Real Numbers:

- **Proof by Contradiction that $\sqrt{2}$ is Irrational:**
  - Assume the opposite i.e. that $\sqrt{2}$ is not irrational => $\sqrt{2}$ can be written in the form $\frac{a}{b}$

    => $\sqrt{2} = \frac{a}{b}$ (where a and b have no common factor)

    => $2 = \frac{a^2}{b^2}$

    => $2b^2 = a^2$

  - As $2b^2$ is an even number => a^2 must be even

    => 'a' can be written in the form 2k

    => $2b^2 = (2k)^2$

    => $2b^2 = 4k^2$

    => $b^2 = 2k^2$, which means b is even as well.

  - If 'a' and 'b' are even, then 2 must divide into both => Contradiction

    => $\sqrt{2}$ is irrational

- **Reminder of Proof by Contradiction in Geometry:**

* Assume opposite is true and prove that the opposite is impossible.

- Example 1:

Prove that an equilateral Δ is also an acute-angled Δ (i.e. has no angle bigger than $90°$)

Proof:

Assume opposite true i.e. equilateral Δ is NOT acute-angled

$\Rightarrow$ it has one angle bigger than $90°$

As Δ is equilateral, all angles are equal

$\Rightarrow$ it has 3 angles bigger than $90°$

But...

Sum of 3 angles $= 180°$ so this is impossible!