Vectors Revision Worked Solutions

Q1. i) $v = u + at$

$\frac{m}{s} = \frac{m}{s} + \frac{m}{s^2}(s)$

$\frac{m}{s} = \frac{m}{s}$ ✓

ii) $v^2 = u^2 + 2as$

$\left(\frac{m}{s}\right)^2 = \left(\frac{m}{s}\right)^2 + 2\left(\frac{m}{s^2}\right)(m)$

$\frac{m^2}{s^2} = \frac{m^2}{s^2} + \frac{m^2}{s^2}$

$\frac{m^2}{s^2} = \frac{m^2}{s^2}$ ✓

iii) $P = \frac{W}{t}$

$kg.\frac{m^2}{s^3} = \frac{kg.\frac{m^2}{s^2}}{s}$

$kg.\frac{m^2}{s^3} = kg.\frac{m^2}{s^3}$

iv) $F = \frac{mv^2}{r}$

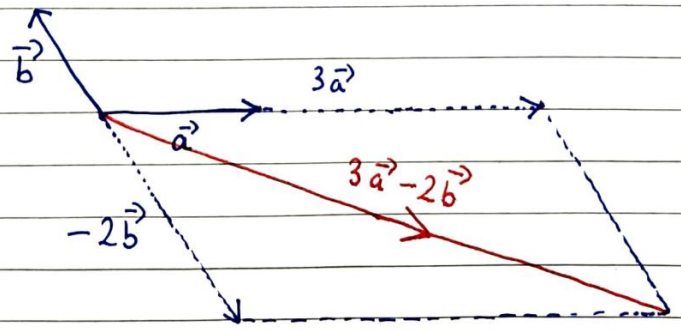
$kg.\frac{m}{s^2} = \frac{kg.\left(\frac{m}{s}\right)^2}{m}$

$kg.\frac{m}{s^2} = \frac{kg.\frac{m^2}{s^2}}{m}$

$kg.\frac{m}{s^2} = kg.\frac{m}{s^2}$ ✓

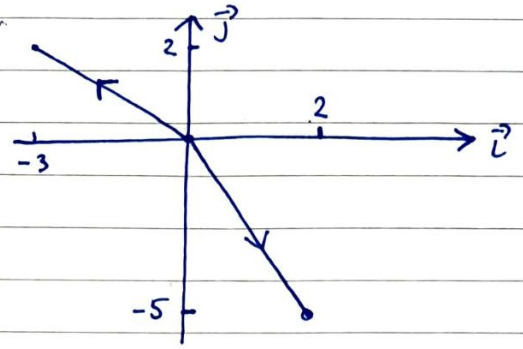
Q2. A vector quantity has a magnitude and a direction.
A scalar quantity has a magnitude only.
Vector examples: force, acceleration....
Scalar examples: mass, speed.....

Q3.



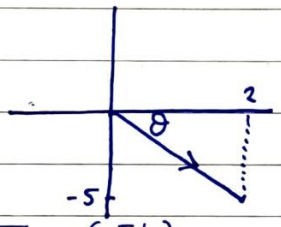
Q4. $\vec{x} = 2\vec{i} - 5\vec{j}$ $\vec{y} = -3\vec{i} + 2\vec{j}$

i)



ii) $|\vec{x}| = \sqrt{(2)^2+(-5)^2} = \boxed{\sqrt{29}}$ $|\vec{y}| = \sqrt{(-3)^2+(2)^2} = \boxed{\sqrt{13}}$

iii) $\vec{x}$



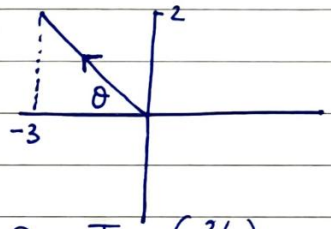
$\theta = \tan^{-1}(5/2)$

$= 68.2°$

$\Rightarrow$ $\boxed{E\,68.2°\,S}$

or $\boxed{S\,21.8°\,E}$

$\vec{y}$



$\theta = \tan^{-1}(2/3)$

$= 33.7°$

$\Rightarrow$ $\boxed{W\,33.7°\,N}$

or $\boxed{N\,56.3°\,W}$

iv) $2\vec{x} - \vec{y}$

$= 2(2\vec{i} - 5\vec{j}) - (-3\vec{i} + 2\vec{j})$

$= 4\vec{i} - 10\vec{j} + 3\vec{i} - 2\vec{j}$

$= \boxed{7\vec{i} - 12\vec{j}}$

v) $|2\vec{x} - \vec{y}|$

$= |7\vec{i} - 12\vec{j}|$

$= \sqrt{(7)^2+(-12)^2}$

$= \boxed{\sqrt{193}}$

Q5. $\vec{x} = 6\vec{i} - 5\vec{j}$ $\vec{y} = 4\vec{i} + 3\vec{j}$ $\vec{z} = 2\vec{i} + \vec{j}$

i) $\vec{x} \cdot \vec{y} = (6\vec{i} - 5\vec{j})(4\vec{i} + 3\vec{j})$

$= (6)(4) + (-5)(3)$

$= 24 - 15$

$= \boxed{9}$

ii) $\vec{x} \cdot \vec{z} = (6\vec{i} - 5\vec{j})(2\vec{i} + \vec{j})$

$= (6)(2) + (-5)(1)$

$= 12 - 5$

$= \boxed{7}$

iii) $\vec{x}(\vec{y} + \vec{z})$

$\vec{y} + \vec{z} = 4\vec{i} + 3\vec{j} + 2\vec{i} + \vec{j}$

$= 6\vec{i} + 4\vec{j}$

$\Rightarrow (6\vec{i} - 5\vec{j})(6\vec{i} + 4\vec{j})$

$= (6)(6) + (-5)(4)$

$= 36 - 20$

$= \boxed{16}$

Q6

i) $|5\vec{i} + 12\vec{j}| = |12\vec{i} + 5\vec{j}|$

$= \sqrt{(12)^2 + (5)^2}$

$= 13$

$(5\vec{i} + 12\vec{j})(12\vec{i} + 5\vec{j})$

$= (5)(12) + (12)(5)$

$= 60 + 60$

$= 120$

$\Rightarrow \cos\theta = \dfrac{\vec{x} \cdot \vec{y}}{|\vec{x}| \cdot |\vec{y}|}$

$= \dfrac{120}{(13)(13)}$

$= \dfrac{120}{169}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{120}{169}\right)$

$= \boxed{44.76°}$

ii) $|8\vec{i} - \vec{j}| = \sqrt{(8)^2 + (-1)^2} = \sqrt{65}$

$|-7\vec{i} + 4\vec{j}| = \sqrt{(-7)^2 + (4)^2}$

$= \sqrt{65}$

$(8\vec{i} - \vec{j})(-7\vec{i} + 4\vec{j})$

$= (8)(-7) + (-1)(4)$

$= -56 - 4$

$= -60$

$\cos\theta = \dfrac{\vec{x} \cdot \vec{y}}{|\vec{x}| \cdot |\vec{y}|}$

$= \dfrac{-60}{\sqrt{65}\sqrt{65}}$

$= -\dfrac{12}{13}$

$\Rightarrow \theta = \cos^{-1}\left(-\dfrac{12}{13}\right) = 157.38°$

$\Rightarrow$ Smaller angle $= \boxed{22.62°}$

Q7.

i) $\vec{p} \perp \vec{q}$?

$(4\vec{i} - 2\vec{j})(\vec{i} + 2\vec{j})$

$= (4)(1) + (-2)(2)$

$= 4 - 4$

$= 0 \Rightarrow \vec{p} \perp \vec{q}$

ii) $\vec{p} \perp \vec{r}$?

$(4\vec{i} - 2\vec{j})(2\vec{i} + 4\vec{j})$

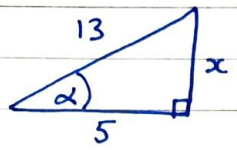
$= (4)(2) + (-2)(4)$

$= 8 - 8$

$= 0 \Rightarrow \vec{p} \perp \vec{r}$.

Q8

$\cos\alpha = 5/13$



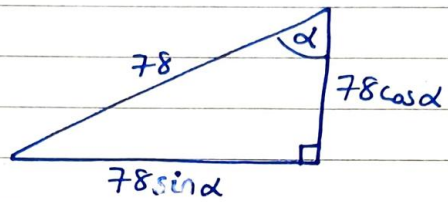
$(13)^2 = (5)^2 + (x)^2$

$169 - 25 = x^2$

$144 = x^2$

$x = 12$

$\Rightarrow \sin\alpha = \frac{12}{13}$

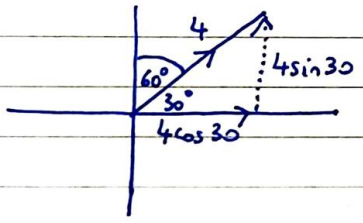


$|pq| = 78\sin\alpha$

$= 78\left(\frac{12}{13}\right)$

$= \boxed{72\text{m}}$

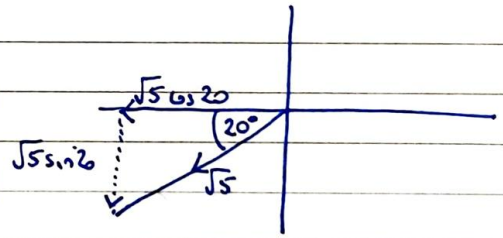
Q9. i) Sketch:



$\Rightarrow 4\cos 30\vec{i} + 4\sin 30\vec{j}$

$= \boxed{2\sqrt{3}\vec{i} + 2\vec{j}}$

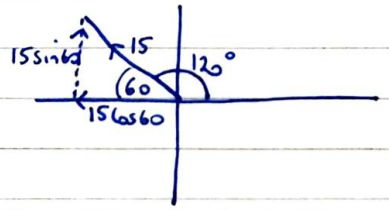
ii) Sketch:



$\Rightarrow -\sqrt{5}\cos 20\vec{i} - \sqrt{5}\sin 20\vec{j}$

$\boxed{-2.1\vec{i} - 0.72\vec{j}}$

iii) Sketch:



$\Rightarrow -15\cos 60\vec{i} + 15\sin 60\vec{j}$

$\boxed{-7.5\vec{i} + 13\vec{j}}$

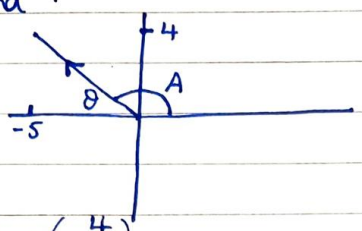
OR Using calculator

Rec (15, 120)

$= \boxed{-7.5\vec{i} + 13\vec{j}}$

Q10. i) $-5\vec{i} + 4\vec{j}$

By hand:



$\theta = \tan^{-1}\left(\frac{4}{5}\right)$

$= 38.7° \Rightarrow A = 141.3°$

$|-5\vec{i} + 4\vec{j}| = \sqrt{(-5)^2+(4)^2}$

$= \sqrt{41}$

$\Rightarrow$ $\sqrt{41}$ | $141.3°$

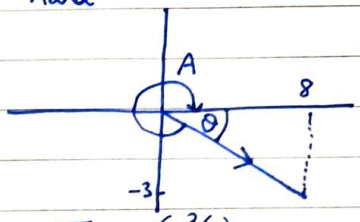
OR By calc:

Pol (-5, 4)

= $\sqrt{41}$ | $141.3°$

ii) $8\vec{i} - 3\vec{j}$

By hand:



$\theta = \tan^{-1}\left(\frac{3}{8}\right)$

$= 20.6° \Rightarrow A = 339.4°$

$|8\vec{i} - 3\vec{j}| = \sqrt{(8)^2+(-3)^2}$

$= \sqrt{73}$

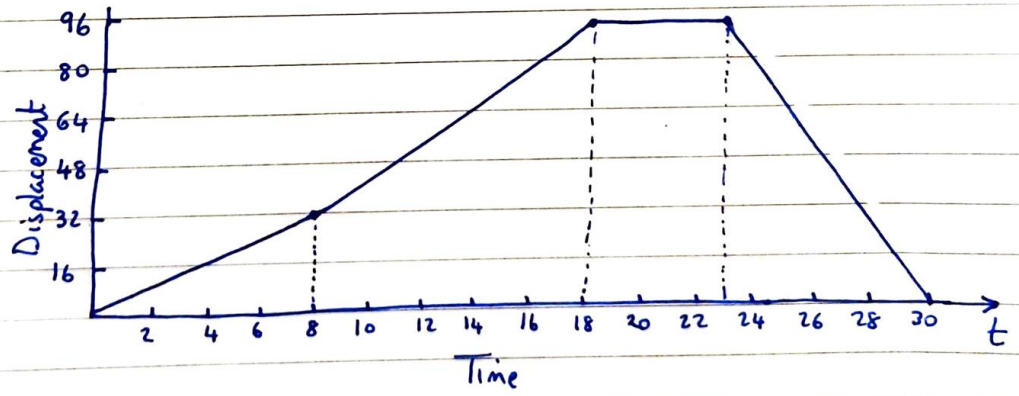
$\Rightarrow$ $\sqrt{73}$ | $339.4°$

OR By calc:

Pol (8, -3)

= 8.544 | $-20.6°$

Q11.



Q12. $\vec{x} = 3\vec{i} - \vec{j}$ $\vec{y} = 9\vec{i} + 4\vec{j}$

i) $-3\vec{x} + \vec{y}$

$= -3(3\vec{i} - \vec{j}) + (9\vec{i} + 4\vec{j})$

$= -9\vec{i} + 3\vec{j} + 9\vec{i} + 4\vec{j}$

$= 0\vec{i} + 7\vec{j}$

No $\vec{i}$ component $\Rightarrow$ along $\vec{j}$ axis

ii) $k\vec{x} + \vec{y}$

$= k(3\vec{i} - \vec{j}) + 9\vec{i} + 4\vec{j}$

$= 3k\vec{i} - k\vec{j} + 9\vec{i} + 4\vec{j}$

$= (3k+9)\vec{i} + (-k+4)\vec{j}$

Along $\vec{i}$ axis $\Rightarrow$ $\vec{j}$ component $= 0$

$\Rightarrow -k + 4 = 0$

$\boxed{k = 4}$

Q13. $|t\vec{i} + 24\vec{j}| = |15\vec{i} - 20\vec{j}|$

$\sqrt{(t)^2 + (24)^2} = \sqrt{(15)^2 + (-20)^2}$

$\sqrt{t^2 + 576} = 25$

Square both sides:

$t^2 + 576 = 625$

$t^2 = 625 - 576$

$t^2 = 49$

$\Rightarrow t = \sqrt{49} = \boxed{\pm 7}$

Q14. $\vec{a} = 3\vec{i} + 2\vec{j}$ $\vec{b} = 4\vec{i} - \vec{j}$

$l(3\vec{i} + 2\vec{j}) + k(4\vec{i} - \vec{j}) = 2\vec{i} + 5\vec{j}$

$3l\vec{i} + 2l\vec{j} + 4k\vec{i} - k\vec{j} = 2\vec{i} + 5\vec{j}$

$(3l + 4k)\vec{i} + (2l - k)\vec{j} = 2\vec{i} + 5\vec{j}$

$\vec{i} = \vec{i}$ $\vec{j} = \vec{j}$

$3l + 4k = 2$: A $2l - k = 5$: B

Solving:

A: $3l + 4k = 2$

B×4: $8l - 4k = 20$

$11l = 22$

$\boxed{l = 2}$

A: $3(2) + 4k = 2$

$6 + 4k = 2$

$4k = -4$

$\boxed{k = -1}$

Q15. $\vec{p} = 3\vec{i} - (k-1)\vec{j}$ $\quad \vec{q} = (k+1)\vec{i} + 4\vec{j}$

i) $\vec{p} \perp \vec{q} \Rightarrow \vec{p}.\vec{q} = 0$

$$(3)(k+1) + (4)(-(k-1)) = 0$$
$$3k + 3 - 4k + 4 = 0$$
$$-k + 7 = 0$$
$$\boxed{k = 7}$$

ii) $\vec{p} + \vec{q} = 3\vec{i} - 6\vec{j} + 8\vec{i} + 4\vec{j}$
$= \boxed{11\vec{i} - 2\vec{j}}$

iii) $|\vec{p} + \vec{q}| = \sqrt{(11)^2 + (-2)^2} = 5\sqrt{5}$ $\quad |\vec{q}| = \sqrt{(8)^2 + (4)^2} = 4\sqrt{5}$

$$(\vec{p} + \vec{q})(\vec{q}) = (11\vec{i} - 2\vec{j})(8\vec{i} + 4\vec{j})$$
$$= (11)(8) + (-2)(4) = 80$$

$$\Rightarrow \cos\theta = \frac{\vec{x}.\vec{y}}{|\vec{x}|.|\vec{y}|} = \frac{80}{(5\sqrt{5})(4\sqrt{5})} = \boxed{\frac{4}{5}}$$