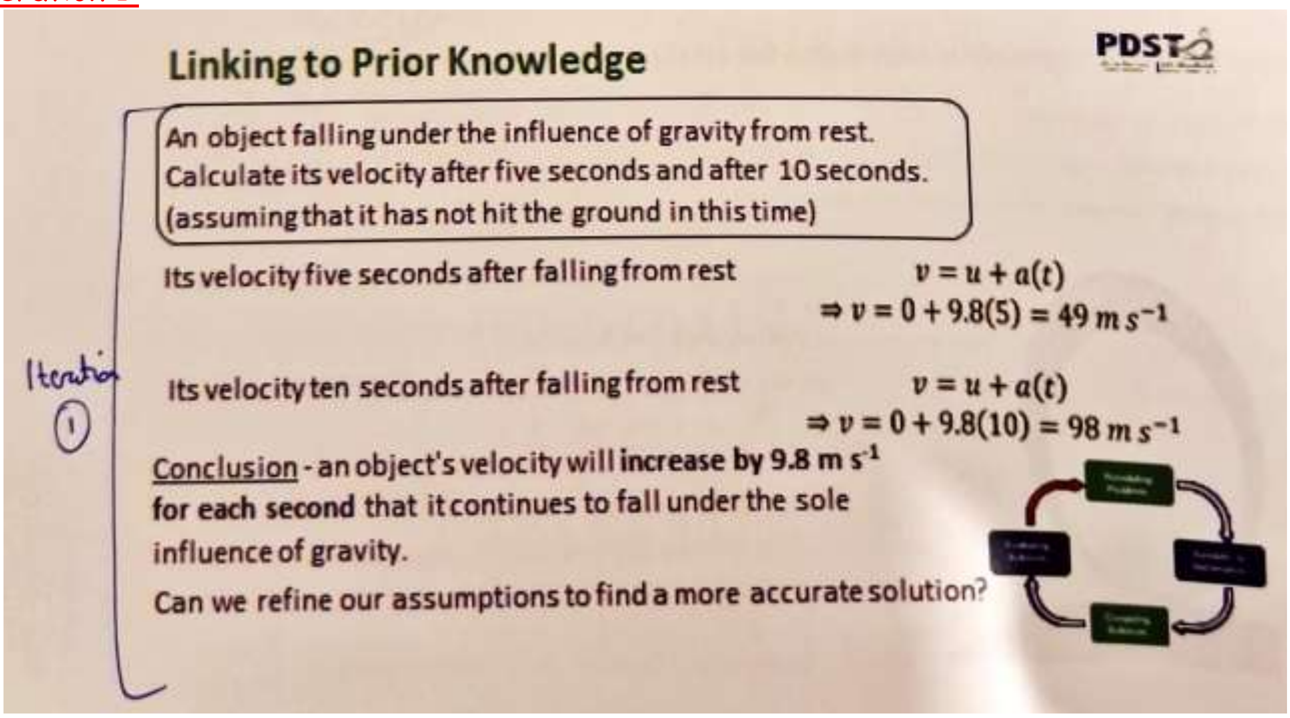


Iteration 1:



Evaluation: The student originally considered the force of gravity as the only force being experienced by the falling object. On evaluating the solution the student considered that there is another force in play, namely, air resistance (drag). This would influence the student re-formulating the problem and moving to their next iteration.

Iteration 2:

Determining the Terminal Velocity without Calculus

The net force = mass x acceleration

= weight – drag

= m.g $-(0.24\,v^2)$

$$\Rightarrow \quad 90.a = 90.g \quad - 0.24\,v^2$$

Multiplying both sides of the equation by 100

$$9000.a = 9000.g \quad - 24\,v^2$$

Divide by 24

$$\Rightarrow \quad 375.a = 375.g \quad - v^2$$

Terminal Velocity is the fastest you will fall in a given configuration. This is when air resistance balances your weight. So the parachutist net acceleration is zero.

$$v^2 = 375.g \quad \Rightarrow \quad v = 35.\sqrt{3}\ m\,s^{-1}$$

This is the equivalent of just over **60 m s⁻¹**

This value as a **second iteration** will have to be evaluated .

Can we refine our assumptions further to find a more accurate solution?

Iteration 3:

Determining the Terminal Velocity using Calculus

The net force = mass x acceleration

= weight – drag

= m.g $-(0.24\,v^2)$

$$\Rightarrow \quad 90.a = 90.g \quad - 0.24\,v^2$$

We developed this earlier to :

$$375.a = 375.g \quad - v^2$$

$$375.\frac{dv}{dt} = 375.g - v^2$$

Multiplying both sides by dt $\Rightarrow$ $375\,dv = (375.g - v^2)\,dt$

Dividing both sides by $375.g - v^2$

$$\Rightarrow \quad \frac{\mathbf{375.dv}}{\mathbf{(375.g - v^2)}} = \mathbf{1.dt}$$

Now we are ready to integrate both sides of this equality

If we integrate both sides of the equality with respect to their derivatives

$$\frac{375.dv}{(375.g - v^2)} = 1.dt$$

$$\Rightarrow \int \frac{375.dv}{(375.g - v^2)} = \int 1.dt \quad \Rightarrow \quad 375 \int \frac{1.dv}{(375.g - v^2)} = \int 1.dt$$

The LHS of the integral is of the form

$$375 \int \frac{1.dv}{(35\sqrt{3})^2 - (v)^2)} = \int 1.dt$$

$$375.\frac{1}{2(35\sqrt{3})} \ln\left|\frac{35\sqrt{3} + v}{35\sqrt{3} - v}\right| = t + c$$

How do we determine 'c'? We have one unknown 'c', so we require one condition

When t=0, v=0 *Substituting into* : $375.\frac{1}{2(35\sqrt{3})} \ln\left|\frac{35\sqrt{3} + v}{35\sqrt{3} - v}\right| = t + c$

We get $\Rightarrow$ $375.\frac{1}{2(35\sqrt{3})} \ln|1| = 0 + c \Rightarrow 0 = c$

$$375.\frac{1}{2(35\sqrt{3})} \ln\left|\frac{35\sqrt{3} + v}{35\sqrt{3} - v}\right| = t + c \quad \text{becomes} \quad 375.\frac{1}{2(35\sqrt{3})} \ln\left|\frac{35\sqrt{3} + v}{35\sqrt{3} - v}\right| = t$$

So we now an expression for time and velocity

$$375.\frac{1}{35\sqrt{3}} \ln\left|\frac{35\sqrt{3}+v}{35\sqrt{3}-v}\right| = t$$

To identify the terminal velocity, we must find the value of v as time tends to infinity.

RHS of the equation
As time tends to infinity: $\lim_{t\to\infty}(t) = \infty$

LHS of the equation
The denominator must tend to zero: $35\sqrt{3} - v \to 0$

$$\Rightarrow \quad v = 35\sqrt{3} \sim 60\ m\ s^{-1}$$

$$\Rightarrow \textbf{Terminal Velocity} = 35\sqrt{3} \sim 60\ m\ s^{-1}$$

This completes our third iteration which we will have to evaluate.

Graphical Representation of Terminal Velocity