1) Factorising and Manipulation of Formulae:

Factorising:

1. Taking out the HCF (taking out what's common)

e.g.s

i) $2x - 10$
$= 2(x - 5)$

ii) $3x^2 - 18x$
$= 3x(x - 6)$

2. Grouping (always has 4 terms)

e.g.s

i) $ax + ay + bx + by$
$= a(x + y) + b(x + y)$
$= (x + y)(a + b)$

ii) $3p - 3q - pk + kq$
$= 3(p - q) - k(p - q)$
$= (p - q)(3 - k)$

3. Quadratic (always has 3 terms x^2, x, a)

e.g.s

i) $x^2 + 5x + 6$
$= (x + 3)(x + 2)$

ii) $x^2 - 3x - 18$
$= (x - 6)(x + 3)$

4. Difference of 2 Squares (always 2 terms with a '-' between)

Note: Watch for square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81….

e.g.s

i) $x^2 - 9y^2$
$= (x)^2 - (3y)^2$
$= (x - 3y)(x + 3y)$

ii) $16a^2 - 25b^2$
$= (4a)^2 - (5b)^2$
$= (4a - 5b)(4a + 5b)$

2) Solving Quadratic Equations:

a) Solving Quadratic Eqns by factorising: (Equations with an x^2)

Steps:

1. Bring all terms to the left-hand side (LHS) and leave '0' on the RHS
2. Factorise the LHS (See section on Factorising in previous tab)
3. If LHS can't be factorised the 'Quadratic Formula' needs to be used (See Example 3 on the right)
4. Let each factor be = 0
5. Solve the two simple equations to find the two answers.

Example 1: $x^2 - 3x - 18 = 0$

$(x - 6)(x + 3) = 0$

$x - 6 = 0 \quad or \quad x + 3 = 0$

=> $x = 6 \quad or \quad x = -3$

Example 2: $4x^2 - 25 = 0$

$(2x - 5)(2x + 5) = 0$

$2x - 5 = 0 \quad or \quad 2x + 5 = 0$

=> $2x = 5 \quad or \quad 2x = -5$

=> $x = \frac{5}{2} \quad or \quad x = \frac{-5}{2}$

b) Solving Quadratic Eqns using the "-b Formula":

Note: This method can be used for ALL quadratic equations.

If $ax^2 + bx + c = 0$ is a quadratic equation, then the roots of the equation are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

See Tables pg 20

Example 3: Solve $x^2 - 2x - 5 = 0$.

In this case: a = 1, b = -2 and c = -5

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$\Rightarrow x = \frac{2 \pm \sqrt{24}}{2}$$

$$\Rightarrow x = 3.45 \quad or \quad x = -1.45$$

c) Quadratic Eqns with fractions:

Example: Solve $\frac{2}{x+1} - \frac{3}{x-2} = \frac{5}{2}$

Method 1: (Multiply across by common denominator)

In this case the common denominator would be $2(x + 1)(x - 2)$:

$$2(x+1)(x-2)\frac{2}{x+1} - 2(x+1)(x-2)\frac{3}{x-2} = 2(x+1)(x-2)\frac{5}{2}$$

2(x - 2)(2) - 2(x + 1)(3) = 5(x + 1)(x - 2)

4x - 8 - 6x - 6 = $5x^2$ - 5x - 10

- $5x^2$ + 3x - 4 = 0

$5x^2$ - 3x + 4 = 0…….and solve this as before.

Method 2: (Tidy up both sides into single fractions and cross multiply) (See Section 2 - Example 2)

$$\frac{2}{x + 1} - \frac{3}{x - 2} = \frac{5}{2}$$

$$\frac{2(x - 2) - 3(x + 1)}{(x + 1)(x - 2)} = \frac{5}{2}$$

$$\frac{-x - 7}{(x + 1)(x - 2)} = \frac{5}{2}$$

$$2(-x - 7) = 5(x + 1)(x - 2)$$

$$-2x - 14 = 5(x^2 - x - 2)$$

$$-2x - 14 = 5x^2 - 5x - 10$$

$5x^2 - 3x + 4 = 0$ etc.

d) Forming Quadratic Equation from the roots:

Method 1:

Steps:

1. Let x = both of the roots.
2. Create two factors that are = 0.
3. Multiply the two factors together using "split and repeat".

Example: Find the quadratic equation with roots -1 and 3.

$x = -1 \quad or \quad x = 3$

$x + 1 = 0 \quad or \quad x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x(x - 3) + 1(x - 3) = 0$

$x^2 - 3x + x - 3 = 0$

$x^2 - 2x - 3 = 0$

Method 2: Use the formula

Need to know to use this method.

$$x^2 - (sum\ of\ roots)x + (product\ of\ roots) = 0$$

Example: Find the quadratic equation with roots -1 and 3.

$x^2 - (sum\ of\ roots)x + (product\ of\ roots) = 0$

$x^2 - (-1 + 3)x + ((-1)(3)) = 0$

$x^2 - 2x - 3 = 0$