Revision Sheet 10: Surds and Indices (3rd Year – Unit 4)

Q1. Surds & Indices – The Basics

Note: No calculator allowed for any questions in Q1.

a) Simplify: (i) $x^4 \times x^3$ (ii) $\frac{a^8}{a^2}$ (iii) $(y^2)^3$ (iv) q^0 (v) $16^{\frac{1}{2}}$ (vi) 2^{-3}

b) Express the following in their simplest form: (i) $\sqrt{12}$ (ii) $\sqrt{50}$ (iii) $\sqrt{48}$ (iv) $\sqrt{200}$

c) Find the value of: (i) $8^{\frac{2}{3}}$ (ii) $16^{\frac{3}{4}}$ (iii) $100^{\frac{3}{2}}$ (iv) $64^{-\frac{1}{3}}$ (v) $(\frac{9}{25})^{-\frac{1}{2}}$ (vi) $(\frac{27}{125})^{-\frac{2}{3}}$

d) Simplify the following: (i) $\sqrt{80} + 2\sqrt{5}$ (ii) $\sqrt{20} + \sqrt{45} - 2\sqrt{5}$ (iii) $\sqrt{32} + 2\sqrt{8}$

e) Write the following as a power of 2: (i) 32 (ii) 1 (iii) $\frac{1}{8}$ (iv) $\sqrt{32}$ (v) $\frac{1}{\sqrt{8}}$ (vi) 8^{100}

f) Write the following as a power of 3: (i) 9 (ii) $\sqrt{27}$ (iii) $\frac{1}{9}$ (iv) $\frac{1}{\sqrt{27}}$ (v) $\frac{81}{\sqrt{3}}$ (vi) 27^{40}

g) Simplify the following: (i) $\sqrt{3}(\sqrt{3} - 2)$ (ii) $\sqrt{2}(5\sqrt{2} - \sqrt{7})$ (iii) $(\sqrt{3} + 1)(\sqrt{3} - 1)$

h) (i) Express $\frac{81^{\frac{1}{2}} \times 9^{-1}}{27^{\frac{2}{3}}}$ in the form 3^x, where $x \in Z$.

(ii) Evaluate $\frac{3^3 \times 27^{\frac{2}{3}}}{3^{-3} \times 81^{\frac{3}{4}}}$ and give your answer in the form 3^k, where $k \varepsilon N$.

(iii) Simplify $\frac{4^2 \times 16^{\frac{1}{2}}}{4^3 \times 64^{\frac{2}{3}}}$ and give your answer in the form 4^p, where $p \varepsilon Z$.

i) Simplify the following: (i) $(1 - 2\sqrt{3})(1 + 2\sqrt{3})$ (ii) $(3 - \sqrt{2})^2$

j) Express $\frac{2\sqrt{45}}{\sqrt{10}}$ in the form $k\sqrt{2}$, where $k \in N$.

Q2. Indices – Solving Equations

a) Solve the following equations for x:

(i) $3^x = 81$ (ii) $4^x = 32$ (iii) $9^x = 27$ (iv) $16^x = 32$ (v) $2^x = \frac{1}{8}$ (vi) $5^x = \frac{1}{125}$

b) Solve the following equations for x:

(i) $27^x = 81$ (ii) $2^{-x} = 16$ (iii) $3^{x+2} = 81$ (iv) $2^x = \frac{\sqrt{2}}{8}$

c) Solve the following equations for x:

(i) $2^{2x-2} = \frac{16}{\sqrt{8}}$ (ii) $9^{3-x} = \frac{\sqrt{27}}{81}$ (iii) $16^{x-1} = 2\sqrt{32}$ (iv) $8^{3x} = 16^{30}$

Q3. Extra Challenge and Problem Solving

a) Show that $(\frac{27}{8})^{\frac{1}{3}} + (\frac{100}{9})^{-\frac{1}{2}} = \frac{9}{5}$, without the use of a calculator.

b) Express $2^{10} + 2^{10} + 2^{10} + 2^{10}$ in the form 2^n, where $n \in N$.

c) Given that $p = \sqrt{5} + \sqrt{2}$ and $q = \sqrt{5} - \sqrt{2}$, simplify $p^2 - q^2$.

d) $(\sqrt{x} + \frac{2}{\sqrt{x}})(\sqrt{x} - \frac{2}{\sqrt{x}})$ can be written in the form $x - \frac{p}{x}$. Find down the value of p.

e) Remove the brackets and simplify $b^{\frac{1}{2}}(b^{\frac{1}{2}} + b^{-\frac{1}{2}})$.

f) Given that $t = k\sqrt{x + 5}$, find the value of k when $x = \frac{5}{4}$ and $t = \frac{1}{4}$, without using a calculator.

g) If $x = \sqrt{a^2 + b^2}$, find the value of x when $a = \sqrt{2}$ and $b = \sqrt{7}$, without using a calculator.

Revision Sheet 10 Solutions:

Q1.

a) (i) x^7 (ii) a^6 (iii) y^6 (iv) 1 (v) 4 (vi) $\frac{1}{8}$ b) (i) $2\sqrt{3}$ (ii) $5\sqrt{2}$ (iii) $4\sqrt{3}$ (iv) $10\sqrt{2}$

c) (i) 4 (ii) 8 (iii) 1000 (iv) $\frac{1}{4}$ (v) $\frac{5}{3}$ (vi) $\frac{25}{9}$ d) (i) $6\sqrt{5}$ (ii) $3\sqrt{5}$ (iii) $8\sqrt{2}$

e) (i) 2^5 (ii) 2^0 (iii) 2^{-3} (iv) $2^{\frac{5}{2}}$ (v) $2^{-\frac{3}{2}}$ (vi) 2^{300} f) (i) 3^2 (ii) $3^{\frac{3}{2}}$ (iii) 3^{-2} (iv) $3^{-\frac{3}{2}}$ (v) $3^{\frac{7}{2}}$ (vi) 3^{120}

g) (i) $3-2\sqrt{3}$ (ii) $10-\sqrt{14}$ (iii) 2 h) (i) 3^{-2} (ii) 3^5 (iii) 4^{-2}

i) (i) -11 (ii) $11-6\sqrt{2}$ j) $3\sqrt{2}$

Q2.

a) (i) $x=4$ (ii) $x=\frac{5}{2}$ (iii) $x=\frac{3}{2}$ (iv) $x=\frac{5}{4}$ (v) $x=-3$ (vi) $x=-3$

b) (i) $x=\frac{4}{3}$ (ii) $x=-4$ (iii) $x=2$ (iv) $x=-\frac{5}{2}$

c) (i) $x=\frac{9}{4}$ (ii) $x=\frac{17}{4}$ (iii) $x=\frac{15}{8}$ (iv) $x=\frac{40}{3}$

Q3.

a) $\frac{9}{5}$ b) 2^{12} c) $2\sqrt{10}$ d) $p=4$ e) $b+1$ f) $k=\frac{1}{10}$ g) 3