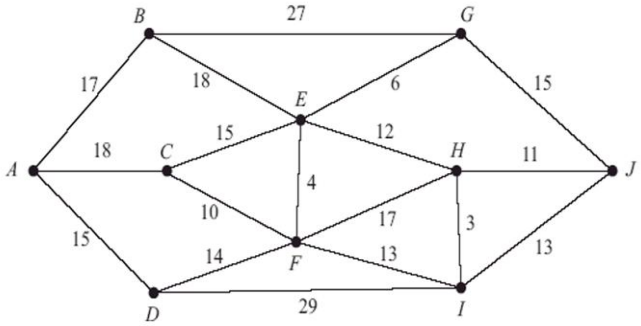
End of Chapter Revision Sheet: Optimal Paths/Dynamic Programming 5th Year

Q1.

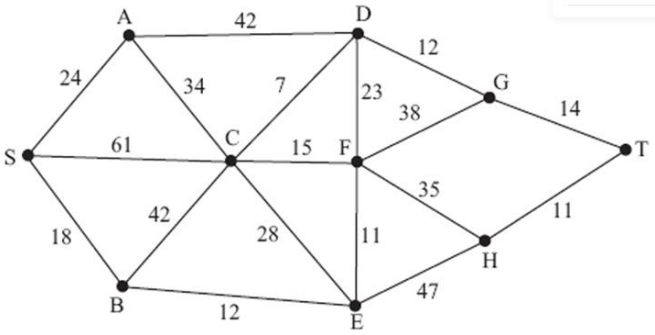
(i) The figure below shows a network of roads connecting towns A – J together. The number on each arc represents the length of that road in km.



(a) Use **Dijkstra's algorithm** to find the shortest route from A to J. State your shortest route and its length.

(b) The road from C to F will be closed next week for repairs. Find a shortest route from A to J that does **not include** CF and state its length.

(ii) The diagram below shows a network of cycle tracks within a national park. The number on each arc represents the time taken, in minutes, to cycle along the corresponding track.



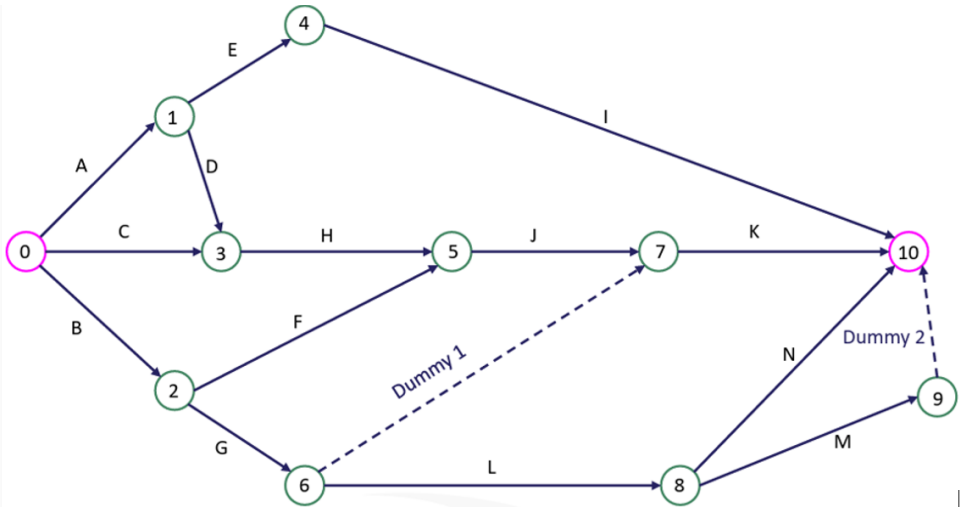
Use **Dijkstra's algorithm** to find the shortest route from A to J. **State** your shortest route and its length.

Q2.

Using the precedence table on the right, **draw the activity network** to represent it in the space provided below, using at most two dummies.

Activity	Depends on
A	-
B	A
C	A
D	A
E	B
F	B, C
G	D, F
H	D
I	G, H

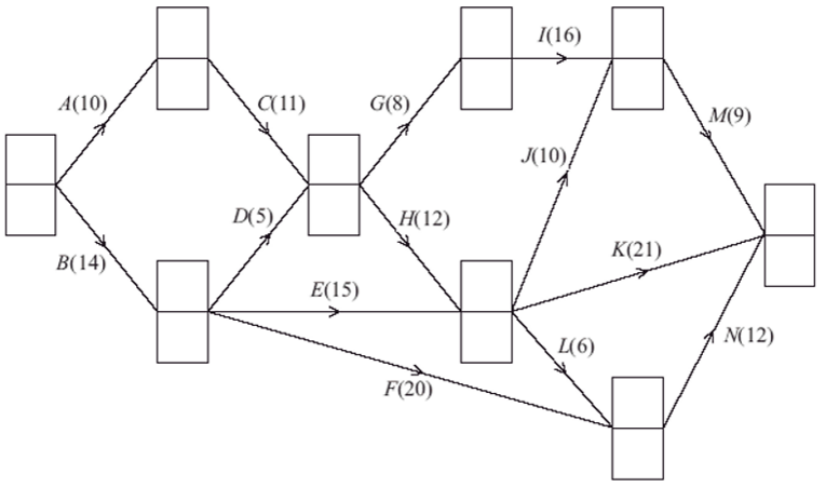
Q3. Using the activity network below, **complete the precedence table** on the right



Activity	Depends on
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	
K	
L	
M	
N	

Q4. An engineering project is modelled by the activity network shown in the figure below. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest time.

(a) Calculate the **early time** and **late time** for each event.



KEY

Early event time
Late event time

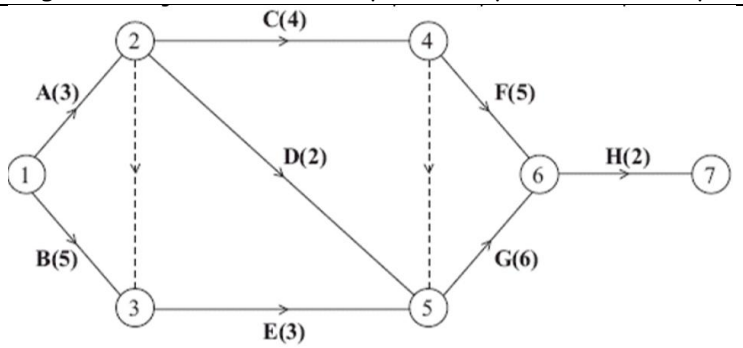
(b) Find the **total float** on activities D and F. Show your workings.

(c) State the **critical** activities.

(d) Draw a **Cascade (Gantt)** chart for this project.

(e) The chief engineer visits the project on day 15 and day 25 to check the progress of the work. Given that the project is on schedule, which activities **must** be happening on each of these two days?

Q5. The diagram below is the activity network relating to a building project. The number in brackets on each arc gives the time taken, in days, to complete the activity.

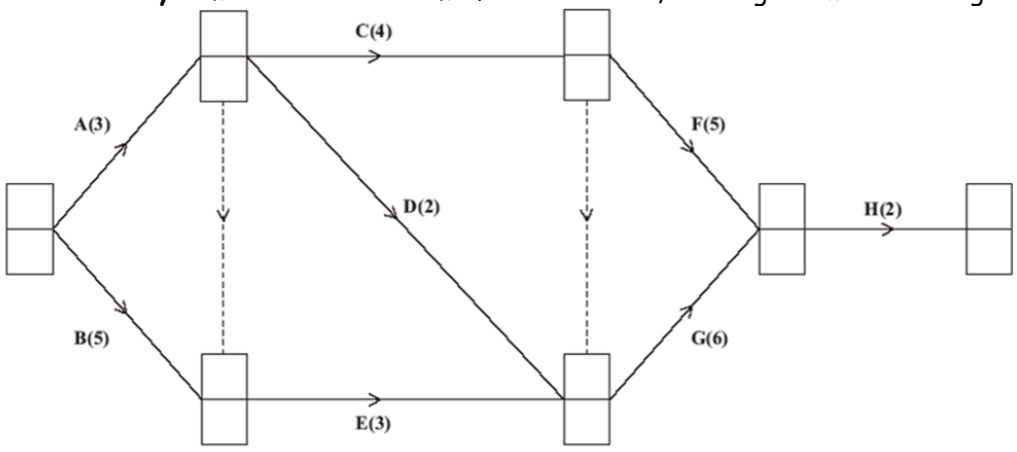


Activity	Depends on
A	
B	
C	
D	
E	
F	
G	
H	

(a) Complete the **precedence table**.

(b) **Explain** the significance of the dotted line from event 2 to event 3.

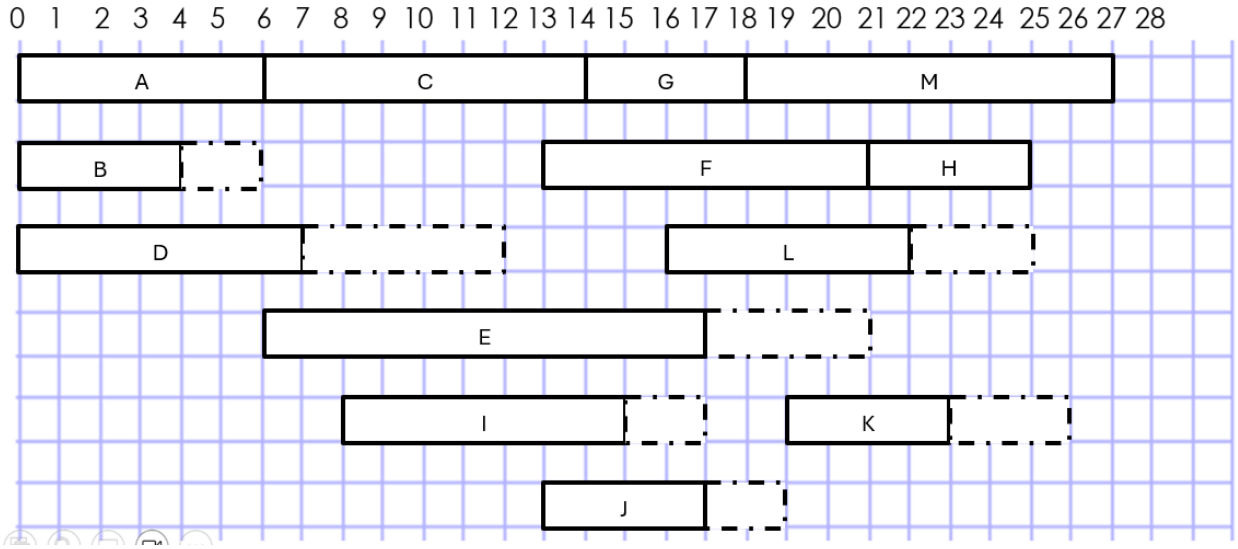
(c) Calculate the **early time** and the **late time** for each event, showing them on the diagram below.



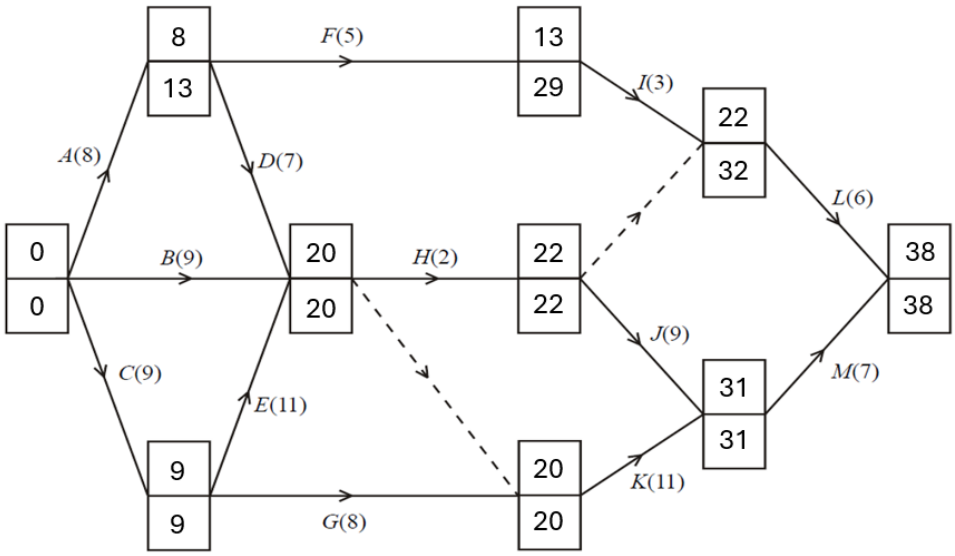
(d) Determine the **critical activities** and the length of the **critical path**.

(e) Draw a **cascade (Gantt)** chart for the project.

Q6. From the following cascade chart produce a **scheduling diagram**. The times are shown in days. What is the **smallest number** of workers needed to complete the project in the minimum time?



Q7. The diagram below is the activity network for a project. All times are in days.



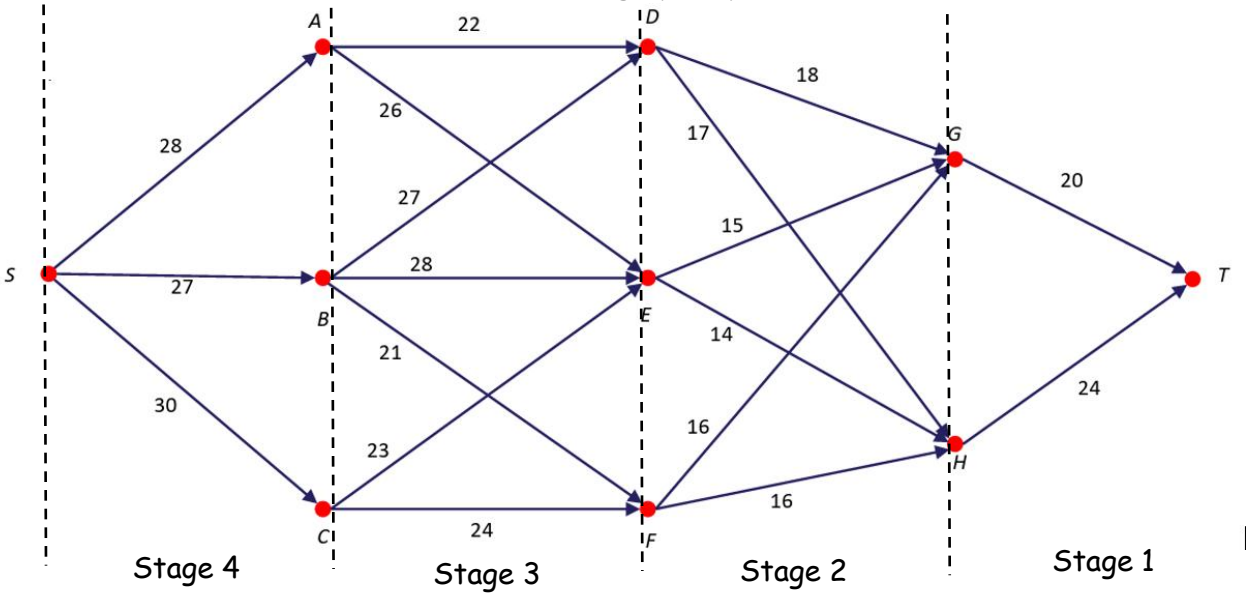
(i) What is the **shortest** (critical) **time** in which the project can be completed?

(ii) Which activities are on the **critical path(s)**?

(iii) What is the **lower bound** for the number of workers needed to complete the project in the critical time?

(iv) Only two workers are available for the project. Complete a **scheduling diagram** for these two workers, and from it determine the shortest time in which they can complete the project.

Q8. A company that owns an apartment building is planning upgrades and maintenance for the building over the next four years. The maintenance company took all the researched costs and information and translated them into a mathematical representation. The network represents the decisions associated with the maintenance and the company wish to minimise their costs over the 4-year period. The number on each edge represents the **cost in €1000s** corresponding to each decision. Determine **which decisions the company should make to minimise the costs** and calculate the average yearly cost based on those decisions.



Q9. Rita grows her own strawberries and uses them to produce jam, ice cream and tartlets. In a particular year, she has 5 kilogrammes of strawberries collected and wants to allocate them to the three products she makes, to maximise her profit. The table below shows the profit she makes, depending on how many kilogrammes she uses for each. Use **dynamic programming** to advise Rita on the possible ways to **allocate** the 5 kilogrammes of strawberries to maximise her profit, and state what the maximum profit would be.

Kilogrammes	1	2	3	4	5
Jam	75	145	230	315	420
Ice cream	150	185	215	220	255
Tartlets	125	265	330	350	395

Q10. Joe has a small company that manufactures dog houses. He can make up to 4 dog houses in any one month, but if he has to build more than 3, he must hire additional help at a cost of €600 per month. He can store a max of 3 dog houses in his own shed at a cost of €80 per month. The overhead costs for any month that he makes dog houses are €1200 a month. Dog houses are delivered at the end of every month, and you can assume he has none in stock at the start of March and at the end of July. His order book for the five months is:

Month	March	April	May	June	July
Number ordered	2	3	7	3	4

(a) Use **dynamic programming** to establish a schedule that will **minimise** Joe's costs and state the minimum production cost.

(b) The cost of materials for each doghouse is €120 and he sells the dog houses for €800 each. Calculate his **profit** for the five months above.

Q11. Amy makes her own pottery and sells it at fairs all over the country. She visits a different fair every week and needs to decide which three to visit over the next three weeks to maximise her profit. She starts from home at the start of the three-week period and finishes at home at the end. Her plan is to spend one week at each of the three fairs she chooses and travel directly from one fair to the next. The three tables below give the fairs she can visit in a particular week, the expected profits from them and the travel costs between them.

Table 1:

Week	1	2	3
Fairs	A,B	C,D,E	F,G,H

Table 2:

Festival	A	B	C	D	E	F	G	H
Expected Profit	700	600	900	1200	1100	400	900	500

Table 3:

Travel Cost (€)	A	B	C	D	E	F	G	H
Home	150	70				70	100	80
A			130	160	140			
B			110	120	130			
C						210	190	180
D						220	150	140
E						180	110	130

Use **dynamic programming** to find the schedule that **maximises** the total expected profit, considering the travel costs. State the maximum profit.

Q12. A local branch of DHL has a fleet of vans it uses for its delivery drivers. They have just built a new warehouse and bought a new fleet of vans to operate out of the warehouse at a cost of €50,000. At the start of every year, they have to decide whether to sell the vans or keep them. The annual costs for maintaining the vans are shown in the table below. The vans become worthless after 4 years.

Use **dynamic programming** to figure out a strategy for DHL to minimise the cost of the vans over the next 6 years. The resale prices of the vans are shown in the table below, for each age.

State **which year(s)** DHL should replace the vans, and what the **minimum** cost of the vans will be over the 6 years.

Age of Vans	1 yr old	2 yrs old	3 yrs old	4 yrs old
Maintenance	4000	7000	11000	16000
Resale value	37000	30000	18000	0

Answers:

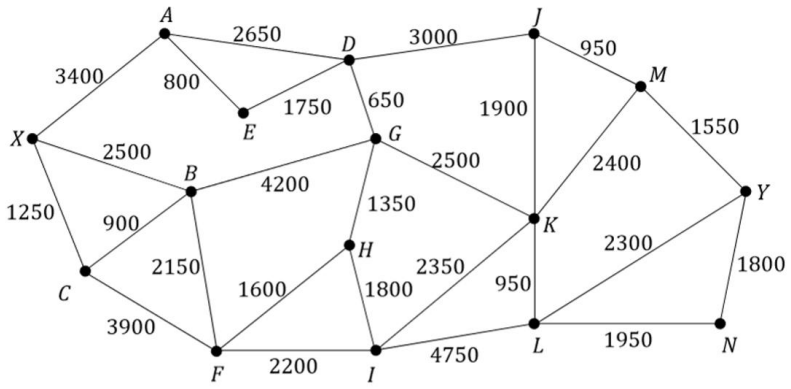
Q1. (i) (a) ACFEGJ, 53 km (b) ADFEGJ or ACEGJ, 54 km (ii) SBEFHT, 87 mins	
Q4. (b) D: 2, F: 8 (c) Critical: A, C, G, H, I, K, M (e) Day 15: C, Day 25: G, H, E, F	
Q5. (b) Dummy activities (d) Critical: B, E, G, H, Length = 16	Q6. 4 workers
Q7. (i) 38 days (ii) Critical: CEHJM and CEHKM (iii) 3 workers (iv) 51 days	Q8. SBFGT, €84000, €21000
Q9. 2 for Jam, 1 for Ice cream, 2 for Tartlets = €560	
Q10. (a) 4 in March, April, May and July, 3 in June = €8800 (b) €4120	Q11. Home -> B -> D -> G -> Home, €2260
Q12. Sell every 2 years, Min cost = €81000	

Past Exam Questions:

2023 Q2

Question 2

(a) A university has decided to improve the paths on its campus. In the network shown below the nodes labelled with the letters X and Y represent the two entrances to the campus and the nodes labelled with the letters A to N represent the key buildings on the campus.
The edges represent the paths, with the weight of each edge representing the cost (in €) of carrying out the improvement work for that path.



The university decides that the first part of the work will be to provide an improved route between entrance X and entrance Y. Use Dijkstra's algorithm to find the route between X and Y that is cheapest to improve. Calculate the cost of carrying out such improvements. Relevant supporting work must be shown.

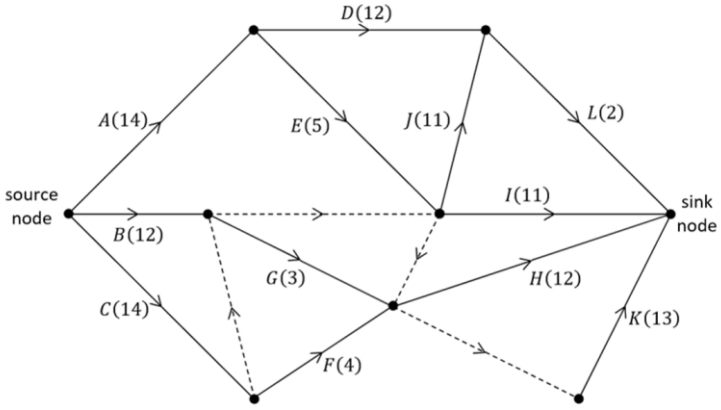
2023 Q9

Question 9

The manager of a regional hospital decides to arrange for the refurbishment of one of the hospital wards. The diagram below shows the scheduling network for the project.

The edges of the network represent the activities that have to be completed as part of the project and are labelled with the letters A to L. The duration, in days, of each activity is represented by the number in brackets. The unlabelled edges (shown with dashed lines) do not represent real activities but they help explain the order in which the activities must happen. The letters used to label the edges should **not** be taken as representing the order in which the activities happen.

The nodes of the network represent events or points in time during the project. The source node is the time when the project begins and the sink node is the time when the project ends.



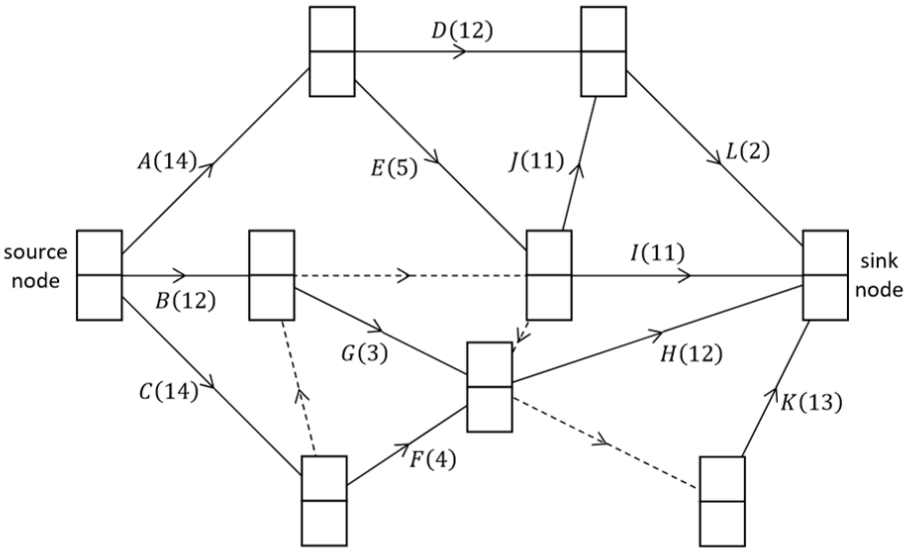
(i) Complete the table on the next page by listing, for each activity, the other activities on which it depends directly. That is, for each activity $X \in \{A, B, C, \ldots, L\}$, write the smallest possible list of other activities which need to be completed before activity X can begin.

Use the space below to show relevant supporting work, if necessary.

Activity	Depends directly on ...	Activity	Depends directly on ...
A		G	
B		H	
C		I	
D		J	
E		K	
F		L	

(ii) Calculate the early time and the late time for each event.

Complete the diagram below by writing the early time (upper box) and late time (lower box) at the node representing each event.

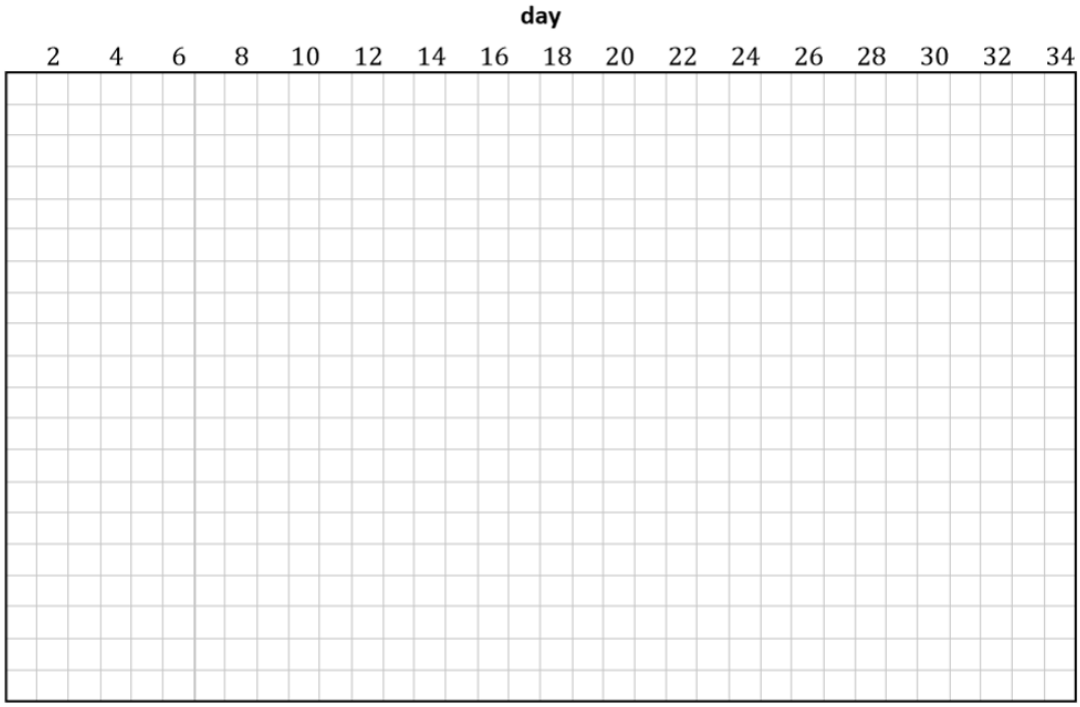


Use the space below and on the next page to show relevant supporting work, if necessary.

(iii) Write down the critical path(s) for the network.

A cascade chart (Gantt chart) is a type of bar chart which may be used to represent a project's schedule. The duration of each activity is represented by the width of the horizontal bar for that activity, with time on the horizontal axis. The float time for an activity is represented by a rectangle drawn using dotted lines to the right of the bar for that activity. The top row of a cascade chart is used for a critical path.

(iv) Draw a cascade chart or similar bar chart to represent the schedule for this project.



The hospital manager visits the project on day 18 to check the progress of the work, which is on schedule.

(v) Write down the activities which may be happening on day 18.

Past Exam Questions:

2023 Q2 Path: XAEDJMY Cost: €11450	2023 Q9 (iii) AEJL or AEK including 2 dummies (v) D, E, F, and G