Revision Sheet 3 Solutions

Q1. i) $x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x+5 = 0$ or $x-3 = 0$

$\boxed{x = -5}$ $\boxed{x = 3}$

ii) $2x^2 + 13x - 45 = 0$

$(2x-5)(x+9) = 0$

$2x-5 = 0$ or $x+9 = 0$

$2x = 5$ $\boxed{x = -9}$

$\boxed{x = 5/2}$

iii) $144x^2 - 16 = 0$

$(12x)^2 - (4)^2 = 0$

$(12x+4)(12x-4) = 0$

$12x+4 = 0$ or $12x-4 = 0$

$\frac{12x}{12} = \frac{-4}{12}$ $\frac{12x}{12} = \frac{4}{12}$

$\boxed{x = -\frac{1}{3}}$ $\boxed{x = \frac{1}{3}}$

iv) $6 - 31x = -5x^2$

$5x^2 - 31x + 6 = 0$

$(5x-1)(x-6) = 0$

$5x-1 = 0$ or $x-6 = 0$

$\frac{5x}{5} = \frac{1}{5}$ $\boxed{x = 6}$

$\boxed{x = \frac{1}{5}}$

v) $\frac{x^2}{4} + \frac{7}{20}x + \frac{1}{10} = 0$

$20\left(\frac{x^2}{4}\right) + 20\left(\frac{7}{20}x\right) + 20\left(\frac{1}{10}\right) = 20(0)$

$5x^2 + 7x + 2 = 0$

$(5x+2)(x+1) = 0$

$5x+2 = 0$ or $x+1 = 0$

$\frac{5x}{5} = \frac{-2}{5}$ $\boxed{x = -1}$

$\boxed{x = -\frac{2}{5}}$

Q2 i) $x^2 - 8x + 9 = 0$

$a = 1 \quad b = -8 \quad c = 9$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(9)}}{2(1)}$

$= \boxed{4 + \sqrt{7} \text{ or } 4 - \sqrt{7}}$

ii) $2x^2 - 3x - 23 = 0$

$a = 2 \quad b = -3 \quad c = -23$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-23)}}{2(2)}$

$= \boxed{4.22, -2.72}$

Q3 i) $\frac{1}{x+4} + \frac{1}{x+1} = \frac{1}{2}$

$\frac{1(x+1)+1(x+4)}{(x+4)(x+1)} = \frac{1}{2}$

$\frac{x+1+x+4}{(x+4)(x+1)} = \frac{1}{2}$

$\frac{2x+5}{(x+4)(x+1)} = \frac{1}{2}$

$1(x+4)(x+1) = 2(2x+5)$

$x(x+1)+4(x+1) = 4x+10$

$x^2+x+4x+4 = 4x+10$

$x^2+5x+4-4x-10 = 0$

$x^2+x-6 = 0$

$(x+3)(x-2) = 0$

$x+3=0$ or $x-2=0$

$\boxed{x=-3}$ $\boxed{x=2}$

ii) $\frac{4}{2x-5} - \frac{3}{5x-1} = -3$

$\frac{4(5x-1)-3(2x-5)}{(2x-5)(5x-1)} = \frac{-3}{1}$

$\frac{20x-4-6x+15}{(2x-5)(5x-1)} = \frac{-3}{1}$

$\frac{14x+11}{(2x-5)(5x-1)} = \frac{-3}{1}$

$-3(2x-5)(5x-1) = 1(14x+11)$

$-3[10x^2-25x-2x+5] = 14x+11$

$-30x^2+75x+6x-15 = 14x+11$

$-30x^2+81x-14x-15-11 = 0$

$-30x^2+67x-26 = 0$

$30x^2-67x+26 = 0$

$(15x-26)(2x-1) = 0$

$15x-26=0$ or $2x-1=0$

$\frac{15x}{15} = \frac{26}{15}$ $\frac{2x}{2} = \frac{1}{2}$

$\boxed{x = 26/15}$ $\boxed{x = 1/2}$

Q4 i) $x=0$ or $x=7$

$x-7=0$

$\Rightarrow x(x-7)=0$

$\boxed{x^2-7x=0}$

ii) $x=-8$ or $x=8$

$x+8=0$ $x-8=0$

$\Rightarrow (x+8)(x-8)=0$

$x(x-8)+8(x-8)=0$

$x^2-8x+8x-64=0$

$\boxed{x^2-64=0}$

iii) $x=\frac{1}{2}$ or $x=3$

$2x=1$ $x=3$

$2x-1=0$ $x-3=0$

$\Rightarrow (2x-1)(x-3)=0$

$2x^2-x-6x+3=0$

$\boxed{2x^2-7x+3=0}$

Q5 i) $L: x = 5 - y$

$C: x^2 + y^2 = 17$

Using L

$* \quad x = 5 - y$

Put * into C

$C: x^2 + y^2 = 17$

$(5-y)^2 + y^2 = 17$

$25 - 10y + y^2 + y^2 = 17$

$2y^2 - 10y + 25 - 17 = 0$

$2y^2 - 10y + 8 = 0$

$y^2 - 5y + 4 = 0$

$(y - 4)(y - 1) = 0$

$y - 4 = 0$ or $y - 1 = 0$

$y = 4$ $\quad$ $y = 1$

Put y into *

If $y = 1$

$\Rightarrow x = 5 - y$

$x = 5 - 1$

$x = 4$

If $y = 4$

$\Rightarrow x = 5 - 4$

$x = 1$

$\Rightarrow$ Ans: $\boxed{(4,1) \text{ and } (1,4)}$

ii) $L: 5x - y = -13$

$C: x^2 + y^2 = 13$

Using L

$* \quad y = 5x + 13$

Put * into C

$C: x^2 + y^2 = 13$

$x^2 + (5x+13)^2 = 13$

$x^2 + 25x^2 + 130x + 169 - 13 = 0$

$26x^2 + 130x + 156 = 0$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x + 3 = 0$ or $x + 2 = 0$

$x = -3$ $\quad$ $x = -2$

Put x into *

If $x = -2$

$\Rightarrow y = 5x + 13$

$= 5(-2) + 13$

$= 3$

If $x = -3$

$\Rightarrow y = 5x + 13$

$= 5(-3) + 13$

$= -2$

$\Rightarrow$ Ans: $\boxed{(-2,3) \text{ and } (-3,-2)}$

iii) $L: 2x = y + 3$

$C: x^2 + y^2 - 3xy = -45$

Using L

$* \quad y = 2x - 3$

Put * into C

$C: x^2 + y^2 - 3xy = -45$

$x^2 + (2x-3)^2 - 3x(2x-3) = -45$

$x^2 + 4x^2 - 12x + 9 - 6x^2 + 9x + 45 = 0$

$-x^2 - 3x + 54 = 0$

$x^2 + 3x - 54 = 0$

$(x+9)(x-6) = 0$

$x + 9 = 0 \quad \text{or} \quad x - 6 = 0$

$x = -9 \qquad x = 6$

Put x into *

If $x = -9$

$\Rightarrow y = 2x - 3$

$= 2(-9) - 3$

$= -21$

If $x = 6$

$\Rightarrow y = 2x - 3$

$= 2(6) - 3$

$= 9$

$\Rightarrow$ Ans: $\boxed{(-9, -21) \text{ and } (6, 9)}$

Past Exam Questions

Q6. $L: x - 5y = -13$

$C: x^2 + y^2 = 13$

Using L:

$x - 5y = -13$

$x = 5y - 13$ (*)

Put * into C

$C: x^2 + y^2 = 13$

$(5y - 13)^2 + y^2 = 13$

$25y^2 - 65y - 65y + 169 + y^2 = 13$

$26y^2 - 130y + 156 = 0$

$y^2 - 5y + 6 = 0$

$(y - 3)(y - 2) = 0$

$y = 3$ or $y = 2$

If $y = 3$

(*) $x = 5(3) - 13$

$= 15 - 13$

$= 2 \Rightarrow \boxed{x = 2 \quad y = 3}$

If $y = 2$

(*) $x = 5(2) - 13$

$= 10 - 13$

$= -3 \Rightarrow \boxed{x = -3 \quad y = 2}$

Q7. $2x^2 - 7x - 3 = 0$

$a = 2 \quad b = -7 \quad c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$$

$$= \frac{7 \pm \sqrt{49 + 24}}{4}$$

$$= \frac{7 \pm \sqrt{73}}{4}$$

$$= \frac{7 + \sqrt{73}}{4} \text{ or } \frac{7 - \sqrt{73}}{4}$$

$$= \boxed{3.89 \text{ or } -0.39}$$

Q8. $3x^2 - 6x - 8 = 0$

$a = 3 \quad b = -6 \quad c = -8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-8)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{36 + 96}}{6}$$

$$= \frac{6 \pm \sqrt{132}}{6}$$

$$= \frac{6 + \sqrt{132}}{6} \text{ or } \frac{6 - \sqrt{132}}{6}$$

$$= 2.91 \text{ or } -0.91$$

$$= \boxed{2.9 \text{ or } -0.9}$$

Q9. $L: y + 5 = 2x$

$C: x^2 + y^2 = 25$

Using L:

$y = 2x - 5$ (*)

Put * into C:

$C: x^2 + (2x - 5)^2 = 25$

$x^2 + 4x^2 - 10x - 10x + 25 = 25$

$5x^2 - 20x + 25 - 25 = 0$

$5x^2 - 20x = 0$

$5x(x - 4) = 0$

$5x = 0 \quad \text{or} \quad x - 4 = 0$

$x = 0 \qquad x = 4$

If $x = 0$

$y = 2(0) - 5$ (*)

$y = -5 \Rightarrow \boxed{x = 0 \quad y = -5}$

If $x = 4$

$y = 2(4) - 5$ (*)

$= 8 - 5$

$= 3 \Rightarrow \boxed{x = 4 \quad y = 3}$

Q10. $\frac{5}{x+3} - \frac{1}{x} = \frac{1}{2}$

LCM of denominators is $2(x)(x+3)$ so mult each term by that:

$$2(x)(x+3)\left(\frac{5}{x+3}\right) - 2(x)(x+3)\left(\frac{1}{x}\right) = 2(x)(x+3)\left(\frac{1}{2}\right)$$

$$2(x)(5) - 2(x+3)(1) = x(x+3)(1)$$

$$10x - 2x - 6 = x^2 + 3x$$

$$0 = x^2 + 3x - 8x + 6$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x - 2 = 0$$

$$\boxed{x = 3} \qquad \boxed{x = 2}$$