Question 3 **(30 marks)**

(a) $ABCD$ is a parallelogram.

$|AB| = 10$ cm, $|BC| = 13$ cm, and $|\angle ABC| = 110°$.

Find the area of $ABCD$, correct to the nearest cm^2.

Q3	Model Solution – 30 Marks
(a)	Area of $ABC = \frac{1}{2}(10)(13)\sin 110 = 61{\cdot}08..$ Area of $ABCD = 2(61{\cdot}08\ ...)$ $= 122{\cdot}16\ ...$ $= 122$ [cm^2] [nearest cm^2] **OR** Area of $ABCD = (10)(13)\sin 110$ $= 122{\cdot}16\ ...$ $= 122$ [cm^2] [nearest cm^2]

(b) X is an angle, with $0° \leq X \leq 360°$, and

$$\cos(2X) = \frac{\sqrt{3}}{2}$$

Find **all** the possible values of X.

(b)	Reference angle in 1st and 4th quadrants $\cos^{-1}\frac{\sqrt{3}}{2} = 30°$ or $330°$ $2X = 30°, 330°, 390°, 690°$ $X = 15°, 165°, 195°, 345°$

(c) KLM is a triangle where $|MK| = 15\sqrt{3}$ cm, $|ML| = 45$ cm, and $|\angle KLM| = 25°$.
θ is the angle $\angle LKM$.

Work out the **two** possible values of θ, for $0° < \theta < 180°$.
Give each answer correct to the nearest degree.

(c)	$\frac{\sin\theta}{45} = \frac{\sin 25}{15\sqrt{3}}$ $\sin\theta = \frac{45(0.4226)}{15\sqrt{3}} = 0.73196$ $\sin\theta = 0.73196\ ...\ 47.05°$ angle in 1st and 2nd quadrants $\theta = 47.05°$ or $\theta = 132.95°$ $\theta = 47°$ or $\theta = 133°$ [nearest degree]

Olga is a cyclist.

(a) The diagram below shows a road $[AB]$, which is not to scale.
AC is horizontal and BC is vertical.
$|BC| = 9$ m and $|AB| = 70$ m.

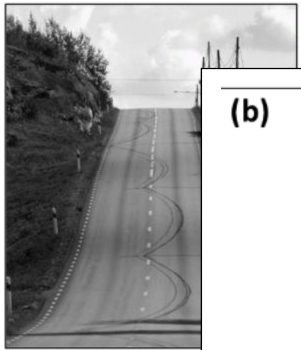
(a)	$|AC|^2 + 9^2 = 70^2$ $|AC|^2 = 70^2 - 9^2$ $|AC|^2 = 4819$ $|AC| = \sqrt{4819}$ Gradient $= \frac{9}{\sqrt{4819}} \times 100 = 12.96..$ $=13\%$ [nearest percent]

The gr

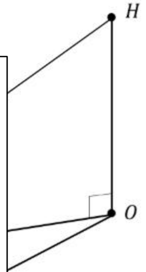
Find th

(b) Olga wants to measure the vertical height of a hill. The point H is at the top of the hill. The points R and P are 20 m apart on horizontal ground, at the bottom of the hill.

Olga measures the angle of elevation from R to H.
Taking O to be the point directly below H that is horizontal with R and P, Olga also measures the angles $\angle OPR$ and $\angle ORP$.
All of these are shown in the diagram below (not to scale).



Source: www.bikeforums.net/



(b)

$|\angle POR| = 5°$

$$\frac{|RO|}{\sin 87} = \frac{20}{\sin 5}$$

$$|RO| = \frac{20 \sin 87}{\sin 5}$$

$|RO| = 229 \cdot 16$ m

$$\tan 17 = \frac{|HO|}{229 \cdot 16}$$

$|HO| = 229 \cdot 16 \tan 17$

$|HO| = 70$ [m]

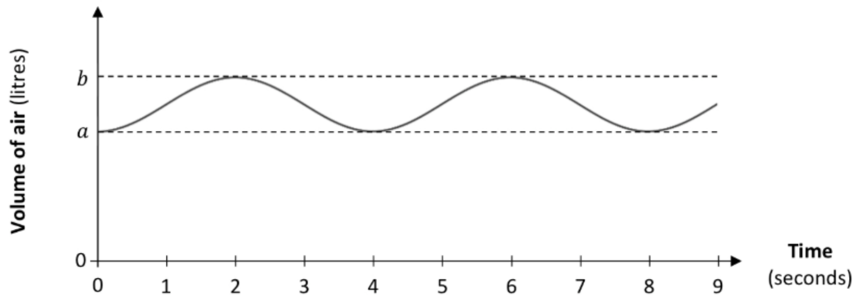
Work out the distance | ... tive to the points R and P. Give your ans ...

Olga has some tests done to measure her lung capacity.
When she is resting, the volume of air, V, in her lungs after t seconds can be modelled by:

$$V(t) = 2 - 0{\cdot}4\cos\left(\frac{\pi}{2}t\right),$$

where V is in litres, $t \geq 0$ is the time in seconds from a given point in time, and $\frac{\pi}{2}t$ is in **radians**.

The diagram below shows the graph of the function $y = V(t)$ for the first 9 seconds.



(c) Find the values marked ... imum values of V.

(c) **(d)**

(c) Range $= 2 \pm 0 \cdot 4$

$a = 1 \cdot 6 \quad b = 2 \cdot 4$

OR

$V'(t) = -0 \cdot 4\left[\left(-\sin\frac{\pi}{2}t\right)\left(\frac{\pi}{2}\right)\right] = 0$

$\sin\frac{\pi}{2}t = 0$

$\frac{\pi}{2}t = 0$ or $\frac{\pi}{2}t = \pi$

$t = 0$ or $t = 2$

$V(0) = 1.6 \quad V(2) = 2.4$

$a = 1 \cdot 6 \quad b = 2 \cdot 4$

(d) What is the connection ... ther Olga is breathing in or breathing out?

(d) If $V'(t) > 0$ then the volume of air is increasing so she is breathing in.
If $V'(t) < 0$ then the volume of air is decreasing so she is breathing out.

(e) Use the formula $V(t) = 2 - 0{\cdot}4\cos\left(\frac{\pi}{2}t\right)$ to find each of the following, when Olga is resting. Give each answer correct to 3 decimal places.

(i) Find the volume of air in Olga's lungs, half a second after $t = 0$.

(e) (i)

$V(0 \cdot 5) = 2 - 0 \cdot 4\cos\frac{\pi}{2}(0 \cdot 5)$

$= 1 \cdot 7171 \ldots$

$= 1 \cdot 717$ litres [3 d.p.]

(ii) Find the rate at which the volume of air in Olga's lungs is increasing, half a second after $t = 0$.

(e) (ii)

$V'(t) = -0 \cdot 4\left[\left(-\sin\frac{\pi}{2}t\right)\left(\frac{\pi}{2}\right)\right]$

$V'(0 \cdot 5) = 0 \cdot 4\left(\frac{\pi}{2}\right)\left[\sin\frac{\pi}{2}(0 \cdot 5)\right]$

$= 0 \cdot 4442..$

$= 0 \cdot 444$ litres/sec [3 d.p.]

Question 2 **(30 marks)**

(a) Prove that $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

$\sin A = \cos(90 - A)$

$\sin(A+B) = \cos(90-(A+B)) = \cos((90-A)-B)$

$= \cos(90-A)\cos B + \sin(90-A)\sin B$ ... by $\cos(A-B)$ formula

$= \sin A \cos B + \cos A \sin B$

(b) Using the formula in **part (a)**, and without using a calculator, find the value of $\sin 75°$. Give your answer in surd form.

(b) $\sin(30+45) = \sin 30 \cos 45 + \cos 30 \sin 45$

$$\sin 75 = \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right) + \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$$

$$\sin 75 = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

(c) Find all solutions of the following equation in t, for $0° \le t \le 360°$:

$$\sin t = \sin(2t)$$

$$sin2t - sint = 0$$

$$2cos\frac{3t}{2}sin\frac{t}{2} = 0$$

$$cos\frac{3t}{2} = 0$$

$t = 60°, 180°$ *and* $300°$

$$sin\frac{t}{2} = 0$$

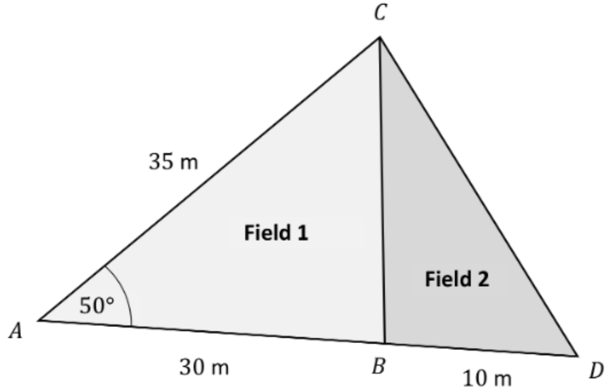
$t = 0°$ *and* $360°$

Question 9 **(50 marks)**

Oscar is taking some measurements and is using trigonometry to work out some angles, distances, and areas.

First, Oscar takes measurements of two adjacent triangular fields, **Field 1** (ABC) and **Field 2** (BDC), as shown in the diagram below (not to scale).
B lies on the line AD. $|AB| = 30$ m, $|BD| = 10$ m, $|AC| = 35$ m, and $|\angle CAD| = 50°$.

Note: the angle ABC is **not** a right angle.



(a) Find the area of **Field 1** and, hence, find the area of **Field 2**.
Give each answer correct to the nearest m^2.

Total area $= \frac{1}{2}(35)(40)\sin 50 = 536{\cdot}2$ ...

So, area Field 2 $= 536{\cdot}2 ... - 402{\cdot}1 ...$

$= 134$ [m^2] [nearest m^2]

(b) Find the length of the perimeter of **Field 1**.
Give your answer correct to the nearest metre.

(b) $|CB|^2 = 35^2 + 30^2 - 2(35)(30)\cos 50$

$= 775{\cdot}14$ ...

$|CB| = \sqrt{775{\cdot}14 ...} = 27{\cdot}8 ... = 28$ [m] [$\in \mathbb{N}$]

Perimeter $= 35 + 30 + 28 = 93$ [m] [$\in \mathbb{N}$]

Question 4 **(30 marks)**

(a) **(i)** Prove that $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$.

$$\tan(A+(-B)) = \frac{\tan A + \tan(-B)}{1-\tan A \tan(-B)}$$

$$= \frac{\tan A - \tan B}{1+\tan A \tan B}$$

(ii) Write $\tan 15°$ in the form $\frac{\sqrt{a}-1}{\sqrt{a}+1}$, where $a \in \mathbb{N}$.

$$\tan 15 = \tan(60-45)$$

$$= \frac{\tan 60 - \tan 45}{1+\tan 60 \tan 45}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

(b) The triangle ABC is shown in the diagram below.
$|AC| = |BC|$ and $|\angle ACB| = 45°$. $|AB| = 10\sqrt{2-\sqrt{2}}$, as shown.
Find the length

Sine Rule:

$$\frac{180-45}{2} = 67{\cdot}5°$$

$$\frac{x}{\sin 67{\cdot}5} = \frac{10\sqrt{2-\sqrt{2}}}{\sin 45}$$

$$x = \frac{10\sqrt{2-\sqrt{2}}\ \sin 67{\cdot}5}{\sin 45} = 10$$

Cosine Rule:

$$\left(10\sqrt{2-\sqrt{2}}\right)^2 = x^2 + x^2 - 2(x)(x)\cos 45°$$

$$(2-\sqrt{2})x^2 = 100(2-\sqrt{2})$$

$$x = 10 \ \ [\text{as } x > 0]$$

(b) The voltage, $V(t)$, (in Volts) of a certain alternating current is given by the function:

$$V(t) = 110\sqrt{2}\sin(120\pi t),$$

where t is in seconds.

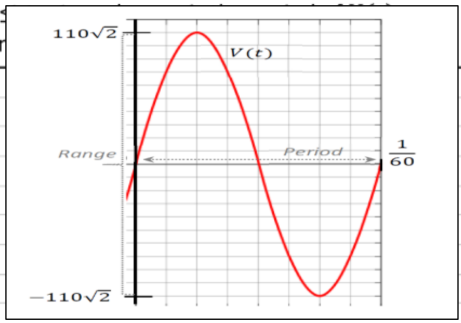
(i) Find the period and range of the function $V(t)$.

Period:

Range: $[-110\sqrt{2},\ 110\sqrt{2}]$

Period: $\frac{2\pi}{120\pi}$ or $\frac{1}{60}$

(ii) Sketch the function for 0
Indicate the period and ra



(iii) Use $V(t)$ to find the voltage when $t = 6{\cdot}67$ seconds.
Give your answer correct to two decimal places.

$$V(6{\cdot}67) = 110\sqrt{2}\sin 120\pi 6{\cdot}67$$

$$= 147{\cdot}949 \ldots = 147{\cdot}95 \text{ [Volts] [2 D.P.]}$$

(iv) Find one value for t where the voltage is 110 Volts.
Give your answer in the form $\frac{a}{b}$ where $a, b \in \mathbb{N}$.

Accept any value of t satisfying $t = \frac{1+8n}{480}$ *or* $t = \frac{3+8n}{480}$, *as long as in the correct form.*

$$V(t) = 110\sqrt{2}\sin 120\pi t = 110$$

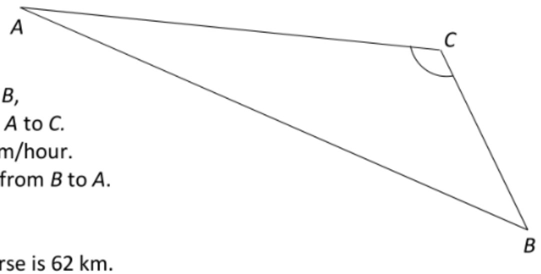
So $\sin 120\pi t = \frac{1}{\sqrt{2}}$

So $120\pi t = \frac{\pi}{4}$, i.e. $t = \frac{1}{480}$ [seconds]

or $120\pi t = \frac{3\pi}{4} \Rightarrow t = \frac{3}{480}$ [seconds]

Question 7 **(50 marks)**

The diagram (Triangle ABC) shows the 3 sections of a level triathlon course. In order to complete the triathlon, each contestant must swim 4 km from C to B, cycle from B to A, and then run 28 km from A to C. Mary can cycle at an average speed of 25 km/hour. It takes her 1 hour and 12 minutes to cycle from B to A.



(a) Show that the **total length** of the course is 62 km.

$$25 \times 1{\cdot}2 = 30$$
$$30 + 28 + 4 = 62 \text{ [km]}$$

(c) Show that $|\angle ACB| = 116{\cdot}5°$, correct to 1 decimal place.

$$30^2 = 28^2 + 4^2 - 2(28)(4)\cos C$$
$$\cos C = \frac{28^2 + 4^2 - 30^2}{2(28)(4)}$$
$$\cos C = -\frac{100}{224}$$
$$C = 116{\cdot}51\ldots = 116{\cdot}5 \text{ [1 D.P.]}$$

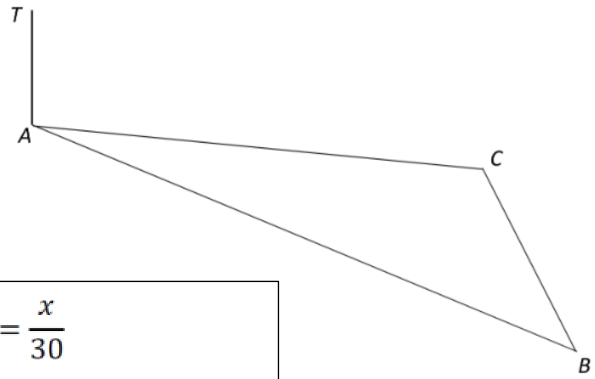
(d) To comply with safety regulations, the region inside the triangular course must be kept clear of people. Find the area of this region. Give your answer, in km², correct to 1 decimal place.

$$\text{Area} = \tfrac{1}{2} A\, B \sin C$$
$$\text{Area} = \tfrac{1}{2}(28)(4)\sin 116{\cdot}5°$$
$$= 50{\cdot}11\ldots = 50{\cdot}1 \text{ [km}^2\text{] [1 D.P.]}$$

(e) Find the shortest distance from the point C to the side AB. Give your answer in km, correct to 1 decimal place.

$$\text{Area} = \tfrac{1}{2}\text{base} \times d$$
$$50{\cdot}1 = \tfrac{1}{2}(30)d$$
$$d = \frac{50{\cdot}1}{15} = 3{\cdot}34 = 3{\cdot}3 \text{ [km] [1 D.P.]}$$

(f) The course is viewed from a camera tower which rises vertically from point A. The top of the tower is point T. The angle of elevation of T from B is $0{\cdot}05°$. Find $|AT|$, the vertical height of the tower. Give your answer correct to the nearest metre.



$$\tan(0{\cdot}05) = \frac{x}{30}$$
$$30 \times \tan(0{\cdot}05) = x$$
$$x = 0{\cdot}02617\ldots \text{ km}$$
$$= 26 \text{ [m] } [\in \mathbb{N}]$$

Question 4 **(30 marks)**

(a) (i) Prove that $\cos 2A = \cos^2 A - \sin^2 A$.

$$\cos(A + B) = \cos A . \cos B - \sin A . \sin B$$
$$\cos(A + A) = \cos A . \cos A - \sin A . \sin A$$
$$\cos(2A) = \cos^2 A - \sin^2 A$$

(ii) $\sin\frac{\theta}{2} = \frac{1}{\sqrt{5}}$, where $0 \le \theta \le \pi$.

Use the formula $\cos 2A = \cos^2 A - \sin^2 A$ to find the value of $\cos\theta$.

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\Rightarrow \cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$$

using a right-angled triangle $\cos\frac{\theta}{2} = \frac{2}{\sqrt{5}}$

$$\cos\theta = \left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{3}{5}$$

(b) Solve the equation:

$$\tan(B + 150°) = -\sqrt{3},$$

for $0° \le B \le 360°$.

$\tan(\text{angle}) = -\sqrt{3}$, so reference angle $= 60°$

$\mathbf{150° \le B + 150° \le 510°}$

In Quad's 2 or 4, so angles are $\mathbf{300°}$ or $\mathbf{480°}$

$B + 150 = 300$ **or** $B + 150 = 480$

So $B = 150°$ **or** $B = 330°$

Question 4 **(25 marks)**

(a) Find the two values of θ for which $\tan\frac{\theta}{2} = -\frac{1}{\sqrt{3}}$, where $0 \le \theta \le 4\pi$.

Reference angle: $\frac{\pi}{6}$

2nd Quadrant: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$$\frac{\theta}{2} = \frac{5\pi}{6} + 2n\pi$$

$$\theta = \frac{5\pi}{3} + 4n\pi$$

$n = 0 \Rightarrow \theta = \frac{5\pi}{3} = 300°$

4th Quadrant: $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

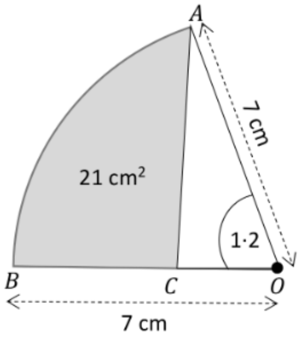
$$\frac{\theta}{2} = \frac{11\pi}{6} + 2n\pi$$

$$\theta = \frac{11\pi}{3} + 4n\pi$$

$n = 0 \Rightarrow \theta = \frac{11\pi}{3} = 660°$

(b) The diagram shows OAB, a sector of a circle of radius 7 cm with centre O.
In the sector, $|\angle BOA| = 1{\cdot}2$ radians.
The area of the shaded region is 21 cm^2.

Find $|BC|$.
Give your answer correct to 1 decimal place.



Area of ΔCOA = Area of Sector $- 21$

$= \frac{1}{2}r^2\theta - 21 = 8{\cdot}4$

Area of ΔCOA: $\frac{1}{2}|CO||7| \sin 1{\cdot}2 = 8{\cdot}4$

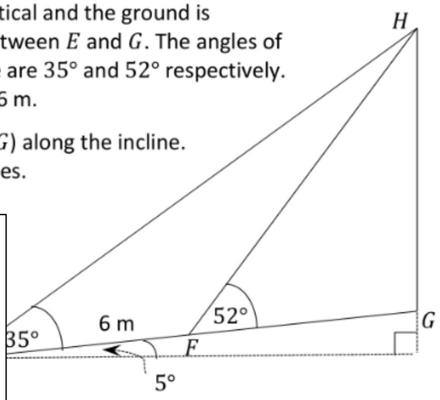
$$|CO| = \frac{8{\cdot}4}{3{\cdot}5 \sin 1{\cdot}2} = 2{\cdot}57$$

$|BC| = 7 - 2{\cdot}6 = 4{\cdot}4$ cm

Question 3 **(25 marks)**

(a) A flagpole $[GH]$, shown in the diagram, is vertical and the ground is inclined at an angle of 5° to the horizontal between E and G. The angles of elevation from E and F to the top of the pole are 35° and 52° respectively. The distance from E to F along the incline is 6 m.

Find how far F is from the base of the pole (G) along the incline. Give your answer correct to two decimal places.



(a)

$$\frac{6}{\sin 17°} = \frac{|HF|}{\sin 35°}$$

$$|HF| = \frac{6 \sin 35°}{\sin 17°} = 11{\cdot}77$$

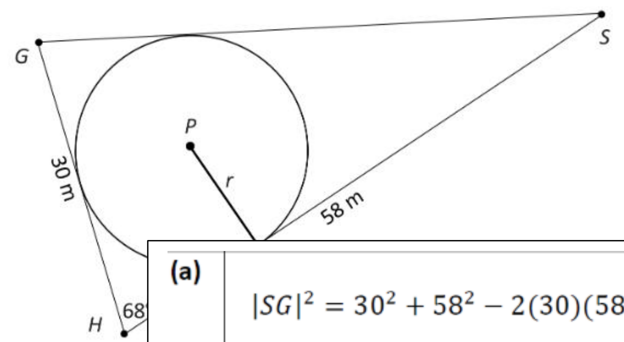
$$\frac{11{\cdot}77}{\sin 95°} = \frac{x}{\sin 33°}$$

$$x = \frac{11{\cdot}77(\sin 33°)}{\sin 95°}$$

$$x = 6{\cdot}43 \text{ m}$$

Question 9 **(55 marks)**

The diagram below shows a triangular patch of ground ΔSGH, with $|SH| = 58$ m, $|GH| = 30$ m, and $|\angle GHS| = 68°$. The circle is a helicopter pad. It is the incircle of ΔSGH and has centre P.



(a) Find $|SG|$. Give your answer i

(a)

$$|SG|^2 = 30^2 + 58^2 - 2(30)(58)(\cos 68)$$

$$= 2960{\cdot}369$$

$$|SG| = 54{\cdot}409 \text{ m}$$

$$|SG| = 54{\cdot}4$$

(b) Find $|\angle HSG|$. Give your answer in degrees, correct to 2 decimal places.

$$\frac{54{\cdot}4}{\sin 68} = \frac{30}{\sin \angle HSG}$$

$$\sin \angle HSG = 0{\cdot}51131$$

$$|\angle HSG| = 30{\cdot}75$$

(c) Find the area of ΔSGH. Give your answer in m^2, correct to 2 decimal places.

Area $\Delta GSH = \frac{1}{2}(30)(58) \sin 68 = 806{\cdot}65$

Also Area ΔGSH:

$$\frac{1}{2}(54{\cdot}4)(58) \sin 30{\cdot}75$$

and

$$\frac{1}{2}(54{\cdot}4)(30) \sin 81{\cdot}25$$

(d) **(i)** Find the area of ΔHSP, in terms of r, where r is the radius of the helicopter pad.

$$\frac{1}{2}(58)(r) \text{ or } 29r$$

(ii) Show that the area of ΔSGH, in terms of r, can be written as $71{\cdot}2r$ m^2.

Area ΔGHS

$$= \frac{1}{2}(30)(r) + \frac{1}{2}(54{\cdot}4)(r) + \frac{1}{2}(58)(r)$$

$$= 15r + 27{\cdot}2r + 29r = 71{\cdot}2r$$

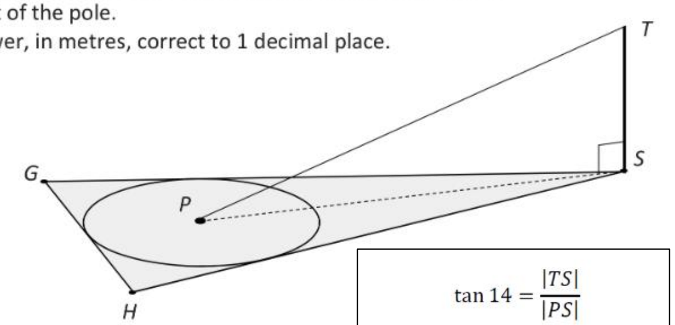
(iii) Find the value of r. Give your answer in metres, correct to 1 decimal place.

$$71{\cdot}2r = 806{\cdot}62$$

$$r = \frac{806{\cdot}62}{71{\cdot}2}$$

$$= 11{\cdot}3289 = 11{\cdot}3$$

(e) $[ST]$ is a **vertical** pole at the point S.
The angle of elevation of the top of the pole from the point P is 14°.
Find the height of the pole.
Give your answer, in metres, correct to 1 decimal place.



$$\tan 14 = \frac{|TS|}{|PS|}$$

$$\sin 15{\cdot}375 = \frac{11{\cdot}3}{|PS|} = 42{\cdot}51$$

$$=> |PS| = 42{\cdot}619$$

$$\tan 14 = \frac{|TS|}{42{\cdot}619}$$

$$|TS| = 10{\cdot}626 = 10{\cdot}6$$

Question 4 **(25 marks)**

(a) Show that $\cos 2\theta = 1 - 2\sin^2\theta$.

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = (1 - \sin^2 A) - \sin^2 A$$

$$\cos 2A = 1 - 2\sin^2 A$$

(b) Find the cosine of the acute angle between two diagonals of a cube.

Let length of side be x

Diagonal of any face $= \sqrt{x^2 + x^2} = \sqrt{2}x$

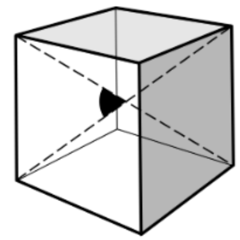
Internal diagonal $= x^2 + \left(\sqrt{2}x\right)^2 = \sqrt{3}x$

By cosine rule:

$$x^2 = \left(\frac{\sqrt{3}x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2}\right)^2 - 2\frac{\sqrt{3}x}{2}\frac{\sqrt{3}x}{2}\cos A$$

$$\cos A = \frac{\left(\frac{\sqrt{3}x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2}\right)^2 - x^2}{2\left(\frac{\sqrt{3}x}{2}\right)\left(\frac{\sqrt{3}x}{2}\right)}$$

$$\cos A = \frac{1}{3}$$



Question 4 **(25 marks)**

(a) Find **all** the values of x for which $\cos(2x) = -\frac{\sqrt{3}}{2}$, where $0° \le x \le 360°$.

$2x = 150 + 360n$ or $2x = 210 + 360n$

$x = 75 + 180n$ $\quad$ $x = 105 + 180n$

$n = 0 \Rightarrow x = 75°$ $\quad$ $n = 0 \Rightarrow x = 105°$

$n = 1 \Rightarrow x = 255°$ $\quad$ $n = 1 \Rightarrow x = 285°$

(b) Let $\cos A = \frac{y}{2}$, where $0° < A < 9$

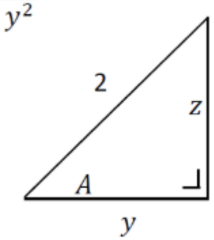
$$2^2 = y^2 + z^2$$

$$z = \sqrt{4 - y^2}$$

$$\sin 2A = 2\sin A\cos A$$

$$2\left(\frac{\sqrt{4-y^2}}{2}\right)\left(\frac{y}{2}\right)$$

$$= \frac{y\sqrt{4-y^2}}{2}$$



Question 9 **(40 marks)**

The depth of water, in metres, at a certain point in a harbour varies with the tide and can be modelled by a function of the form

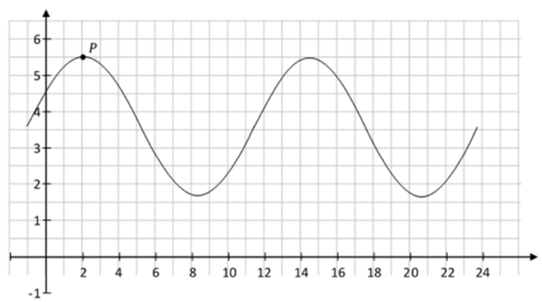
$$f(t) = a + b\cos ct$$

where t is the time in hours from the first high tide on a particular Saturday and a, b, and c are constants. (**Note**: ct is expressed in radians.)

On that Saturday, the following were noted:

- The depth of the water in the harbour at high tide was 5·5 m
- The depth of the water in the harbour at low tide was 1·7 m
- High tide occurred at 02:00 and again at 14:34.

(a) Use the information you are given to add, as accurately as you can, labelled and scaled axes to the diagram below to show the graph of f over a portion of that Saturday.
The point P should represent the depth of the water in the harbour at high tide on that Saturday morning.



(b) **(i)** Find the value of a and the value of b.

$f(t) = a + b\cos ct$

$Range: [(a+b), (a-b)]$

$a + b = 5{\cdot}5 \quad a - b = 1{\cdot}7$

$a = 3{\cdot}6 \quad b = 1{\cdot}9$

(ii) Show that $c = 0{\cdot}5$, correct to 1 decimal place.

Time between two successive high tides is: $12\frac{34}{60}$ hours

$\text{period} = \frac{2\pi}{c}$

$\text{period} = 12\frac{34}{60}$

$c = \frac{2\pi}{12\frac{34}{60}} = 0{\cdot}4999 = 0{\cdot}5$

(c) Use the equation $f(t) = a + b\cos ct$ to find the times on that Saturday **afternoon** when the depth of the water in the harbour was exactly 5·2 m.
Give each answer correct to the nearest minute.

$5{\cdot}2 = a + b\cos ct$

$5{\cdot}2 = 3{\cdot}6 + 1{\cdot}9\cos 0{\cdot}5t$

$0{\cdot}5t = \cos^{-1}\frac{1{\cdot}6}{1{\cdot}9} = 0{\cdot}569621319$

$0{\cdot}5t = 0{\cdot}5696$

$t = 1{\cdot}139$ hours

(before and after high tide at 14:34)

Time = 1 hour 8 minutes

Times: (14:34) ± 1 hour 8 min

$\Rightarrow$ 13:26 and 15:42

Question 8 **(45 marks)**

The height of the water in a port was measured over a period of time. The average height was found to be 1·6 m. The height measured in metres, $h(t)$, was modelled using the function

$$h(t) = 1{\cdot}6 + 1{\cdot}5\cos\left(\frac{\pi}{6}t\right)$$

where t represents the number of hours since the last recorded high tide and $\left(\frac{\pi}{6}t\right)$ is expressed in radians.

(a) Find the period and range of $h(t)$.

Period:

Range:

Period = $\frac{2\pi}{\frac{\pi}{6}}$ = 12 hours

Range =
$[1{\cdot}6 - 1{\cdot}5\,, 1{\cdot}6 + 1{\cdot}5\,] = [0{\cdot}1 \text{ m}, 3{\cdot}1 \text{ m}]$

(b) Find the maximum height of the water in the port.

Max = 1·6 + 1·5(1) =3·1 m.

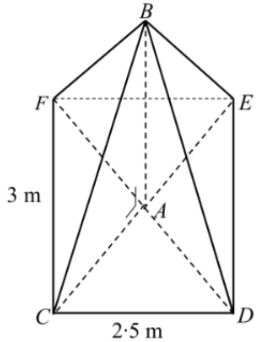
or

3·1 m from range

Question 7 **(55 marks)**

A glass Roof Lantern in the shape of a pyramid has a rectangular base $CDEF$ and its apex is at B as shown. The vertical height of the pyramid is $|AB|$, where A is the point of intersection of the diagonals of the base as shown in the diagram.
Also $|CD| = 2{\cdot}5$ m and $|CF| = 3$ m.

(a) **(i)** Show that $|AC| = 1{\cdot}95$ m, correct to two decimal places.



$$|EC|^2 = 3^2 + 2{\cdot}5^2 = 15{\cdot}25$$

$$|EC| = \sqrt{15{\cdot}25}$$

$$|EC| = 3{\cdot}905$$

$$\Rightarrow |AC| = 1{\cdot}9525$$

$$= 1{\cdot}95$$

(ii) The angle of elevation of B from C is 50° (i.e. $|\angle BCA| = 50°$).
Show that $|AB| = 2{\cdot}3$ m, correct to one decimal place.

$$\tan 50° = \frac{|AB|}{1{\cdot}95}$$

$$|AB| = 1{\cdot}95(1{\cdot}19175) = 2{\cdot}23239$$

$$|AB| = 2{\cdot}3$$

(iii) Find $|BC|$, correct to the nearest metre.

$$|BC|^2 = 1{\cdot}95^2 + 2{\cdot}3^2$$

$$|BC| = 3 \cdot 015377$$

$$|BC| = 3$$

Also: $\sin 40° = \frac{1{\cdot}95}{|BC|}$ **or** $\cos 40° = \frac{2{\cdot}3}{|BC|}$ **or**

$\cos 50° = \frac{1{\cdot}95}{|BC|}$ **or** $\sin 50° = \frac{2{\cdot}3}{|BC|}$

(iv) Find $|\angle BCD|$, correct to the nearest degree.

$$3^2 = 3^2 + 2{\cdot}5^2 - 2(3)(2{\cdot}5)\cos \propto$$

$$15 \cos \propto = 6{\cdot}25$$

$$\propto = 65°$$

(v) Find the area of glass required to glaze all four triangular sides of the pyramid.
Give your answer correct to the nearest m^2.

$A = 2 \times$ isosceles triangle $+ 2 \times$ equilateral triangle

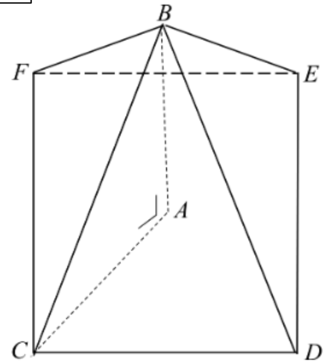
$$= 2\times\left[\tfrac{1}{2}(2{\cdot}5)(3)\sin 65°\right] +$$

$$2\times\left[\frac{1}{2}(3)(3)\sin 60°\right]$$

$$= 14{\cdot}59$$

A=15

(b) Another Roof Lantern, in the shape of a pyramid, has a square base $CDEF$. The vertical height $|AB| = 3$ m, where A is the point of intersection of the diagonals of the base as shown.
The angle of elevation of B from C is 60°
(i.e. $|\angle BCA| = 60°$).
Find the length of the side of the square base of the lantern.
Give your answer in the form $\sqrt{a}$ m, where $a \in \mathbb{N}$.



$$\tan 60° = \frac{3}{|CA|}$$

$$\Rightarrow |CA| = \sqrt{3}$$

$$|CE| = 2\sqrt{3}$$

$$x^2 + x^2 = \left(2\sqrt{3}\right)^2$$

$$x = \sqrt{6}$$